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A comparison between Gaussian plume models and analytical advection-diffusion equation in unstable condition

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सार – इस शोधपत्र में विक्षेपण प्राचालों और भंवर विसरणशील आकृतियों का तीन आयामों में उपयोग करके क्रमशः गॉसीयन प्लूम मॉडल और संवहनीय विसरण समीकरण के घोल की तुलना की गई है। उसके बाद यथा प्रस्तावित गॉसीयन प्लूम मॉडल और मिस्र के परमाणु ऊर्जा प्राधिकरण (EAEA) में मापेगए आयोडीन-135 (I^{135}) की सांद्रता की अस्थिर स्थिति में प्रेक्षित डेटा की तुलना की गई है।

ABSTRACT. In this paper, comparing between the Gaussian plume models and the solution of advection-diffusion equation in three dimensions using different shapes of dispersion parameters and eddy diffusivities respectively. After that, comparing between the Gaussian plume model, as proposed model and observed concentrations data which measuring on the Egyptian Atomic Energy Authority (EAEA) for Iodine-135 (I^{135}) in unstable condition.

Key words – Urban air quality, Gaussian-Plume model, Advection-Diffusion equation, Dispersion parameters, Stability conditions, Iodine-135 (I^{135}).

1. Introduction

The Gaussian-plume models play a major role in the regulatory area. However, they may be almost the best models to use and it was noted at the 15th International Clean Air Conference 2000-Modeling Workshop that particular models are not always chosen on an objective scientific basis Ross (2001).

In artificial applications, the classical Gaussian diffusion models are mostly used in effecting the impacts of finding and proposed sources of air contaminants on local and urban air quality Arya (1999). Homeliness, associated with the Gaussian analytical model, does this approach particularly suitable for organizational usage in mathematical modeling of the air pollution. Indeed, such models are quite useful in short-range forecasting. The lateral and vertical dispersion parameters, respectively σ_y and σ_z, represent the key turbulent parameterization in this approach, once they contain the physical ingredients that describe the dispersion process and, consequently, express the spatial extent of the contaminant plume under the effect of the turbulent motion in the Planetary boundary layer (PBL) Abdul-Wahab (2006).

The atmospheric advection-diffusion equation had long been made to know the transport of pollutants in a turbulent atmosphere was studied by Seinfeld (1986). An analytical dispersion Model for sources in the atmospheric surface layer with dry deposition with the ground Surface has been studied by Kumar and Sharan (2016). Also, investigated the variation of eddy diffusivity on the mimics of behavior of advection-diffusion equation was studied by Essa *et al*. (2018). Essa *et al*. (2020) solved the advection-diffusion equation with variable vertical eddy diffusivity and wind speed using Hankel transform to get the crosswind integrated concentration. Recently Comparison between two analytical solutions of advection-diffusion equation using separation technique and Hankel Transform was studied by Essa and Taha (2021). Also Evaluation of Analytical Solution of Advection Diffusion Equation in Three Dimension was studied by Essa *et al*. (2021).

In this work, we compare between a Gaussian plume models and the solution of Advection-diffusion equation in three dimensions using different shapes of dispersion parameters and eddy diffusivities respectively. After that, we used the Gaussian plume model, proposed model and comparing with observed concentrations data which is taken from the Egyptian Atomic Energy Authority for Iodine-135 (I^{135}) in unstable condition.

2. Mathematical models

2.1. *First case*

Concentration in a Gaussian model can be written by Abd El-Wahab *et al*. (2014):

$$C(x,y,z)=\frac{Q}{2\pi u\sigma_y\sigma_z}e^{\frac{-y^2}{2\sigma_y^2}}\left[e^{\frac{-(z-H)}{z\sigma_z^2}}+e^{\frac{-(z+H)^2}{z\sigma_z^2}}\right]e^{-\frac{vx}{u}} \tag{1}$$

where σ_y and σ_z are the dispersion parameters in crosswind and vertical directions of the plume, Q is the emission rate, H is the effective stack Height; $H = h_s + \Delta h$, h_s is the stack height and Δh is the plume rise, u is the mean wind speed and y, z are the crosswind and vertical coordinates, respectively. $e^{-vx/u}$ is the radioactive decay for isotope, $\upsilon = 2.9 \times 10^{-5}\ s^{-1}$

$$H = h_s + \Delta h = h_s + 3\,(w/u)\,D \tag{2}$$

where, w is the exit velocity of the pollutants and D is the internal stack diameter

The mean concentration of a pollutant plumes emitted from a point source can be assumed to have a Gaussian distribution which are highly idealized, since they require stationary and homogeneous turbulence in the PBL where, the flow may be assumed quasi-stationary for suitable short periods of time (from 10 min to 1 h).

