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Comparison of Geostrophic Winds on constant pressure surfaces with observed winds in India

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ABSTRACT. The paper describes a method of comparing geostrophic winds derived from constant pressure surfaces with observed winds. The differences in heights of the constant pressure surfaces between five pairs of stations in India have been calculated using upper wind data for two levels *viz.* 850 mb and 700 mb. The differences have also been obtained from the data of radio sonde ascents. These values have been compared and correlation coefficients have been worked out. Similar computations have been made for some cases in the British Isles. Some plausible explanations for the comparatively lower values of correlation coefficients for the Indian stations have been given.

1. *Introduction.*

The first attempt to compare winds derived from isobars with observed winds in India was by Ishaque[1]. He compared the direction and velocity of the gradient winds derived from sea level isobars with the observed winds at different levels at Agra and Bangalore in the winter and hot weather periods. At Agra the gradient wind direction agreed very closely with the observed wind direction at 0.5 km. But the correlation between the computed velocity and the observed velocities at 0.5 and 1.0 km was very low, the correlation coefficients being respectively 0.34 and 0.39. At Bangalore the observed wind direction at all levels was very much at variance with that derived from the surface isobars. In the absence of upper air temperature data, Ishaque could not compare the observed winds at upper levels with the isobaric winds at the corresponding levels. Also, his study was confined to the winter and hot weather periods only.

Since 1944 daily upper air temperature and humidity data are available from a network of stations in India and neighbourhood and the contours of the constant pressure surfaces are being studied by forecasters in India. Hence it is of considerable interest to make a quantitative comparison of winds derived from the contours of the constant pressure surfaces and the observed winds.

The network of radio-sonde stations in India is not, however, close enough to construct the contours of the constant pressure surfaces unaided by observed winds. All that is possible is to draw the contours so as to fit the observed winds at pilot balloon stations and the height values (of the constant pressure surface) available for the radio-sonde stations. If this procedure is adopted, experience shows that there is generally no difficulty to fit the observed wind directions with the contour lines. But what is needed is a quantitative comparison between the contour and observed wind directions and velocities. In the present study a method has been evolved for such comparison on the assumption of geostrophic wind relation.

2. *Method of comparison.*

The geostrophic wind component 'v' normal to the direction of 'n' on a constant pressure surface is given by

$$\frac{dz}{dn} = -\frac{2\omega}{g} v \sin\phi \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where $\frac{dz}{dn}$ is the variation of the height of the constant pressure surface per unit distance in the direction 'n'. 'v' is taken as positive when the wind is blowing towards the right hand side of an observer facing to increasing value of 'n'.

If P and Q are any two points on the constant pressure surface,

$$\int_Q^P dz = -\frac{2\omega}{g} \int_Q^P v \sin\phi \, dn \quad \ldots \quad \ldots \quad (2)$$

The integral on the left hand side of the equation gives the difference in heights of the constant pressure surface between two points P and Q and its value is independent of the path followed in integrating the right hand side term in equation (2). If the points P and Q represent two radio-sonde stations, the observed difference ($\triangle Z_o$) in the heights of the constant pressure surface will be available from the radio-sonde ascents. From pilot balloon winds the height difference of the constant pressure surface between the two stations may be calculated on the assumption of geostrophic wind using equation (2). This we designate as calculated difference ($\triangle Z_c$) in the heights. $\triangle Z_o$ and $\triangle Z_c$ are then compared.

Pilot balloon winds are measured in India at stations about 150 miles apart. The path followed for integration is through a chain of pilot balloon stations as close as possible to the straight line between P and Q. This is illustrated in Fig. 1 where A, B and C are pilot balloon stations and P and Q both radio-sonde and pilot balloon stations. The

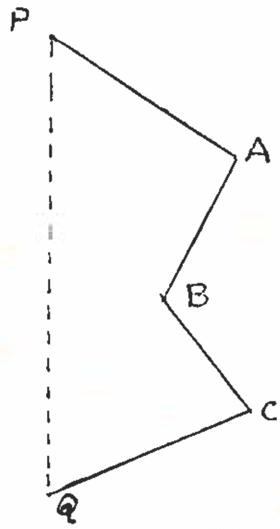

Fig. 1.

