551·577 : 551·509 (545)

Some further results of investigations on Quantitative Precipitation Forecasting over selected area in north India

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(*Received 1 October 1965*)

ABSTRACT. The present paper gives results on Quantitative Precipitation Forecasting involving application of the forecasting model to each of the days of two monsoon months, *viz.*, July and September 1962. Further, in order to simplify these computations and reduce the computational time, partial divergence tables have been prepared for the triangular grid comprising of Allahabad, Calcutta and Gauhati, which covers the area under investigation. In addition a nomogram has been prepared to facilitate further reduction in computational time in estimating precipitation rates from a knowledge of vertical velocities and moisture content in the vertical. The computation of rainfall made at intervals of twelve-hour period for the two months has then been compared with the actuals which have been evaluated as the arithmetic mean of rainfall recorded by a representative number of raingauges within this grid.

The results of this comparison show that the computed amounts are in general, an over estimate over the observed amounts. Although it has not been possible to assess the reasons for such variation, to be of any theoretical importance, the computed values have shown that they account for almost all the actual heavy rainfall situations, this in itself forming an interesting result of the investigations.

1. Introduction

Most of the workers on objective Quantitative Precipitation Forecasting (Q.P.F.) have taken into consideration horizontal convergence, vertical velocity and distribution of humidity mixing ratio (in the vertical) for calculating precipitation in some form or other. Various methods for computing vertical motion are also given by Panofsky (1951). Most of these methods evaluate vertical velocity based on an application of equation of continuity, where it would be necessary first to obtain divergences, calculated either by grid method or triangular method (Bellamy 1949). The Laplacian method given by Petterssen (1956) and later applied by Estoque (1957) in his two-level prediction model, and the advection methods were notable, in this context.

In the present paper, vertical velocities have been computed by the use of equation of continuity which involved computing of divergences. Divergences have been computed by making use of Bellamy's (1949) triangular technique. Since the area of investigation remained the same, the authors have simplified this technique by preparing partial divergence tables for unit wind speed and for all direction ranging from 0°—360°; the map scale taken being 1 : 35000. This reduced computational time considerably. In the same way a nomogram has also been presented, which reads off precipitation rates, against vertical velocities and mixing ratio, which further reduces time of computation.

2. Computations

(*a*) *Preparation of Partial Divergence Tables*

The method of computation of the partial divergence tables is based on Bellamy's method which gives the horizontal divergence directly from wind observations without recourse to streamline analysis of the wind field. Assuming the wind field between the three observational points A, B, C vertices of a triangle to be a linear function of space between these points, Bellamy has suggested that the horizontal divergence $\triangle$ at the centroid of the triangular volume ABC of unit height is equal to the sum of partial divergences $\triangle_A$, $\triangle_B$, $\triangle_C$ due to individual winds at A, B, C respectively.

$$\text{or } \triangle = \triangle_A + \triangle_B + \triangle_C \quad (1)$$

and further defining the partial divergence as the rate of outflow through unit vertical sides of the volume, per unit volume, he has derived —

$$\triangle_A = n_A / h_A \quad (2)$$

where, as in Fig. 1,

h_A = perpendicular distance AD from vertex A on the opposite side BC, a constant for this grid,

n_A = component of wind vector, along the line DA, and perpendicular to BC.

Taking AN as the direction of north, n_A can be calculated by the formula

$$n_A = v_A \cos(\theta_A - \alpha_A) \quad (3)$$

where, v_A is the wind speed at A, θ_A is the angle which the wind vector makes with true north and α_A is the angle (of azimuth) of the perpendicular line DA as measured from north in clockwise direction. This is fixed for a fixed point of a fixed grid.

With the help of Eq. (3), Eq. (2) becomes —

$$\triangle_A = v_A \cos(\theta_A - \alpha_A)/h_A \times 3600 \quad (4)$$

In the grid ACG considered in this paper where

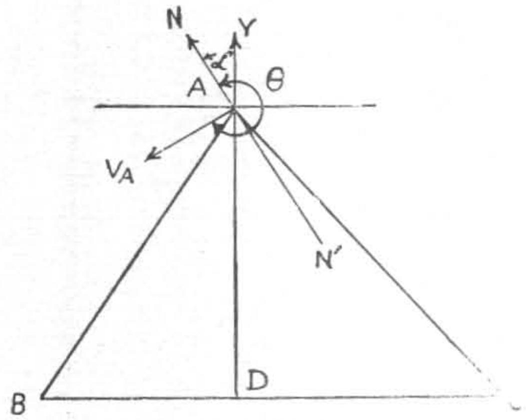

Fig. 1

A stands for Allahabad (Lat. 25° 27′N, Long. 81° 44′E), C for Calcutta (Lat. 22° 32′N, Long. 88° 20′E) and G for Gauhati (Lat. 26° 11′ N, Long. 91° 45′ E) the values of these constants h and α are as follows —