Taking the two different dispersion parameters of $\sigma_y and \sigma_z$ in each case as follows:

(*i*) *The first*

Crosswind and vertical dispersion parameters for convective conditions are taken from Lidiane *et al.* (2008) in the form:

$$\frac{\sigma_y^2}{h^2}=\frac{0.55X^2\psi^{\frac{2}{3}}}{1+\left(2.2X\psi^{\frac{1}{3}}\right)} \tag{3}$$

$$\frac{\sigma_z^2}{h^2}=\frac{0.42X^2\psi^{\frac{2}{3}}}{1+\left(2.9X\psi^{\frac{1}{3}}\right)} \tag{4}$$

where, $\psi=\frac{\varepsilon h}{w_*^3}$; ε is the mean dissipation rate of turbulence kinetic energy per unit time per unit mass of fluid, Field observations in a convective PBL show that 0.65 by Cauchey and Palmer (1979). $X=\frac{xw_*}{uh}$ is a non-dimensional distance defined by the ratio of travel time (x/u) to convective time scale and h is mixing height.

(*ii*) *The second*

Crosswind and vertical dispersion parameters for convective conditions are taken from Lidiane *et al*. (2008) in the form:

$$\frac{\sigma_y^2}{h^2}=\frac{0.66}{\pi^2}\int_0^\infty\frac{\sin^2\left(0.75\pi\psi^{\frac{1}{3}}Xn'\right)}{n'^2\left(1+n'\right)^{\frac{5}{3}}}dn' \tag{5}$$

$$\frac{\sigma_z^2}{h^2}=\frac{0.98}{\pi^2}\int_0^\infty\frac{\sin^2\left(0.98\pi\psi^{\frac{1}{3}}Xn'\right)}{n'^2\left(1+n'\right)^{\frac{5}{3}}}dn' \tag{6}$$

where, $n'=\frac{1.5z}{u\left(f_m^*\right)_i}n$; $\left(f_m^*\right)_i$ is the reduced frequency of the convective spectral peak in the form $\left(f_m^*\right)_i=\frac{z}{h}$.

2.2. *Second case*

The advection-diffusion equation can be written as follows:

$$u\frac{\partial C(x,z)}{\partial x}=\frac{\partial}{\partial z}\left[k_z\frac{\partial C(x,z)}{\partial z}\right] \tag{7}$$

where, $C\ (x,\ z)$ is the crosswind integrated concentration of pollutants, u is the wind speed (m/s) and K_z is vertical eddy diffusivity that is taken as a function of linear vertical distance.

TABLE 1

Power-law exponent 'n' is a function of air stability in urban area

	A	B	C	D	E	F
n	0.85	0.85	0.80	0.75	0.60	0.40

Eqn. (7) is solved under the Boundary conditions as follows:

The null flux conditioning of contaminants on the ground surface and the top at the vertical height are used:

$$\frac{\partial c(x,z)}{\partial z} = 0 \qquad \text{at } z = 0, h \tag{a}$$

where, h is the height of the planetary boundary layer (PBL). In addition, the mass continuity of the source with emission rate Q at the stack height 'h_s'.

$$uc\,(0, z) = Q\delta\,(z - h_s) \tag{b}$$

The vertical eddy diffusivity is taken as follows:

$$k_z = \alpha z \qquad 0 \le h_s \le h \tag{c}$$

where, $\alpha = 0.31\left(\frac{w_*}{u}\right)^2$; w_* is a convective vertical velocity and u is the wind speed at reference 10 m.

Then, the concentration in three dimensions is as follows:

$$C(x, y, z) = \frac{1}{2\pi\sigma_y} C(x, z) e^{-\frac{y}{2\sigma_y^2}} \tag{8}$$

σ_y is the crosswind standard deviation which is calculated from Table (2).

