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The application of the theory of tethered balloons to measurement of low level winds*

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ABSTRACT. A theory of tethered balloons advanced by the author earlier (Saha 1956) for the determination of aerodynamic resistance is developed and applied to the measurement of low level winds up to an average height of about 350 ft above the ground. The effect of the supporting thread is discussed in detail and the difficulty of taking the effect of the aerodynamic resistance of the thread in a variable wind distribution into a consistent theory is pointed out. An approximate solution is offered by assuming that the total resistance of the thread is the sum of the resistances of every 100 ft of the length released in a prevailing wind distribution. Certain assumptions are also made in regard to sag angle, the shape of the thread, the air density, the free lift and the diameter of the balloon etc. Winds computed from data of tethered balloon flights in accordance with the present method are compared with the results of pilot balloon flights taken almost simultaneously. Agreement is found to be fairly close and satisfactory. Divergence in a few cases may be due to the fact that in a turbulent atmosphere a tiny pilot balloon like the one used in the present experiments for its low rate of ascent cannot maintain its uniform rate of ascent as assumed and is liable to yield winds that are not quite dependable.

1. Introduction

Low level winds have been measured by different workers using different methods. Those engaged in micro-meteorological work involving collection of data on crop climate etc, super-refraction, and turbulence and diffusion problems have found it convenient to construct micro-meteorological towers of different height and design (Ramdas 1944, Jehn 1948, Church and Gosline 1948, Best *et al*. 1952, Cramer and Record 1953). Over a height range from a few feet above the ground to 400 ft aloft, hot-wire anemometers or conventional cup-type anemometers of different designs have been fitted up at desired height intervals. In America, the thermistor technique has been developed to a degree which enables its use on captive balloons to determine winds up to a height of over 50 metres (Hales 1948). A hot thermistor responds quickly to changes of wind and the resulting change in the electrical resistance of the element may be calibrated to give a direct scale for the wind velocity. Richardson (1924) used a ballistic method to measure winds aloft. He shot small steel balls upward at an inclination to the vertical so that the prevailing wind distribution could bring it back to the gun. From the time taken by the shot to make the up-and-down journey at the observed angle of projection, Richardson estimated the mean wind distribution. Ordinary pilot balloon methods are not quite suitable to give a detailed structure of low level winds. Even if a very low rate of ascent, say 7 km hr^{-1}, is used and observation be taken on the freely moving balloon at intervals of half a minute, winds can only be obtained at height intervals of 200 ft. Elaborating a theory of tethered balloons which was proposed in an earlier paper (Saha 1956), hereafter referred to as paper 1, on the aerodynamic resistance of tethered balloons, a method is developed which enables a detailed determination of the vertical structure of the wind up to an average height of about 350 ft above the ground. The method may be described as an approximate one as it involves several

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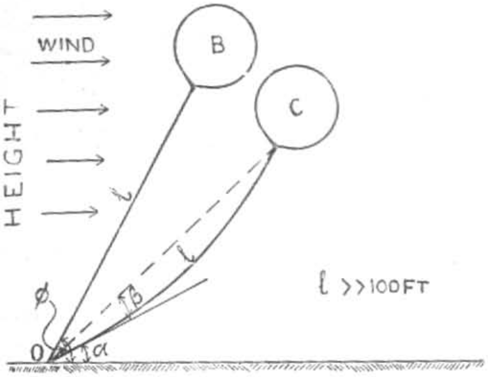

Fig. 1. The pulling-down effect and sag of a long tethering thread

assumptions in regard to the weight, the curvature or sag, and the aerodynamic resistance of the supporting thread in varying wind distributions.