$h_A = 382{\cdot}6$ n. miles, $\alpha_A = 307°$

$h_C = 207{\cdot}3$ n. miles, $\alpha_C = 176°$

$h_G = 277{\cdot}2$ n. miles, $\alpha_G = 29°$

Making use of these constants in Eq. (4), computations have been made for unit value of v and θ from 0° to 360° at 10° interval and presented in a convenient tabular form for quick reference in computation. The divergence values are computed in units of 10^{-5} sec^{-1} . Table 1 gives the values of the partial divergence $\triangle_A$, $\triangle_C$, $\triangle_G$, corresponding to the wind at the stations Allahabad, Calcutta and Gauhati. The algebraic sum of these for given values of wind speed and direction, gives the horizontal divergence valid at the centroid of the total triangular volume ACG of unit height. The dimensions of the triangle ACG were taken from the standard map of Survey of India to scale 1″ = 40 miles (64 km nearly).

(*b*) *Preparation of nomogram for calculation of rates of precipitation*

The nomogram (Fig. 2) for evaluation of precipitation is based on the formula which Showalter has derived on the assumption that the rate of precipitation from a layer of atmosphere is equal to the difference between the inflow of moisture into the layer and outflow of moisture from the same layer.

The formulae used are —

$$I_0 = Vz_0\ \rho_0\ (x_0 - x_1)/7 \qquad (5)$$

and $I_1 = Vz_1\ \rho_1\ (x_1 - x_2)/7$ and so on

and $I = I_0 + I_1 + I_2 + \ \ldots\ldots\ldots$

where I_0, I_1, I_2, I_3 are the rates of precipitation in inch/hr, ρ_0, ρ_1, ρ_2 etc are the corresponding densities in gm per m^3 and x_0, x_1, x_2 .... are corresponding mixing ratios in gm per kg. Vz_0, Vz_1, Vz_2 are the vertical velocities in m/sec in between the corresponding individual layers represented by subscripts 0, 1, 2 etc. The vertical velocities are calculated by considering divergences within the layers expressed by the formulae —

$$Vz_1 = \rho_0/\rho_1\ Vz_0 - \tfrac{1}{2}\ (\rho_0/\rho_1\ D_0 + D_1)\ \triangle_z \qquad (6)$$

and $Vz_2 = \rho_1/\rho_2\ Vz_1 - \tfrac{1}{2}\ (\rho_1/\rho_2\ D_1 + D_2)\ \triangle_z$

and so on where D_0, D_1, D_2 etc are the divergences at different levels and $\triangle z$ is the thickness of the corresponding layer.

To construct a nomogram of the type presented, it was essential to first obtain a linear relation between I_0, V_0, ρ_0 and $(x_0 - x_1)$. This was done by taking logarithms of both sides of Eq. (5). For instance, considering a layer with subscript 0,

$$\log I_0 = \log Vz_0 + \log \rho_0 + \log (x_0 - x_1) - -\log 7 \qquad (7)$$

I_0 being dependent on three variables suggest a correlation diagram with two coaxial system of axis is evolved so that the resultant of one system is considered as one of the variables in the other system of axes. The first system consisted of values of $\log (x_0 - x_1)$ and $\log \rho_0$ as the axes. The point of intersection obtained in this system of axes is then taken on to the next system of axes made up of vertical velocity and points of intersection of the first system.

To obtain the rates of precipitation the points of intersection of the second system of axes are first found out. This point of intersection is then read off against the equal rate of precipitation lines which are drawn inclined in the second system of axes (*see* Fig. 2).

Method of use of Nomogram

The following three steps have to be followed to read off the value of rate of precipitation —

1. Read the H.M.R. difference value in the first system of axes and see where it intersects the corresponding densities.
2. From this point of intersection go along the parallel contours numbered 1, 2, 3 etc on to the second system of axes.
3. The point meeting the corresponding vertical velocity, reads the value of the precipitation rate as represented by the second set of parallel straight contours, inclined to the second system of axes.