wind at each pilot balloon station is taken to represent the wind conditions half-way up to the next pilot balloon station and integration of equation (2) is done by the method of summation. Thus in the example in Fig. 1,

$$\Delta Z_c = -\frac{2\omega}{g}\cdot\frac{1}{2}\left\{\left(v_{PA}^{P}+v_{PA}^{A}\right)PA.\sin\phi_1 + \left(v_{AB}^{A}+v_{AB}^{B}\right)AB.\sin\phi_2 + \left(v_{BC}^{B}+v_{BC}^{C}\right)BC.\sin\phi_3 + \left(v_{CQ}^{C}+v_{CQ}^{Q}\right)CQ.\sin\phi_4\right\} \quad \ldots (3)$$

The symbols v_{AB}^{A} and v_{AB}^{B} represent the wind velocities as measured at stations A and B normal to the line AB. PA, AB, BC and CQ are linear distances. ϕ_1, ϕ_2, ϕ_3, and ϕ_4 are the mean values of the latitude between P and A, A and B, B and C and C and Q.

The constant pressure surfaces studied are of 850 and 700 millibars. Strictly the upper winds used in computing $\triangle Z_c$ should be the wind at the level of the constant pressure surface. But since pilot balloon winds are computed for some fixed heights only, 5000 and 10,000 feet winds have been respectively used in computing $\triangle Z_c$ for the 850 and 700 mb surfaces. In the cases studied here, the lowest and the greatest heights attained by the 850 mb surface are 4428 and 5347 feet and by the 700 mb surface are 9914 and 11,400 feet. These are extreme values and generally the heights were nearer to 5000 and 10,000 feet.

3. *Is the assumption of Geostrophic wind valid for low latitudes?*

The method of comparison adopted here involves the assumption of geostrophic wind over Indian latitudes. It may be argued that the gradient wind equation should be used for these latitudes instead of the geostrophic wind equation. It is shown below that the assumption is as valid in the cases under study as in middle latitudes.

The equation for gradient wind is

$$2\,\omega\, v \sin\phi \pm \frac{v^2}{r} = -\frac{1}{\rho}\frac{\delta p}{\delta n} \quad \ldots \ldots \ldots (4)$$

where r is the radius of curvature of the path of the parcel of air. In deriving geostrophic wind $\frac{v^2}{r}$ is neglected in comparison to $2\omega v \sin\phi$ and hence the geostrophic wind equation is valid whenever $\frac{v^2}{r}/2\,\omega v \sin\phi$ or $v/2\,\omega r \sin\phi$ is small. It has been verified by various workers that the geostrophic wind is very highly valid in middle latitudes. Hence if the factor $v/2\omega r \sin\phi$ is of the same magnitude in Indian and middle latitudes, geostrophic wind should be theoretically as much valid in India as in middle latitudes. Let the suffixes 1 and 2 respectively represent conditions in middle latitudes and India. Then for our assumption to be valid

$$\frac{v_2}{2\omega r_2 \sin \phi_2} = \text{or} < \frac{v_1}{2\omega r_1 \sin \phi_1}$$

or $$\frac{v_2}{r_2 \sin\phi_2} = \text{or} < \frac{v_1}{r_1 \sin\phi_1}$$

Leaving aside tropical cyclones, there is no reason to think that on the average r_2 is less than r_1. Hence we regard $r_2 = r_1$. So the condition reduces to

$$\frac{v_2}{\sin \phi_2} = \text{or} < \frac{v_1}{\sin \phi_1}$$

It will be seen in the results presented here that comparison between calculated and observed values of ΔZ has also been carried out for some cases in Great Britain and the correlation coefficient is about 0.9 for these cases. The mean value of $\overline{v}/\sin \phi$ for the cases in Great Britain and India under study here are given in the table of results. It will be seen that $\overline{v}/\sin\phi$ is of the same order in Indian cases as in those from Great Britain. Hence it appears that the cyclostrophic component may be neglected in India with as much justification as in Great Britain. The Indian cases under study are limited to the north of 18°N latitude.

4. *Results*.

ΔZ_c has been computed and compared with observations for the following pairs of radio-sonde stations—(1) Delhi-Nagpur (2) Delhi-Allahabad (3) Allahabad-Calcutta (4) Karachi -Veraval (5) Veraval-Poona. Comparison is made separately for the four periods, December to March, April and May, June to September and October and November to bring out any seasonal peculiarities.

The pilot balloon stations used in computing ΔZ_c are shown in Fig. 2. Only such cases as when pilot balloon winds are available at all the pilot balloon stations between the two radio-sonde stations have been used for comparison.