Then (7) can be written as:

$$u\frac{\partial c(x,z)}{\partial x} = \frac{\partial}{\partial z}\left[\alpha z \frac{\partial c(x,z)}{\partial z}\right] \tag{9}$$

Eqn. (9) becomes as follows:

$$\alpha z \frac{\partial^2 c(x,z)}{\partial z^2} + \alpha \frac{\partial c(x,z)}{\partial z} - u\frac{\partial c(x,z)}{\partial x} = 0 \tag{10}$$

TABLE 2

Contains the crosswind standard deviation σ_y values through different stabilities

Stability classes	Values of σ_y
A	$\sigma_y = 0.40x^{0.91}$
B	$\sigma_y = 0.40x^{0.91}$
C	$\sigma_y = 0.36x^{0.86}$
D	$\sigma_y = 0.32x^{0.78}$

Multiplying Eqn. (10) by $\frac{z}{\alpha}$, one can get in the form:

$$Z^2 \frac{\partial^2 c(x,z)}{\partial z^2} + z\frac{\partial c(x,z)}{\partial z} - \frac{u}{\alpha} Z \frac{\partial c(x,z)}{\partial x} = 0 \tag{11}$$

Changing the independent variable z to ξ by substitution:

$$\xi = z^{\frac{1}{2}}$$

$$d\xi = 0.5z^{-0.5}dz \rightarrow dz = 2z^{0.5}d\xi$$

$$d^2\xi = -0.25z^{-\frac{3}{2}}d^2z$$

$$z = \xi^2 \Rightarrow \frac{\partial \xi}{\partial z} = \frac{1}{2}z^{\frac{-1}{2}} = \frac{1}{2}\left(\xi^2\right)^{\frac{-1}{2}} = \frac{1}{2}\xi^{-1}$$

$$\frac{\partial C}{\partial z} = \frac{\partial C}{\partial \xi}\frac{\partial \xi}{\partial z} = \frac{1}{2}\xi^{-1}\frac{\partial C}{\partial \xi}$$

$$\Rightarrow z\frac{\partial C}{\partial z} = \frac{1}{2}\xi^2\xi^{-1}\frac{\partial C}{\partial \xi} = \frac{1}{2}\xi\frac{\partial C}{\partial \xi}$$

$$\frac{\partial^2 C}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial C}{\partial z}\right) = \frac{\partial}{\partial \xi}\left(\frac{\partial C}{\partial z}\right)\frac{\partial \xi}{\partial z}$$
$$= \frac{1}{2}\xi^{-1}\left[\frac{1}{2}\xi^{-1}\frac{\partial^2 C}{\partial \xi^2} - \frac{1}{2}\xi^{-2}\frac{\partial C}{\partial \xi}\right]$$

$$\Rightarrow z^2\frac{\partial^2 C}{\partial z^2} = \frac{1}{2}\xi^4\xi^{-1}\left[\frac{1}{2}\xi^{-1}\frac{\partial^2 C}{\partial \xi^2} - \frac{1}{2}\xi^{-2}\frac{\partial C}{\partial \xi}\right]$$
$$= \frac{1}{4}\xi^2\frac{\partial^2 C}{\partial \xi^2} - \frac{1}{4}\xi\frac{\partial C}{\partial \xi}$$

$$\frac{1}{4}\xi^2\frac{\partial^2 C}{\partial\xi^2}-\frac{1}{4}\xi\frac{\partial C}{\partial\xi}+\frac{1}{2}\xi\frac{\partial C}{\partial\xi}-\frac{u}{\alpha}\xi^2\frac{\partial C}{\partial x}=0$$

$$\frac{1}{4}\xi^2\frac{\partial^2 C}{\partial\xi^2}+\frac{1}{4}\xi\frac{\partial C}{\partial\xi}-\frac{u}{\alpha}\xi^2\frac{\partial C}{\partial x}=0$$

$$\xi^2\frac{\partial^2 C}{\partial\xi^2}+\xi\frac{\partial C}{\partial\xi}-\frac{4u}{\alpha}\xi^2\frac{\partial C}{\partial x}=0$$

$$\xi^2\frac{\partial^2 c(x,z)}{\partial\xi^2}+\xi\frac{\partial c(x,z)}{\partial\xi}-\frac{4u}{\alpha}\xi^2\frac{\partial c(x,z)}{\partial x}=0$$

$$\xi^2\frac{\partial^2 C(x,z)}{\partial\xi^2}+\xi\frac{\partial C(x,z)}{\partial\xi}-\frac{4u}{\alpha}\xi^2\frac{\partial C(x,z)}{\partial x}=0 \qquad (12)$$

$$\frac{\partial^2 C(x,z)}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial C(x,z)}{\partial\xi}-\frac{4u}{\alpha}\frac{\partial C(x,z)}{\partial x}=0 \qquad (13)$$