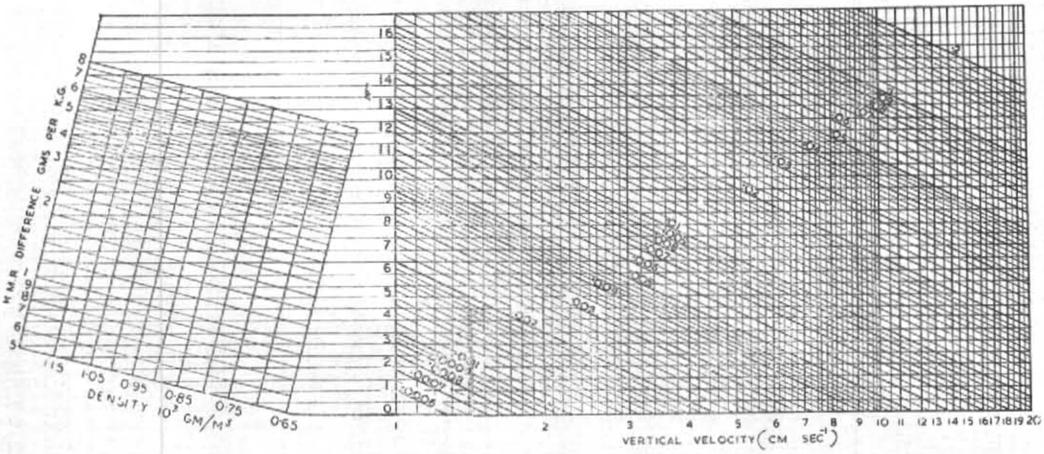

Fig. 2. Nomogram for precipitation in inches per hour

(c) Computed and Observed Rainfall

The day-to-day computed precipitation have been shown in Table 2. The table shows also the actual observed rates of precipitation, which have been obtained by taking the arithmetic mean of the rainfall at stations surrounding the centroid covering an area of about 9000 sq. miles (mentioned as centroid in the table). Since the observed 24-hr amounts are those recorded at 0830 IST (0300 GMT) while the forecast amounts are valid from 0530 IST (0000 GMT) to 0530 IST some discrepancy between the computed and observed rainfall on this account is likely to arise, one of the reasons being the monsoon rainfall predominantly occur in the morning. Table 3 shows a sample computation showing the various steps in which computations are done. According to these a forecast of quantitative precipitation valid for 24 hours ending at 0530 IST on any day is carried out in 2 steps — (*i*) from 0530 to 1730 IST of previous day and (*ii*) from 1730 IST of previous day to 0530 IST of date. This step is introduced to take into account the trend of changes. The rainfall recorded during past 24 hours at 0830 IST of the day is then compared with the computed rainfall.

3. Discussion of Results

(a) General

Precipitation rate has been computed for a continuous period of about three months July to September 1962. In cases of moderate to heavy rainfall situations, the vertical velocities attained in the present computations are of the order of 15 cm $\sec^{-1}$, at a height of 6 km. The exception to the rule of vertical velocity being directly proportional to rainfall amounts have been found to be in those cases where moisture is very low throughout the vertical column and consequently less precipitation even if strong vertical velocities are predicted. It would be seen from results that the computed values are generally an over-estimate over the observed values. There are of course some exceptions here also, possibly for following reasons. Some situations which gave good rate or ascent of air, but produced very little 'observed' rain on the one hand, and some situations where observed rainfall was considerable but computed vertical velocities are quite low. On the other hand due to the fact that an air mass might have advected into a region of convergence from a region of subsidence and *vice versa*, thus retaining its previous characteristics for some period, causing rainfall in subsidence area due to air mass advection, or giving no rain in convergence area for a part of the forecast period. Further, the moisture inflow rate and outflow rate are also not precisely known, as well as the retention of droplets inside the clouds. These factors preclude one from giving a completely satisfactory explanation for the discrepancies between computed and observed precipitation rates.