In order to see the validity of the method adopted here, similar comparison has been made between (1) Stornoway and Aldergrove and (2) Aldergrove-Valentia, stations in Great Britain for one month January 1948. The location of these stations and pilot balloon stations made use of, are represented in Fig. 3. The pilot balloon winds

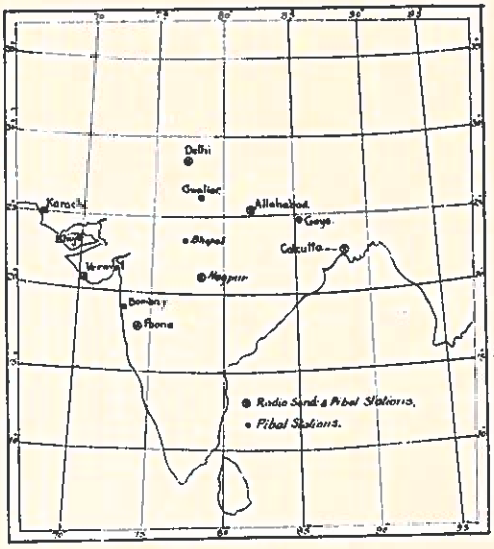

Fig. 2.

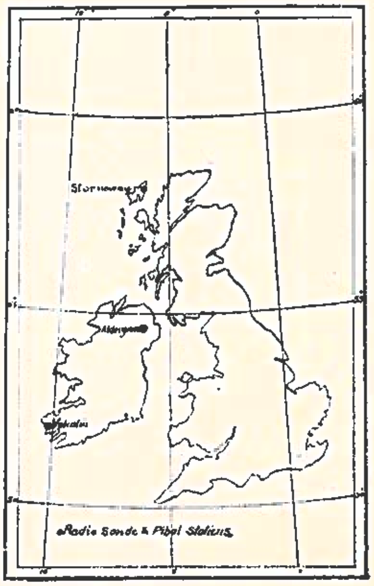

Fig. 3.

used are nearest in time to the time of radio-sonde ascents, if they are not synchronous with the radio-sonde ascents.

Correlation coefficients have been calculated between Δz_e and Δz_o in all cases and the results are given in Tables 1 to 7. The times of radio-sonde ascents and the times of pilot balloon ascents used are given in the tables. In the cases of Stornoway-Aldergrove, Aldergrove-Valentia and Delhi-Nagpur (May and June 1948) both the radio-sonde and pilot balloon ascents are of the same time and strictly comparable. In all other cases the radio-sonde ascents are made 3½ hours later than the pilot balloon ascents. It is assumed in all these cases that the pilot balloon data collected 3½ hours earlier are also valid at the times of radio-sonde ascents.

In the four cases from Great Britain studied, the correlation coefficients are between 0.86 and 0.93 which represent a very high correlation. The number of observations in each case are few only (24 to 26) but the uniformly high correlation leaves no doubt as to the relationship and the correctness of the method of comparison adopted here. The difference between the mean values of Δz_e and Δz_0 is also very small, the maximum difference being only 10%.

From the tables the following features will be noticed about the Indian data:—

(i) The correlation coefficients vary between the wide limits of 0.02 and 0.66, some cases in Table 2 giving even small negative coefficients.

(ii) Between the same two stations, the correlation coefficients vary widely during the different months of the year. The only exception is between Veraval and Poona where the correlation coefficients are consistent and of moderate value.

(iii) The pibal data are of ascents made 3½ hours earlier than the radio-sonde ascents. Opportunity was taken to compare Δz_e and Δz_o between Delhi and Nagpur for May—June 1948 when radio-sonde and pibal data were available for the same time, *viz.* 0700 hours I.S.T. It will be seen in Table 1 that this case gives a much lower correlation coefficient than the evening data.

(iv) Mean values of Δz_e (without reference to sign) are smaller for Indian data than for British data. This is to be expected on account of the weaker winds over the subtropics compared to the middle latitudes.

(v) In most of the cases for the Indian stations (except Delhi-Nagpur) the mean values and standard deviations (without taking sign) of Δz_o are very much greater than the mean values of Δz_e. Even with Poona-Veraval where the correlation coefficients are consistent and of moderate value, Δz_o is generally much greater than Δz_e which makes the significance of the correlation coefficient in this case doubtful. For example with an average of 117 observations in case of 700 mb surface mean Δz_o between Poona and Veraval is 88 feet while the Δz_e is only 25 feet. It is also seen that the standard deviations of Δz_o (taking into account sign) are mostly much more than that of Δz_e.

5. *Effect of errors of observation on the correlation coefficients.*

Both Δz_e and Δz_o must be subject to errors of measurement. Hence a discussion is given below how errors of measurement will affect the correlation coefficients.