2. An approximate method for measuring low level winds

The formula given in Paper 1 to find the coefficient of aerodynamic resistance of spheres using a tethered balloon is—

$$k = F_g \mid \rho \ d^2 \ V^2 \tan \phi \qquad (1)$$

where k = coefficient of aerodynamic resistance of spheres

F = free lift of the balloon

g = acceleration due to gravity

ρ = air density

d = diameter of the balloon

V = wind speed relative to the balloon in the steady state

and ϕ = angle of altitude of the balloon in the steady state

In deriving equation (1), a few simplifying assumptions were made. These were (*i*) the weight of the thread was negligible in comparison with the free lift of the balloon, (*ii*) the aerodynamic resistance of the supporting thread was negligibly small compared to that of the balloon and (*iii*) the thread released was free from curvature or sag. Further, the balloon was assumed to retain its size and shape under the experimental conditions. As discussed in paper 1, it was permissible to admit these assumptions without involving serious error so long as the thread length was kept shorter than 100 feet. When greater lengths were released some of these assumptions became clearly untenable. The weight and the aerodynamic resistance of the thread became appreciable and there was perceptible sag in the thread. On all these counts, there was a pulling-down effect on the balloon which could no longer be neglected. On account of this effect, the angle of altitude of the balloon was under-read as shown in Fig. 1 which illustrates the effect of a long thread on the balloon. Let B be the position at angle of altitude ϕ which the balloon would have occupied if the thread was weightless and free from air resistance. The actual position of the balloon is, however, at C where it has a sag angle β and an observed angle of altitude α. The true angle of altitude of the balloon in position C is, therefore, $(\alpha + \beta)$.

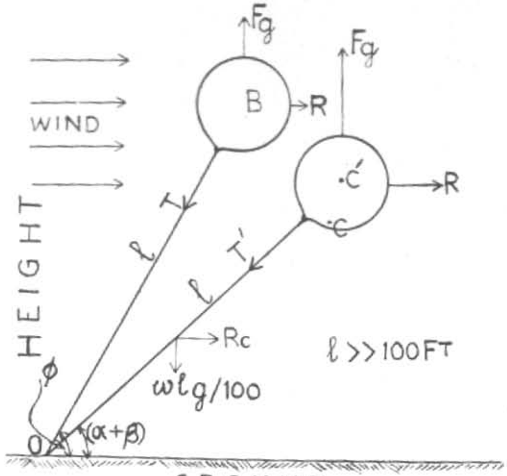

Fig 2. Hypothetical deflection of an inflated balloon tethered by a long inflexible thread

To find the modification in equation (1) caused by the effect of the thread, let it be assumed that only so much thread length be released as would produce a small curvature in the thread under the prevailing

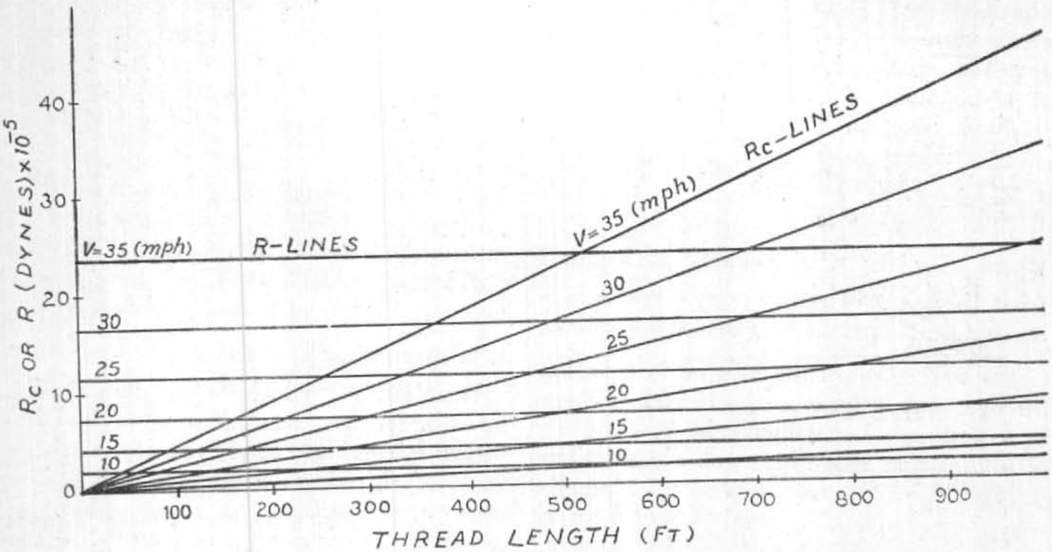

Fig. 3. The aerodynamic resistance of a tethering thread as function of thread-length and wind velocity, compared with that of a tethered balloon at corresponding wind velocities

wind distribution. The balloon and the thread may then be imagined as having moved like a cylindrical rod under the action of the forces on the thread. The position of the balloon C′ under this assumption would be somewhat removed from C as shown in Fig. 2.