(b) Some typical cases

Nevertheless the computations made so far reveal some cases where, a physical explanation is possible. An analysis of two such cases is presented here. The computed rainfall is much overestimated on 17 and 18 September 1962. It was found that this is due to the predominent effect of winds at Calcutta over the combined effect of the winds at Allahabad and Gauhati (being the other two points of the triangle) as regards the computation of total convergence. A trough of low pressure has formed near west central Bay of Bengal outside the triangle (near Calcutta), but the cyclonic circulation of this low pressure area has given rise to convergence at all levels at Calcutta,

TABLE 1

Partial Divergence Table (10^{-5} sec^{-1})

Valid at centroid of grid (Allahabad, Calcutta, Gauhati)

Degrees*	360 180	10 190	20 200	30 210	40 220	50 230	60 240	70 250	80 260
	Station value of divergence for 1 knot								
Allahabad	±·0437	±·0330	±·0212	±·0089	±·0038	±·0163	±·0284	±·0395	±·0495
Calcutta	±·1337	±·1300	±·1224	±·1111	±·0964	±·0788	±·0587	±·0369	±·0140
Gauhati	±·0876	±·0947	±·0990	±·1002	±·0984	±·0935	±·0859	±·0756	±·0631
Degrees*	90 270	100 280	110 290	120 300	130 310	140 320	150 330	160 340	170 350
Allahabad	±·0580	±·0647	±·0694	±·0721	±·0725	±·0707	±·0668	±·0609	±·0531
Calcutta	±·0094	±·0324	±·0545	±·0749	±·0931	±·1084	±·1204	±·1288	±·1333
Gauhati	±·0486	±·0326	±·0157	±·0018	±·0191	±·0359	±·0516	±·0657	±·0779

* $\xi+180=\theta$, where ξ is the direction of wind reported and θ is as shown in Fig. 1

Note — The table can be used for any wind speed by multiplying the wind speed value with partial divergence value, for the corresponding direction given in ddff

TABLE 2

Day-to-day computed and observed rainfall (mm) for the months of July, August and September 1962

Date	July (a)	July (b)	August (a)	August (b)	September (a)	September (b)	Date	July (a)	July (b)	August (a)	August (b)	September (a)	September (b)
1	*		6·1	29·0	2·8	4·1	16	*	27·4	23·0	5·0	9·4	2·4
2	Nil	18·2	35·0	36·0	2·3	3·7	17	*	29·4	16·0	15·0	34·5	4·5
3	0·6	0·6	53·0	23·0	Nil	0·9	18	8·5	13·9	22·3	21·3	*	1·8
4	7·9	1·5	49·0	10·0	Nil	Nil	19	Nil	3·0	25·0	24·4	*	*
5	14·3	2·4	8·0	11·0	2·1	11·0	20	1·4	3·4	20·1	22·7	Nil	10·1
6	0·4	3·8	21·0	6·0	Nil	24·9	21	15·9	6·5	9·6	22·1	28·6	38·7
7	*	0·1	7·0	5·0	3·0	9·9	22	7·9	11·9	*	*	61·7	52·5
8	*	0·2	2·0	4·0	4·2	2·9	23	Nil	6·8	*	*	35·7	2·7
9	1·0	Nil	Nil	Nil	18·0	2·7	24	4·3	20·7	*	*	6·2	1·5
10	8·3	0·2	5·0	1·0	Nil	1·2	25	41·2	21·7	*	*	6·6	1·2
11	33·6	0·7	14·0	6·0	Nil	Nil	26	15·0	10·1	*	*	Nil	Nil
12	3·0	0·1	7·0	6·0	Nil	0·9	27	3·7	7·2	*	*	Nil	Nil
13	*	0·2	6·0	10·0	0·4	Nil	28	28·2	17·6	*	*	Nil	Nil
14	*	1·6	4·0	13·0	1·5	0·3	29	0·6	13·9	14·0	10·8	Nil	Nil
15	12·7	15·0	6·0	6·0	1·1	3·6	30	1·8	2·6	18·9	9·6	Nil	Nil

*Data not available Day-to-day vertical velocities not shown in the table

(a) Computed 24 hours rainfall (b) Average observed near centroid covering 9000 sq. miles

whereas Allahabad and Gauhati are unaffected and showed divergences (Fig. 3) at all levels. However, the integration of divergences at these three places have given rise to negative value of divergence at all levels due to very large contribution of negative divergence at Calcutta, thereby predicting large vertical motion and consequent computed precipitation. Th's would explain the large disparity between computed and observed precipitation on 17 and 18 September. The above analysis also indicates that certain weightage has to be given to such cases where one station's divergence values have over-riding influence on the divergence value of other two stations in the