Let x and y denote the departures from the mean of the true values (without errors of observation) of two quantities,

X and Y the departures from the mean of the actual observed values (including errors)

and a and b the errors of observation respectively, referred to above.

It may be assumed that the errors are not correlated either with values of observations or among themselves. For the sake of simplicity, the departures from the mean have been considered instead of the actual values.

Obviously $X = x+a$ .. (5)

and $Y = y+b$ .. (6)

Using the standard symbols r for the correlation coefficient and σ for the standard deviation

$$r = \frac{\Sigma XY}{n\sigma_X\sigma_Y} = \frac{\Sigma(x+a)\,(y+b)}{n\sigma_X\sigma^Y} \quad (7)$$

and $$r_{xy} = \frac{\Sigma xy}{n\sigma_x\sigma_y} \quad \ldots \quad \ldots \quad (8)$$

$$\therefore \frac{r_{XY}}{r_{xy}} = \frac{\sigma_x\sigma_y}{\Sigma xy}\cdot\frac{\Sigma(x+a)\,(y+b)}{\sigma_X\,\sigma_Y} \quad (9)$$

Now $\Sigma(x+a)(y+b)=\Sigma xy+\Sigma xb+\Sigma ya+\Sigma ab$

$$=\Sigma xy \quad \ldots \quad \ldots \quad (10)$$

since the errors are not correlated with the observations and among themselves and the three terms Σxb, Σya and Σab vanish.

Also
$$n\,\sigma_x^2 = \Sigma(x+a)^2 = \Sigma x^2 + \Sigma a^2 = n(\sigma_x^2+\sigma_a^2)$$

or
$$\sigma_x^2 = \sigma_X^2 - \sigma_a^2 \quad \ldots \quad (11)$$

Similarly
$$\sigma_y^2 = \sigma_Y^2 - \sigma_b^2 \quad \ldots \quad (12)$$

Substituting the above values in equation (9) we have

$$\frac{r_{XY}}{r_{xy}} = \frac{\sigma_x \cdot \sigma_y}{\sigma_X \cdot \sigma_Y}$$

$$= \frac{\sqrt{(\sigma_X^2-\sigma_a^2)}\cdot\sqrt{(\sigma_Y^2-\sigma_b^2)}}{\sigma_X \cdot \sigma_Y}$$

$$= \left\{1-\frac{\sigma_a^2}{\sigma_X^2}\right\}^{\frac{1}{2}} \cdot \left\{1-\frac{\sigma_b^2}{\sigma_Y^2}\right\}^{\frac{1}{2}} \quad (13)$$

6. *Explanation for the low correlation observed.*

The standard deviations of $\triangle Z_c$ and $\triangle Z_o$ (taking into account sign of the values) are given in the tables. These correspond respectively to σ_X and σ_Y. To estimate σ_a (corresponding to $\triangle Z_c$) computations of $\triangle Z_c$ round the closed circuit Delhi, Gwalior, Allahabad, Bareilly and Delhi have been made. $\triangle Z_c$ should be zero if the wind is geostrophic. Corresponding to 60 cases in November and December 1949, at the 5000 feet level for 0700 hours I.S.T. pibal ascents, the mean value of $\triangle Z_c$ is zero with a standard deviation of 16 feet. The deviation of $\triangle Z_c$ from zero round a closed circuit would be the effect of the deviation of the wind from geostrophic assumption and errors in measurement of pilot balloon winds. This standard deviation σ_a will increase with the length of the circuit. The closed circuit Delhi-Gwalior-Allahabad-Bareilly-Delhi is longer than any of the routes for which $\triangle Z_c$ and $\triangle Z_o$ are correlated. So 16 feet may be taken as the σ_a for $\triangle Z_c$.

If $\sigma_X = 2\sigma_a$, equation (13) becomes

$$\frac{r_{XY}}{r_{xy}} = (0.75)^{\frac{1}{2}}\left\{1-\frac{\sigma_b^2}{\sigma_Y^2}\right\}^{\frac{1}{2}}$$

$$=0.87\left\{1-\frac{\sigma_b^2}{\sigma_Y^2}\right\}^{\frac{1}{2}} \quad \ldots \quad (14)$$

Whenever the standard deviation of $\triangle Z_c$ is more than 32, we should expect r_{XY} to be ($0.87\,r_{xy}$) or more if there were no errors in $\triangle Z_o$. But it will be seen that there are many cases of very poor correlation even when the standard deviation of $\triangle Z_c$ is more than 32 feet.