Eqn. (13) can be solved for $C\,(x, z)$ by using Hankel Transform which is defined as :

$$\mathcal{H}_{\mathrm{m}}\left[f(z)\right]=\tilde{f}(s)\equiv\int_0^\infty f(z)\mathrm{J}_{\mathrm{m}}(sz)z\;dz$$

where, the Bessel differential operator is defined as :

$$\Delta_m f(z)\equiv\frac{d^2 f(z)}{dz^2}+\frac{1}{z}\frac{df(z)}{dz}-\left(\frac{m}{z}\right)^2 f(z)$$

which, has the Hankel Transform given by :

$$\mathcal{H}_{\mathrm{m}}\left[\Delta_m f(z)\right]\equiv -s^2\tilde{f}(s)$$

Applying the Hankel Transform to (13), one finds that:

$$\mathcal{H}_0\left[\Delta_0\psi=\frac{4u}{\alpha}\frac{\partial\psi}{\partial x}\right] \qquad (14)$$

One can get:

$$-s^2\tilde{\psi}=\frac{4u}{\alpha}\frac{\partial\tilde{\psi}}{\partial x} \qquad (15)$$

$$-s^2\frac{\alpha}{4u}\int_0^x dx=\int_{\tilde{\psi}(0,s)}^{\tilde{\psi}(x,s)}\frac{\partial\tilde{\psi}}{\tilde{\psi}}$$

$$\ln[\tilde{\psi}(x,s)]=\ln[\tilde{\psi}(0,s)]-s^2\frac{\alpha}{4u}x$$

$$\tilde{\psi}(s,x)=\tilde{\psi}(0,s)\exp\left(-\frac{\alpha}{4u}xs^2\right)$$

From boundary condition "b"

$uc\,(0, z) = Q\delta\,(z – h_s)$ (b)

$$\mathcal{H}_{\mathrm{m}}\left[\psi(0,\xi)\right]=\tilde{\psi}(0,s)=\frac{Q}{u}\int_0^\infty\delta\left(\xi^2-h_s\right)J_m(s\xi)\xi\,d\xi$$

Let $I=\int_0^\infty\delta\left(\xi^2-h_s\right)J_m(s\xi)\xi\,d\xi$

Suppose that

$w = \xi^2 – h_s \Rightarrow dw = 2\xi d\xi$

$w + h_s = \xi^2$

$$I=\int_{-h_s}^\infty\delta(w)J_m\left[s(w+h_s)^{\frac{1}{2}}\right]\xi d\xi$$

$$=\frac{1}{2}\int_{-h_s}^\infty\delta(w)J_m\left[s(w+h_s)^{\frac{1}{2}}\right]dw$$

$$I=\frac{1}{2}J_m\left(sh_s^{\frac{1}{2}}\right)$$

$$\tilde{\psi}(s,x)=\frac{1}{2}\frac{Q}{u}J_m\left(sh_s^{\frac{1}{2}}\right)\exp\left(-\frac{\alpha}{4u}xs^2\right)$$

The inverse of Hankel transform is as follows:

$$\mathcal{H}_{\mathrm{m}}^{-1}[\tilde{\psi}(s,x)]=\psi(x,z)\equiv\int_0^\infty\tilde{\psi}(s,x)J_m(sz)s\;ds$$

$$\psi(x,z)=\frac{1}{2}\frac{Q}{u}\int_0^\infty\exp\left(-\frac{\alpha}{4u}xs^2\right)J_m\left(sh_s^{\frac{1}{2}}\right)J_m(s\xi)sds$$

From Table of Integrals, Series and Products page 739, 6.633 and 2.