In Fig. 2, let R and R_c be the aerodynamic resistances of the balloon and the thread respectively in a wind V which is assumed uniform with height. Let the tension T on the thread in position B change to T' in the pulled-down position C′, both passing through the origin.

Then, for balloon position B, equation (1) takes the form

$$\tan \phi = F_g/R \qquad (2)$$

where $\quad R = k\rho d^2 V^2$

If the thread be assumed uniform and cylindrical in shape

$$R_c = k_c \rho (d_c l) V^2 \qquad (3)$$

where k_c and d_c are respectively the coefficient of aerodynamic resistance and the diameter of the thread and l the length of the thread and the wind is assumed to act normally on the thread.

If in the steady state at C′, the resultant weight and the aerodynamic resistance of the thread are assumed to act at the middle point of the thread and if moments be taken of the forces about the origin, the following simple relation is obtained

$$\tan(\alpha+\beta) = (F - wl/200)g/(R + \tfrac{1}{2}R_c) \qquad (4)$$

It is of interest to know the magnitude of R_c in relation to R in a uniform wind-distribution as given by equation (3) for varying thread lengths released. This information is plotted in Fig. 3. The horizontal lines give the values of R. But in the atmosphere a uniform wind velocity up to the height of the balloon when a thread length much greater than 100 feet has been released is a rather rare occurrence owing to ground friction and many other factors and the assumption of a uniform V over the whole length of the thread released in such a case can hardly be said to conform to actual conditions. In the light of this consideration, equation (3) for the aerodynamic resistance of the supporting thread requires modification. The perfect step in this direction, of course, lies in using an integral equation of the type

$$R_c = \rho\, k_c\, d_c \int_0^L v_l^2\, \delta l \qquad (5)$$

where v_l is the wind velocity at length l from the point of release and other factors are assumed constant over the total length L.

The evaluation of the integral in equation (5) over any length of the thread released is a well-nigh impossibility in the absence of information regarding v_l as a continuous function of l. It may, however, be practicable to arrive at an approximate evaluation of the same quantity in the following manner.

Let v_0 be the wind velocity at 50 feet from the point of release, and V_1, V_2, V_3, ... the respective wind velocities at the end of the 100, 200, 300 feet and so on. It is assumed that the wind velocity is uniform at v_0 over the length 0 to 100 feet, V_1 over the length 100 to 200 feet, V_2 over the length 200 to 300 feet, V_3 over the length 300 to 400 feet, and so on. On this assumption it is possible to write equation (5) in the form

$$R_c = S\, \rho\, k_c\, d_c \times 100 \sum_{0}^{p=n-1} v_p^2 \qquad (6)$$

where n is an integer representing the number of 100^s of feet of the thread released, and S a conversion factor for length.

Substituting the values of R and R_c from equations (2) and (6) in equation (4), the following relation for V at any 100 ft is obtained, after some slight rearrangement.

$$V = \left[\frac{(F - wl/200)g}{kd^2 \rho \tan(\alpha+\beta)} - 50\, \frac{S k_c\, d_c}{kd^2} \sum_{0}^{p=n-1} v_p^2 \right]^{\frac{1}{2}} \qquad (7)$$

The height of the balloon at which this wind speed V is experienced is approximately $l \sin(\alpha+\beta) \cos\beta$ if β is assumed to be small and the shape of the sagging thread in Fig. 1 be assumed to be an arc of a circle.

Obviously, equation (7) provides an approximate method for determining V in steps of 100 ft of the thread released. The height intervals would, of course, depend upon the values of α and β as well as l.

3. Detailed working of the method

Equation (7) which forms the working formula for the determination of low level winds relates the wind velocity with the angle of altitude of a tethered balloon and some other parameters whose values are required to be known accurately. These parameters are the coefficients of the aerodynamic resistances of the balloon and the thread, the air density, the sag angle and the shape of the curve, the free lift and the diameter of the balloon etc under conditions of varying length of the thread released and the wind velocity distribution. To arrive at working values of these parameters let us first deal with them individually.