Two alternatives are open to explain the poor correlation between $\triangle Z_o$ and $\triangle Z_c$.

(i) The error in measurement of $\triangle Z_o$ is large. The fact that the mean value of $\triangle Z_o$ and its standard deviation (neglecting the signs) are greater than the corresponding values of $\triangle Z_c$ lends support to this view. The degree of accuracy that is required in the heights of constant pressure surfaces can be derived from the mean values of $\triangle Z_c$ (neglecting sign). The lowest values of mean $\triangle Z_c$ for any pair of stations in any season out of the cases studied here is 15 feet for 850 mb surface. In order that the heights of constant pressure surfaces may be capable of quantitative interpretation, the probable error in height measurements must be less than the mean value of $\triangle Z_c$ indicated before. In the case of 850 mb surface, it is equivalent to an accuracy of 1°C in temperature measurements and still greater accuracy, in the case of 700 mb surface, assuming that errors are only in temperature measurements. Owing to the very much weaker winds in sub-tropics, the accuracy required in height measurements for constant pressure surface is much greater than in the middle latitudes.

(ii) Deviations from geostrophic wind in our latitudes are large. Geostrophic wind neglects the cyclostrophic and tangential acceleration. From a comparison of the values of $v/\sin\phi$, it was shown before that the cyclostrophic acceleration may be neglected here with as much justification as in higher latitudes. So the deviations must be chiefly due to the tangential acceleration.

7. *Acknowledgment.*

The work was undertaken at the suggestion of Sri Y.P. Rao, whose guidance, counsel and encouragement were invaluable. I wish to record here my most sincere thanks to him. I would also like to express my sincere thanks to Dr. S. Mull, who took lively interest in the work.

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TABLE 1.—(R.S* *DELHI-NAGPUR.*) (P.B § *Delhi-Gwalior-Bhopal-Nagpur*)

Period	Time of ascent R.S. IST	Time of ascent P.B. IST	No. of Obsns.	Mean value ΔZ_c	Mean value ΔZ_0	Standard Deviation ΔZ_c	Standard Deviation ΔZ_0	Correlation coefficient	Mean value (omitting signs) ΔZ_c	Mean value (omitting signs) ΔZ_0	Standard Deviation (omitting signs) ΔZ_c	Standard Deviation (omitting signs) ΔZ_0	$\bar{v}$ / $\sin\phi$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
(a) 850 *mb.*													
Dec-Mar 1947	1930	1600	105	−39	−11	35	57	0.29	44	43	29	39	21.4
Apr-May 1947	"	"	41	−73	−46	47	73	0.61	74	70	45	50	33.5
June-Sept. 1947	"	"	34	−69	−24	55	62	0.62	83	51	38	43	28.8
Oct-Nov. 1947	"	"	50	−9	+29	24	50	0.30	20	47	16	34	20.7
May-June 1948	0700	0700	31	−72	−32	39	67	0.22	71	62	40	41	37.1
(b) 700 *mb.*													
Dec-Mar. 1947	1930	1600	67	−117	−94	46	77	0.24	118	104	44	63	40.4
Apr. & May 1947	"	"	14	−97	−61	55	100	0.42	97	100	54	61	39.2
Oct. & Nov. 1947	"	"	35	−32	−14	44	75	0.31	43	58	32	49	27.1

TABLE 2. (R S: *DELHI-ALLAHABAD*) (P.B: *Delhi-Gwalior-Allahabad*)

Period	Time of ascent R.S. IST	Time of ascent P.B. IST	No. of Obsns.	Mean value ΔZ_c	Mean value ΔZ_0	Standard Deviation ΔZ_c	Standard Deviation ΔZ_0	Correlation coefficient	Mean value (omitting signs) ΔZ_c	Mean value (omitting signs) ΔZ_0	Standard Deviation (omitting signs) ΔZ_c	Standard Deviation (omitting signs) ΔZ_0	$\bar{v}$ / $\sin\phi$
(a) 850 *mb.*													
Dec. 1945–Mar. 1946	1930	1600	91	−14	+12	29	59	0.11	24	47	21	38	27.1
Apr.-May 1946	"	"	23	−3	−26	36	119	−0.07	27	62	24	105	24.0
June-Sept. 1946	"	"	31	+14	+6	27	62	0.57	26	51	17	35	26.2
Oct. 1946	"	"	17	+7	−2	17	92	−0.19	17	71	8	58	21.6
(b) 700 *mb.*													
Dec. 1945–Mar. 1946	1930	1600	68	−48	−10	46	114	0.06	55	84	38	78	46.7