TABLE 3

Sample computation of precipitation rate — 23 September 1962

Level (km)		Upper air data A (ddff)	Upper air data C (ddff)	Upper air data G (ddff)	Partial divergence values ×10⁻⁵ sec⁻¹ A	C	G
Surf.	Morning	0208	1401	2904	—·170	—·108	+·063
	Evening	3210	1803	2702	—·707	—·401	+·097
0·3	Morning	3618	1910	0205	—·787	—1·300	—·495
	Evening	3424	1711	2103	—1·462	—1·466	+·301
0·6	Morning	0226	2114	0106	—·551	—1·555	—·568
	Evening	3624	1812	1903	—1·049	—1·604	+·284
0·9	Morning	3626	2122	0303	—1·136	—2·444	—·301
	Evening	0124	2011	2003	—·792	—1·346	+·297
1·5	Morning	3530	2122	2703	—1·593	—2·444	+·146
	Evening	0326	2015	2704	—·231	—1·836	+·194
2·1	Morning	3427	2128	3602	—1·644	—3·111	—·174
	Evening	0227	1916	2406	—·572	—2·080	+·515
3·0	Morning	3430	2129	1902	—1·827	—3·222	+·189
	Evening	0329	2117	2110	—·258	—1·889	+1·002
4·5	Morning	0226	1824	1612	—·551	—3·209	+·788
	Evening	0220	2212	2208	—·424	—1·157	+·787
5·4	Morning	0217	2027	1713	—·360	—3·305	+1·013
	Evening	0225	1914	1605	—·530	—1·820	+·329

TABLE 3 (*contd*)

Level (km)		Total divergence	Mean divergence	*Computed effective vertical velocity (m/sec)	Density (gm/m³)	Humidity mixing ratio difference (gm/kg)	Computed precipi-tation (inch/hr)
Surf.	Morning	—·215	—·613	—	—	—	Nil
	Evening	—1·011					
0·3	Morning	—2·582	—2·605	—ve	—	—	Nil
	Evening	—2·627					
0·6	Morning	—2·674	—2·521	·0046	1109	·0012	·0009
	Evening	—2·369					
0·9	Morning	—3·881	—2·861	·0107	1076	·0018	·0030
	Evening	—1·841					
1·5	Morning	—3·891	—2·882	·0304	1015	·0018	·0079
	Evening	—1·873					
2·1	Morning	—4·930	—3·533	·0528	956	·0019	·0137
	Evening	—2·137					
3·0	Morning	—4·860	—3·003	·0893	875	·0027	·0301
	Evening	—1·145					
4·5	Morning	—2·972	—1·883	·1457	755	·0009	·0141
	Evening	—·794					
5·4	Morning	—2·652	—2·337	·1815	691	·0008	·0143
	Evening	—2·021					
							Total ·0840 in./hr (2·1336 mm/hr)