(*i*) *The coefficients of the aerodynamic resistances of the balloon and the supporting thread, k and k_c*

In paper 1, values of k were found at different Reynolds numbers. It was seen that k assumed a more or less constant value of $0 \cdot 065$ at Reynolds numbers exceeding about $5 \cdot 0 \times 10^5$ which in the case of an NR-70 balloon with free lift 750 gm would be reached in a wind of about 15 mph. But to enable k values to be found at lower wind speeds, a chart (Fig. 4) was prepared relating the data of wind speed and the angle of altitude presented in Table 2 of paper 1 for an NR-70 balloon with free lift 750 gm.

Fig. 4 gives a scale which could be used to find the wind velocity from the angle of alttiude of the balloon if a suitable experimental thread shorter than 100 feet was used.

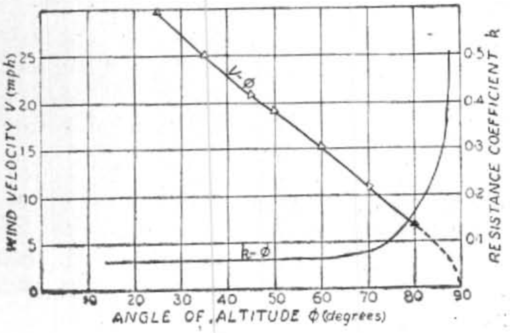

Fig. 4. Wind velocity and the coefficient of aerodynamic resistance of a tethered balloon of free lift 750 gms at different angles of altitude of the balloon

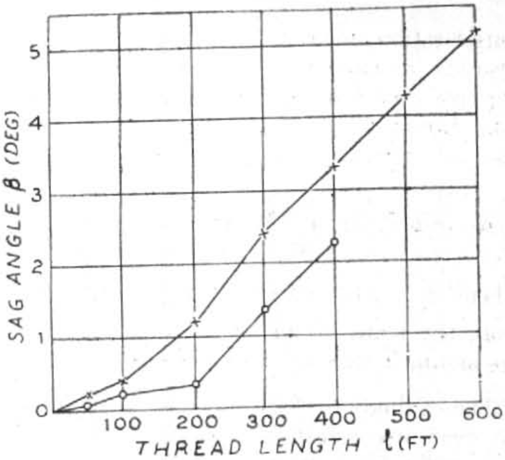

Fig. 5. Variation of the mean sag angle β with thread length l in an average wind of about 15-20 mph

Accurate values of k for use in equation (7) cannot be found without knowing the Reynolds number of the airflow past the balloon which involves a knowledge of the wind velocity which itself it is the object of the paper to determine. But a hint about the value of k to be used may be gathered from the (k-ϕ) relation included in Fig. 4. It has already been mentioned that above 15 mph, k has a constant value of 0·065. It is also seen from Fig. 4 that in the speed range 10 to 15 mph the rate of variation of k is very slow indeed. It would be expected that the pulling down effect in light winds would be small, hence $(\phi-\overline{\alpha+\beta})$ would be small. Therefore the (k-ϕ) curve in Fig. 4 provides a fairly reliable guide to find appropriate values of k from the observed angle of altitude.

Bairstow (1913) has given a curve for the aerodynamic resistance of smooth cylindrical wires at various Reynolds numbers. As already suggested, a straight uniform thread may be regarded as a long cylinder and Bairstow's results may be made applicable to it. In the range of Reynolds numbers found for a braided thread (diameter 0·094 cm) when exposed to a wind velocity ranging from 10 to 30 mph a mean value of 0·56 would be considered appropriate for k_c.

(ii) The sag angle and the shape of the thread

The curvature in the thread when a great length of the thread is released is due partly to the weight of the thread and partly its aerodynamic resistance. When the wind velocity and direction is not uniform at all points along the thread, the centre of gravity of the thread and the point at which the resultant aerodynamic resistance of the thread is supposed to act are likely to deviate from the geometrically middle point of the thread. The curve then produced by the thread may assume a complex shape. But in the present investigation it has been assumed that only so much thread length is released as produces a small sag angle $(0<\beta<5°)$ under all light to moderate wind conditions.