TABLE 3. (R.S: *ALLAHABAD-CALCUTTA*) (P.B: *Allahabad-Asansol-Gaya-Calcutta*)

Period	Time of ascent R.S. IST	Time of ascent P.B. IST	No. of Obsns.	Mean value ΔZ_c	Mean value ΔZ_0	Standard Deviation ΔZ_c	Standard Deviation ΔZ_0	Correlation coefficient	Mean value (omitting signs) ΔZ_c	Mean value (omitting signs) ΔZ_0	Standard Deviation (omitting signs) ΔZ_c	Standard Deviation (omitting signs) ΔZ_0	$\bar{v}$ / $\sin\phi$
(a) 850 *mb.*													
Jan-Mar. 1946	1930	1600	49	+1	−52	35	118	0.66	28	103	21	68	33.9
Apr. & May 1946	"	"	17	+10	−120	17	61	0.06	15	121	13	61	24.3
(b) 700 *mb.*													
Jan-Mar. 1946	1930	1600	37	−15	−58	47	184	0.50	36	139	104	134	57.8

* R.S.=Radio Sonde Stations.

§ P.B.=Pilot Balloon Stations.

Peorid	Times of ascent R.S. IST	Times of ascent P.B. IST	No. of Obsns.	Mean value ΔZ_c	Mean value ΔZ_o	Standard Deviation ΔZ_c	Standard Deviation ΔZ_o	Correlation coefficient	Mean value (omitting signs) ΔZ_c	Mean value (omitting signs) ΔZ_o	Standard Deviation (omitting signs) ΔZ_c	Standard Deviation (omitting signs) ΔZ_o	$\bar{v}$ / $\sin\phi$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
TABLE 4. (R.S: *KARACHI-VERAVAL*) (P.B: *Karachi-Bhuj-Veraval*)													
(a) 850 *mb.*													
Feb-Mar. 1946	1930	1600	53	−16	+83	33	80	0.32	27	87	25	75	25.2
Apr.-May 1946	,,	,,	31	−40	+55	32	67	0.02	42	70	30	50	35.7
(b) 700 *mb.*													
Feb.-Mar. 1946	1930	1600	47	−51	+25	56	97	0.39	58	84	49	54	41.9
Apr. -May 1946	,,	,,	23	−5	+133	25	83	0.15	21	136	15	78	26.5
TABLE 5. (R.S: *VERAVAL-POONA*) (P.B: *Veraval-Bombay-Poona*)													
(a) 850 *mb.*													
Dec. 1945–Mar. 1946	1930	1600	118	+2	−42	19	49	0.45	16	49	10	42	21.9
Apr.-May 1946	,,	,,	44	−5	−27	27	36	0.61	22	35	17	28	35.6
June-Sept. 1946	,,	,,	21	−25	−74	22	132	0.52	27	74	20	132	45.7
Oct. 1946 & Nov. 1945	,,	,,	51	+24	−19	17	34	0.49	25	30	15	24	27.0
(b) 700 *mb.*													
Dec. 1945–Mar. 1946.	1930	1600	117	−9	−76	33	77	0.43	25	88	23	64	34.1
Apr. & May 1946	,,	,,	31	−4	−58	27	56	0.56	23	67	16	44	31.7
Oct. 1946 & Nov.1 945	,,	,,	42	+24	−59	23	62	0.32	28	70	17	50	30.5
TABLE 6. (R.S: *STORNOWAY-ALDERGROVE*) (P.B: *Stornoway-Aldergrove*)													
(a) 800 *mb.**													
Jan. 1948	1500 GMT	1500 GMT	26	−136	−128	135	159	0.91	155	170	111	113	32.9
(b) 700 *mb.*													
Jan. 1948	1500 GMT	1500 GMT	26	−155	−138	153	161	0.93	178	173	126	122	34.3
TABLE 7. (R.S: *ALDERGROVE-VALENTIA*) (P.B: *Aldergrove-Valentia*)													
(a) 800 *mb.**													
Jan. 1948	1500 GMT	1500 GMT	26	−124	−100	151	165	0.90	165	163	105	104	37.9
(b) 700 *mb.*													
Jan. 1948	1500 GMT	1500 GMT	24	−156	−114	163	169	0.88	193	176	116	103	43.0

*Observed winds at 6000′ were made use of for calculating values of ΔZ_c.