A stands for Allahabad, C for Calcutta and G for Gauhati

*Due to Thompson and Collins 1953; figures correct to fourth decimal place

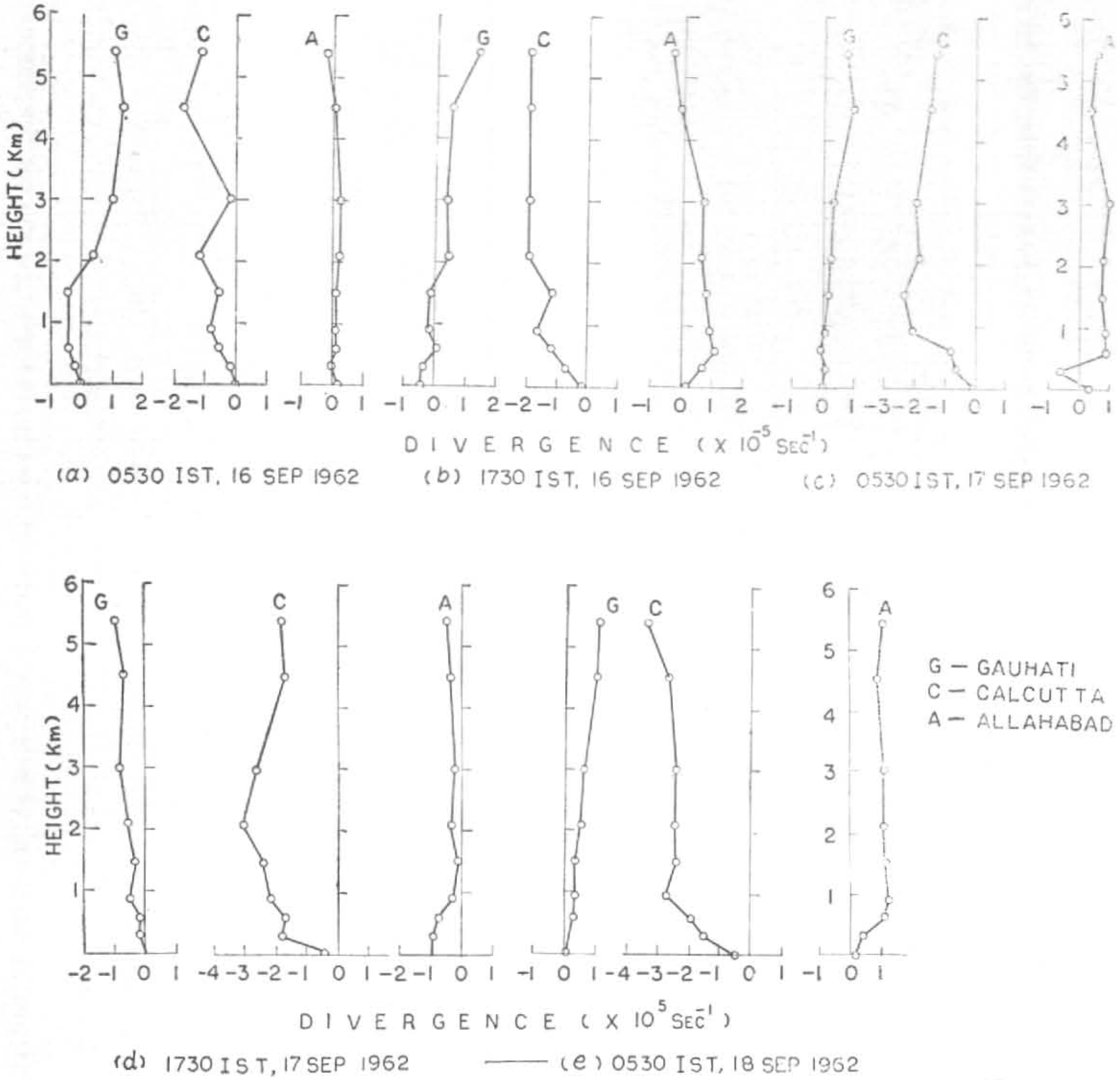

Fig. 3. Case 1 : Computed precipitation was an overestimate of observed rainfall inside the grid

matter of producing total divergence or convergence. Due regard to the position of the depression with respect to the triangle thus becomes inescapable. An examination of case of rainfall on 22–23 September (Figs. 4 and 5) is also made. It shows a very good agreement with observed rainfall, and even with the twelve-hourly computations. In this case, the trough has formed outside the triangle on 21st evening (Fig. 5 c) and is approaching the centroid of the grid from the right of Calcutta. Computation for the 21st morning shows divergence at all levels (Fig. 4 a) but 21st evening 1730 IST computation (Fig. 4 b) shows a completely different picture, *i.e.*, convergence at all levels, which has intensified by 22nd morning as can be seen from Fig. 4 c. By 22nd morning the trough moved well inside the grid, and is near the centroid of the triangle (Fig. 5 d). At this position all the three stations are equally effected in the matter of producing the desired convergence. It is thus evident that validity of the forecast value for a particular area inside the grid can be dependent upon the relative position of the trough. This may, further, lead to a generalisation, *viz.*, that when the depression is somewhere inside the triangle, the forecast values will be in good agreement with the computed ones, whereas when the depression is situated outside the triangle affecting only one of the three stations, the result is an overestimate as compared to actual values. A reasonably correct appreciation of the synoptic situation will apparently increase the accuracy of present method.