$$\int_0^\infty e^{-\sigma^2 s^2}J_m\left(sh_s^{\frac{2-n}{2}}\right)J_m(s\xi)s\;ds$$

$$=\frac{1}{2\sigma^2}\exp\left(-\frac{h_s^{2-n}+\xi^2}{4\sigma^2}\right)I_m\left(\frac{\xi h_s^{\frac{2-n}{2}}}{2\sigma^2}\right)$$

$$\int_0^\infty e^{-\sigma^2 s^2} J_m\left(s h_s^{\frac{2-n}{2}}\right) J_m(s\xi) s\, ds$$

$$= \frac{2u}{\alpha x}\exp\left(-u\frac{h_s+\xi^2}{\alpha x}\right) I_m\left(2u\frac{\xi h_s^{\frac{1}{2}}}{\alpha x}\right)$$

$$\int_0^\infty e^{-\frac{\alpha(2-n)^2}{4u} x s^2} J_0\left(s h_s^{\frac{2-n}{2}}\right) J_0(s\xi) s\, ds$$

$$= \frac{2u}{\alpha x}\exp\left(-u\frac{h_s+\xi^2}{\alpha x}\right) I_0\left(2u\frac{\xi h_s^{\frac{1}{2}}}{\alpha x}\right)$$

$$\text{For}\left[\operatorname{Re} m > -1,\ \left|\arg\left(\sqrt{\frac{\alpha}{4u}X}\right)\right| > \frac{\pi}{4}\right] \qquad (16)$$

$$\psi(x,z) = \frac{Q}{\alpha x}\exp\left(-u\frac{h_s+\xi^2}{\alpha x}\right) I_0\left(2u\frac{\xi h_s^{\frac{1}{2}}}{\alpha x}\right)$$

Then, the crosswind integrated concentration is as follows:

$$c(x,z) = \frac{Q}{\alpha x}\exp\left(-u\frac{h_s+z}{\alpha x}\right) I_0\left[2u\frac{(z h_s)^{\frac{1}{2}}}{\alpha x}\right] \qquad (17)$$

where, I_0 is a Bessel function of order zero. Substituting from Eqn. (17) in Eqn. (8), one can get:

$$C(x,y,z) = \frac{Q}{\sqrt{2\pi\alpha x \sigma_y}} e^{-\frac{y^2}{2\sigma_y^2}} \exp\left(-u\frac{h_s+z}{\alpha x}\right) I_0\left[2u\frac{(z h_s)^{\frac{1}{2}}}{\alpha x}\right] e^{\frac{vx}{u}} \qquad (18)$$

The later analytical model is evaluated by Essa *et al.* (2021) in which the vertical turbulent eddy diffusivity as a function of downwind distance and power law of vertical height and crosswind turbulent eddy diffusivity as a function of downwind distance and invariant wind speed to get the concentration in three dimensions., that is:

$$K_y(x, z) = \beta x u \qquad (19)$$

$$K_z(x, z) = \gamma x z^n, \qquad z \neq 0 \qquad (20)$$

where, $\beta = 0.16\left(\frac{w_*}{u}\right)^2$ and $\gamma = 0.31\left(\frac{w_*}{u}\right)^2$, w_* is the convective vertical velocity and 'n' is a parameter depends on stability conditions (Irwin 1979).

$$C(x, y, z) = \phi(x, y)\, \psi(x, z) \qquad (21)$$

$$= \frac{Q(2-n)^3 (h_s Z)^{\frac{1-n}{2}}}{4\gamma x^2} Exp\left[-\frac{u(1-n)^2\left(h_s^{1-n}+z^{1-n}\right)}{8\gamma x^2} - \frac{vx}{u}\right] \times I_{\frac{1-n}{2-n}}\left[\frac{u(1-n)^2 (h_s Z)^{\frac{2-n}{2}}}{4\gamma x^2}\right] \sum_{l=0}^{\infty} B_l e^{\frac{-\lambda_l^2}{2}x^2} \cos\left(\frac{l\pi}{L_y} y\right) \qquad (22)$$

where, $e^{-vx/u}$ is the radioactive decay for isotope, $\upsilon = 2.9 \times 10^{-5}$ s^{-1}. I_m is a modified Bessel function of the first kind of order m, $m = \frac{1-n}{2-n}$.