Fig. 5 shows the variation of the mean sag angle β with the thread length l for an average wind velocity of about 20 mph. It will be seen that on an average a length 600 ft of the experimental braided thread could be released when tethering an NR-70 balloon with free lift 750 grams to keep the sag angle within the specified limits. The variation of the sag angle with the diameter of the thread is also shown in Fig. 5 in which the sag is considerably less in the case of the nylon thread which is thinner and lighter. It may also be shown that the larger the free lift of the balloon the less is the sag angle for the same thread and the same wind distribution.

The actual shape of the curve in any particular case will, of course, depend upon the vertical wind distribution. Although, observations appear to confirm that in most cases the wind speed increases with height, it is not always so. There are occasions when the speed after going through a maximum falls again after some height or there may be more than one layer of comparatively light winds within the experimental length released. Further, the assumption of a uniform wind direction is also not always correct. Ordinarily there is a slight veering of wind with height but cases of backing are also observed. In such cases the balloon and the thread are not quite in the same vertical plane particularly when a great thread length is released. Different portions of the thread are acted upon by winds in the different layers and the actual shape of the thread may be quite complex. Whatever the wind distribution, the balloon, however, because of its very large free lift takes up its balance position and it is not impossible to have a reliable measure of the angle of altitude except in cases in which the vertical shear of the wind is excessive or there is a complete change of direction with height. In the latter cases, the present method is considered unsuitable.

In cases in which the sag angle is small ($0<\beta<5°$), it may be assumed that the actual shape of the curve does not differ appreciably from that of an arc of a circle. A theory of the shape of the curve may be worked out for a particular wind distribution but as the wind distribution along the length of the thread may be highly variable from case to case, not much attention was paid to the theoretical position in this regard.

(*iii*) *Variation of air density*, ρ

There is a general decrease of air density with height in the atmosphere. However, for finding the values of air density at different steps of the thread-length released, vertical distributions of pressure, temperature and vapour pressure are required to be known in detail. At present, there are few means of doing this. But some estimate can be made of the order of variation likely to occur over the experimental height range of about 350 ft. An average fall of pressure with height is 1 mb per 30 ft, an average temperature lapse rate in an almost saturated adiabatic atmosphere may be assumed to be 6° C per km, and there is likely to be insignificant vertical variation of vapour pressure in the well-mixed monsoon air over the stipulated height range. Under these prevailing conditions it may be easily shown employing the usual density relation that the vertical variation of air density over even the maximum experimental height range is hardly 1 per cent. Under conditions of steep temperature inversion the order of this variation may somewhat increase but in no case exceeds 3 to 4 per cent. In view of this slow decrease of air density with height, it was considered that little error would be involved by neglecting the variation of density with height in the present investigation. The density was, therefore, assumed constant at the value found at the height of the Stevenson screen.

(*iv*) *The balloon free lift F and the diameter d*

The free lift of the balloon being a direct function of its buoyancy as represented by the relation

$$F = \tfrac{1}{6}\,\pi\, d^3\,(\rho - \rho') - W_1$$

where ρ and ρ' are the densities of the outside air and the inside hydrogen respectively and W_1 is the weight of the uninflated balloon, it may be assumed to undergo little change as long as the outside air density does not change markedly. In the foregoing paragraph it was discussed and concluded that the variation of air density over the experimental range of thread length released is of a negligibly small order. Hence it may be concluded here that within this range the free lift and the balloon diameter undergo little change and that in the present experiment their variations were neglected.

(*v*) *An example*

It may be worthwhile to illustrate the present method by working out an example of an actual tethered balloon flight carried out at 0930 IST on 7 July 1954, at Ambala. An NR-70 balloon with free lift 750 gm and diameter 113·2 cm and a braided thread of weight 20 gm per 100 ft and diameter 0·094 cm were used. The following were the flight observations—

l (ft)	Angle of altitude of balloon α (degrees)	Wind direction (degrees)
50	55	130
100	55	130
200	38	130
300	34	130
400	28	130
500	26	130
600	25	130

At the time of the flight, the pressure, temperature and vapour pressure at the height of the stevenson screen were 969 mb, 90°F and 35 mb respectively.