3. Results and discussion

The observed data of I^{135} isotope concentration was obtained from dispersion as experiments conducted in unstable condition air samples which were collected around the Egyptian Atomic Energy Authority. The vertical height is 0.7 m above ground from a stack height of 43 m, for twenty-four hours working, where the air samples were collected during half hour at a height 0.7 m with a roughness length of 0.6 cm. Air samples were collected from 98 to 186 m around the First Research Reactor in Egyptian Atomic Energy Authority (EAEA). The study area is nearly flat, dominated by sandy soil with poor vegetation cover and very little mean annual precipitation (40-80 mm). The study area was divided into 16 sectors (with 22.5° width for each sector), beginning from the north direction. Aerosols were collected at a height of 0.7 m above the ground on a 10.3 cm diameter filter paper with a desired collection efficiency (3.4%) using a high volume air sampler with 220 V/50 Hz bias. The air sampler had an air flow rate of approximately 0.7 m^3/min (25 ft^3/min). Sample collection time was 30 min with an air volume of 21.2 m^3 (750 ft^3). This air volume was corrected to standard conditions (25 °C

TABLE 3

Meteorological data of the nine convective test runs at Inshas site in March and May 2006

Run no.	Working hours of the source	Release rate (Bq)	Wind speed (m s^{-1})	Wind direction (deg)	W_* (ms^{-1})	P-G stability class	h (m)	Vertical distance (m)
1	48	1028571	4	301.1	2.27	A	600.85	5
2	49	1050000	4	278.7	3.05	A	801.13	10
3	1.5	42857.14	6	190.2	1.61	B	973	5
4	22	471428.6	4	197.9	1.23	C	888	5
5	23	492857.1	4	181.5	0.958	A	921	2
6	24	514285.7	4	347.3	1.3	D	443	8.0
7	28	1007143	4	330.8	1.51	C	1271	7.5
8	48.7	1043571	4	187.6	1.64	C	1842	7.5
9	48.25	1033929	4	141.7	2.1	A	1642	5.0

TABLE 4

Observed, Gaussian and predicted concentrations for Run 9 experiments

Test	Downwind distance (m)	Observed conc.(Bq/m^3)	Gaussianconc. Eqns (1, 3, 4) (Bq/m^3)	Gaussianconc. Eqns (1, 5, 6) (Bq/m^3)	Predicted conc. Eq. (18) (Bq/m^3)	Predicted conc. Eq. (22) (Bq/m^3)
1	100	0.025	0.023987	0.039975	0.020607	0.0296
2	98	0.037	0.024223	0.03302	0.044329	0.0197
3	136	0.091	0.041614	0.082803	0.078956	0.0508
4	135	0.197	0.081836	0.166257	0.090608	0.2247
5	106	0.272	0.075292	0.274148	0.331532	0.3339
6	186	0.188	0.123797	0.107066	0.156713	0.1218
7	165	0.447	0.245783	0.216606	0.573346	0.4159
8	154	0.123	0.253737	0.151414	0.125247	0.1500
9	106	0.032	0.024865	0.044721	0.047909	0.0381

and 1013 hPa) (Raymond *et al*., 2000). The filter paper was directly measured by energy and efficiency calibrated HPGe detectors. The measured efficiencies of the detectors relative to 3” × 3” NaI (Tl) detector were 15.6 and 30% measured at 1.332 MeV with source to detector distance of 25 cm. Meteorological data was provided by Environmental Radioactive Contamination Unit’s meteorological, station at the Radiation Protection Department, EAEA. The height of the meteorological tower is 15 m. Vertical temperature gradient (T/Z) was determined by measuring the temperature at 10- and 60-m levels from the multilevel meteorological tower of Inshas, Siting & Environmental Department, National Center for Nuclear Safety & Radiation Control, EAEA. This tower is located very near to the study area.

The values of ‘*n*’ are a function of air stability are taken from Hanna *et al*. (1982) and presented in Table 1. The meteorological data during the experiments are taken from Essa and El-Otaify (2008) and presented in Table 3. The observed concentration of I^{135} isotope and the predicted concentrations by Eqns. (1), (18) and (22) below the plume centerlines are presented in Table 4. A comparison between predicted and observed

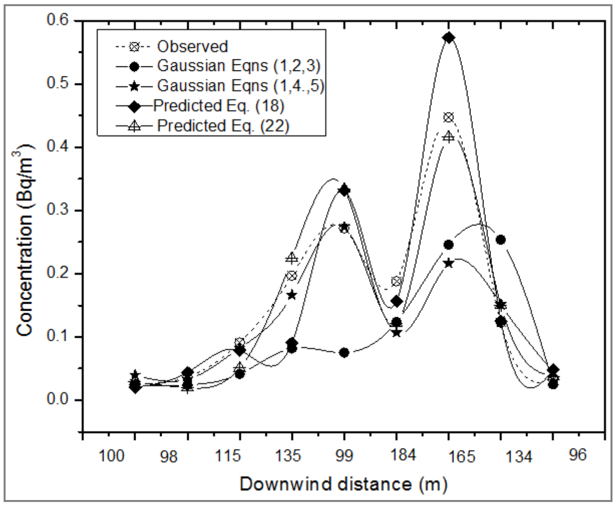

Fig. 1. Relation between two Gaussian and two predicted models and observed Concentrations (Bq/m^3) *via* downwind distances

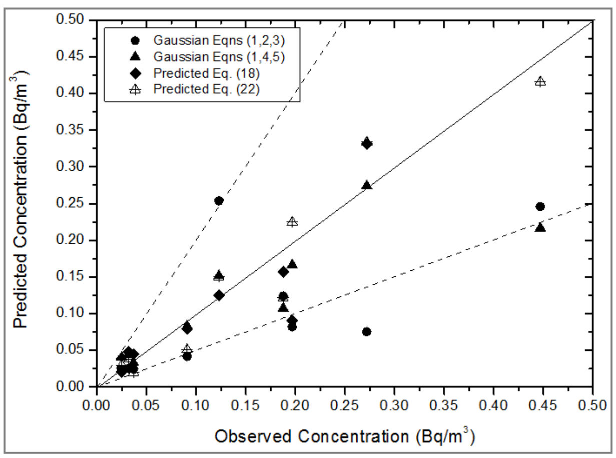

Fig. 2. Achieving between two Gaussians, two predicted concentrations with observed concentration

concentrations of radioactive I^{135} *via* downwind distance in unstable condition at Inshas is shown in both Table 4 and Fig. 1, also, the relation between predicted and observed concentration data is shown in Fig. 2.

From the two figures, one finds that the best proposed model is Eqn. (22) because of the vertical eddy diffusivity as a function of power law in the vertical height, then Eqn. (18) because of the linearity of vertical eddy diffusivity. The predicted two models achieved 98% and 100%for two Eqns. (18 and 22) respectively. Also, Gaussian model (1, 5, 6) gives better results than Gaussian plume model Eqns. (1, 2, 3) because of the strongest of the vertical dispersion for Eqns. (1, 4, 5).

TABLE 5

Comparison between Gaussian, predicted and observed concentrations in unstable condition

Models	NMSE	FB	COR	FAC2
Gauss Eqns (1, 3, 4)	0.83	0.45	0.66	0.63
Gauss Eqns (1, 5, 6)	0.35	0.23	0.84	0.79
Pred Eqn (18)	0.14	-0.40	0.95	1.04
Pred Eqn (22)	0.06	0.02	0.96	0.98

4. Statistical technique

Comparing between Gaussian, predicted and observed concentrations is introduced by (Hanna, 1989). Where, NMSE is the normalized mean square error, FB is the fraction bias, COR is the correlation coefficient and FAC2 is a factor of two. The statistical technique in Table 5 shows that the entire proposed model inside a factor of two with observed concentration data. Also the statistical shows that the predicted model Eqn. (22) is the best for NMSE, FB, COR and FAC2 then, the proposed model Eqn. (18), then Gaussian model Eqns. (1, 4, 5) and the fewest accuracy is Gaussian model Eqns. (1, 2, 3) for homogeneity.

5. Conclusions

The idea of this work, comparing two Gaussian plume models and two predicted models using advection-diffusion equation considering the linearity and power law of vertical height. One finds that the best model is Eqn. (22) because of the vertical eddy diffusivity as a function of power law in the vertical height, then Eqn. (18) because of the linearity of veridical eddy diffusivity. The predicted two models achieved 98% and 100% for two Eqns. (18, and 22) respectively. Also Gaussian model (1, 5, 6) gives better results than Gaussian plume model Eqns. (1, 2, 3) because of the strongest of the vertical dispersion for Eqns. (1, 4, 5).

Disclaimer : The contents and views expressed in this study are the views of the authors and do not necessarily reflect the views of the organizations they belong to.

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