The values used of the other parameters occurring in equation (7) in relation to this particular flight were—

$$k = 0{\cdot}065$$

$$k_c = 0{\cdot}56$$

$$\rho = 1{\cdot}094\times 10^{-3} \text{ gm per c.c.}$$

$$g = 979{\cdot}4 \text{ cm sec}^{-2}$$

and a mean value of β in the following measure of its dependence on thread-length—

l(ft)	50	100	200	300	400	500	600
β(deg)	0·2	0·5	1·2	2·4	3·3	4·3	5·2

Substitution of the foregoing values of the relevant parameters in equation (7) reduces the formula for wind velocity to the simplified form

$$V=\frac{(750-l/10)\,1077}{\tan(\alpha+\beta)}-0{\cdot}09502(v_0^2+v_1^2+\ldots)$$

The wind velocities at the different thread lengths are then found in the following manner—

$l=50$ ft, $v_0=17$ mph

$$=760 \text{ cm sec}^{-1} \quad (\textit{vide} \text{ Fig. 4})$$

$$l=100\text{ ft}, v_1=\frac{740\times 1077}{\tan 55{\cdot}5}-{\cdot}09502(760^2)$$

$$=702 \text{ cm sec}^{-1}=15{\cdot}8 \text{ mph}$$

$$l=200\text{ ft}, v_2=\frac{730\times 1077}{\tan 39{\cdot}2}-{\cdot}09502(760^2+702^2)$$

$$=928 \text{ cm sec}^{-1}=20{\cdot}8 \text{ mph}$$

$$l=300\text{ ft}, v_3=\frac{720\times 1077}{\tan 36{\cdot}4}-{\cdot}09502(760^2+702^2+928^2)$$

$$=940 \text{ cm sec}^{-1}=21{\cdot}0 \text{ mph}$$

and so on, for every successive 100 ft up to 600 ft. These calculations gave at

$l=400$ ft, $v_4=22{\cdot}1$ mph

$l=500$ ft, $v_5=21{\cdot}5$ mph

and $l=600$ ft, $v_6=20{\cdot}8$ mph.

Velocities thus found were plotted against the appropriate heights and are shown in Fig. 7.

4. Experimental arrangement

The arrangement for reading the angle of altitude of the balloon at successive thread lengths was practically the same as in paper 1. A new device was, however, introduced for quick winding and unwinding of the thread. This was a cylindrical wooden drum of effective diameter 14·5 inches which could be rotated rapidly about a horizontal axle passing through the centre of the drum. The steel support of the drum was fixed firmly on to a small stool. A view of the drum with a balloon in process of being released and the pibal theodolite in the background is presented in a photograph shown in Fig. 6. With the help of this drum, a thread length of 100 ft could be released in even less than 10 seconds.

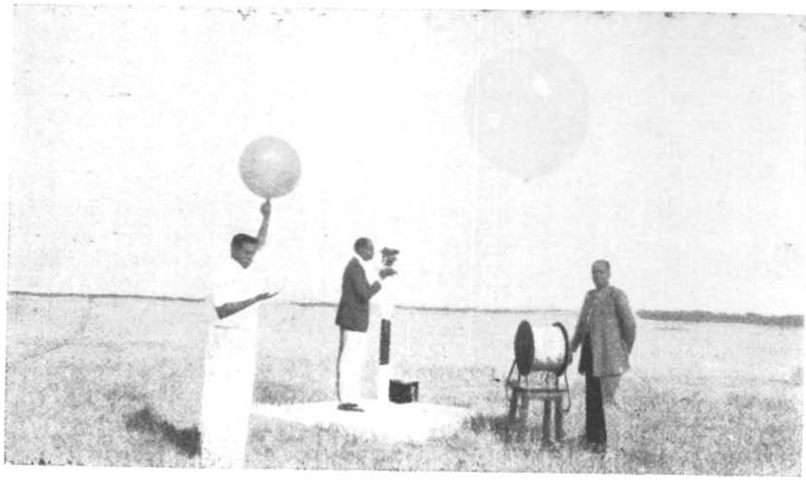

Fig. 6. Photograph showing the release drum with an experimental balloon and the calibration theodolite in the background

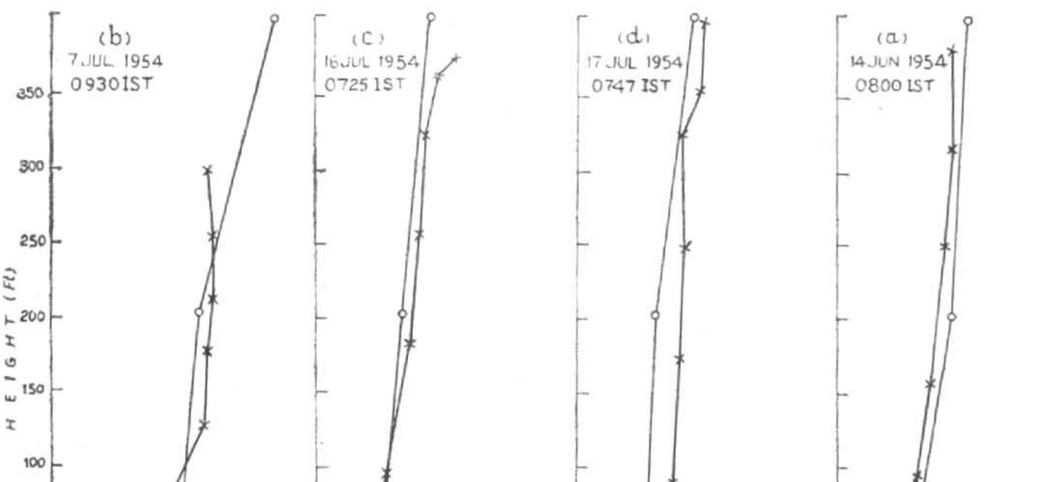

Fig. 7. Comparison of winds determined by the method of tethered balloon flights with simultaneous pibals (o—o pibals, x—x tethered balloon winds)

The theodolite normally used for pilot balloon work was also used for finding the true altitude of the balloon ($\alpha+\beta$) at different thread lengths. The balloon was viewed through the telescope and as soon as it was got at the centre of the grid in the field of view the angle of altitude was read. This gave the true altitude of the balloon. In the case of a turbulent atmosphere it was best to have three persons, one to bring the balloon in the line of sight of the telescope, the second to look through the field of view of the theodolite and call out whenever the balloon was observed at the centre of the grid, and the third to give a prompt reading of the angle of altitude whenever the second called out. There was strict coordination among the three. Simultaneously with determination of the true angle of altitude of the balloon, the angle of altitude of the supporting thread at the ground was read by means of an angle card in the usual manner. The difference gave β. A calibration curve for β at different thread lengths was prepared in this way.

5. Comparison of results with pibal data

To check the results of the present investigation, the computed winds were compared with winds found simultaneously by the normal pilot balloon method using an NR-15 balloon with rate of ascent 7 km hr^{-1} or 400 ft min^{-1}, the balloon position being read at intervals of half a minute. Wind data were thus independently available at height intervals of about 200 ft.

In Fig. 7 are presented the results of four flights for comparison. It will be seen that although the agreement is not perfect in every case, the present method serves to give the structure of winds in greater detail. In judging the closeness of agreement between the two sets of wind data it may be well to bear in mind the various assumptions and limitations imposed on the present method. The pilot balloon method on the other hand suffers from the fact that when the atmosphere is turbulent the rate of ascent of the balloon is vitiated and that its constancy can no longer be assumed without involving error. This error is likely to be quite serious for a tiny balloon like the NR-15 balloon, free lift 18 grams. The powerful eddies of all possible sizes will play mercilessly on it during its ascent and the rate of ascent which is assumed to be constant is bound to be interfered with, resulting in inaccuracy of wind data.

6. Conclusion

In conclusion, it may be stated that although the present method is an approximate one its chief advantage consists, apart from its simplicity and cheapness, in ability to yield a fine structure distribution of the winds. There are not many methods which can do this at present. Perhaps, the only method comparable with present one in yielding the fine structure of the wind in the vertical is that of the sky-high micro-meteorological towers as have been constructed at Richland and Texas in U.S.A. positioning an array of anemometers at close intervals of height. But building high towers is obviously a very costly undertaking and not within the means of even big scientific institutions.

7. Acknowledgement

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