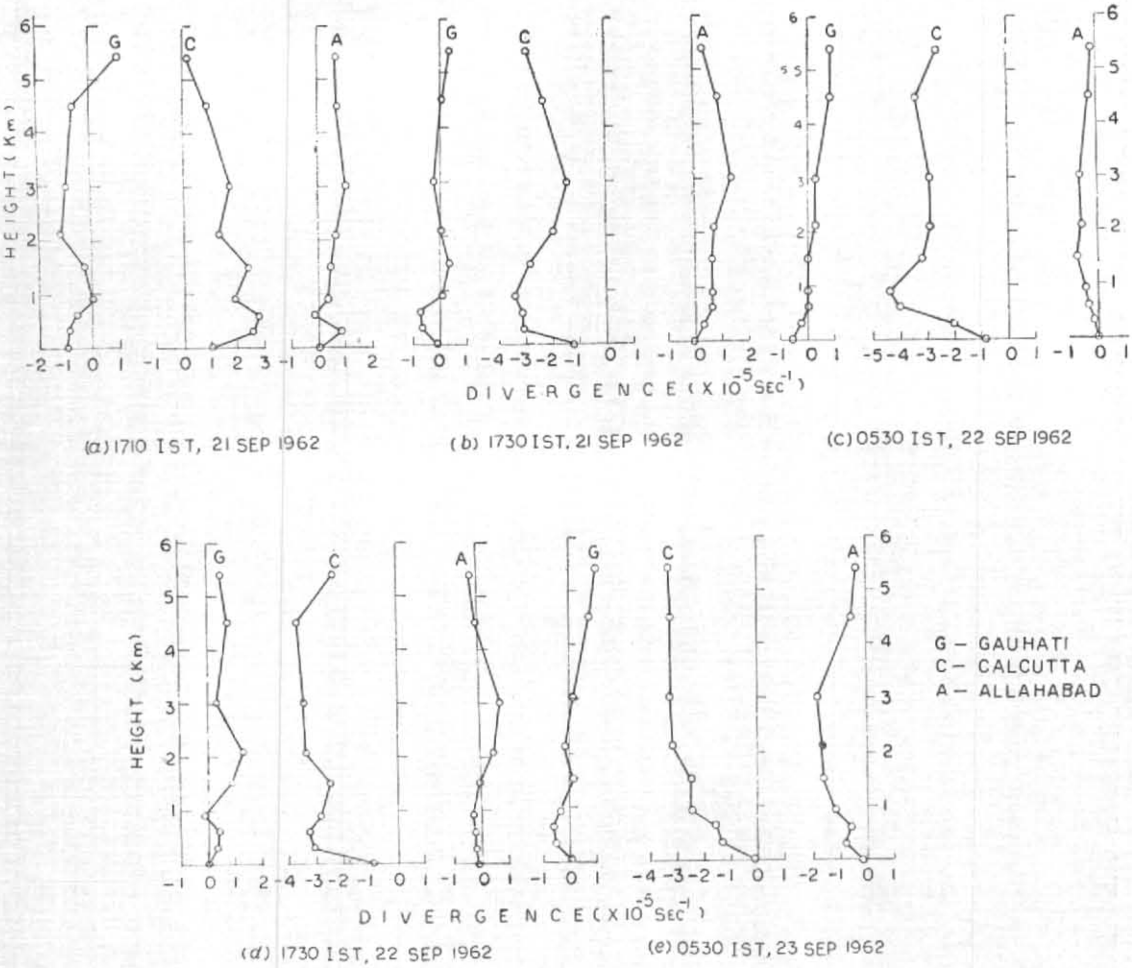

Fig. 4. Case 2 : Computed precipitation was in close agreement with observed precipitation inside the grid

(c) *Layer-wise contribution to total rainfall*

The percentage of total precipitation contributed by each layer has been calculated, based on the computations for the months of July to September. These confirm that the layer 2·1—3·0 km contributes maximum to the total precipitation and its average contribution is about 35 per cent of the total. Fig. 6 shows the percentage contribution made by different layers to the total rainfall. It will be observed that below 900 mb very little contribution is made to the total precipitation. It would appear that even in tropical regions the layer of maximum contribution remains almost the same as that reported by some authors (Myers and Lott 1963), in higher latitudes. This is inspite of the fact, that precipitation in tropics occurs from relatively warmer clouds than in higher latitudes.

4. Conclusions

Results of further study in Quantitative Precipitation Forecasting based on the model for computing precipitation rates developed by some of the authors have led to the following conclusions—

1. The investigations confirm that the layer 2·1 to 3·0 km gives maximum contribution, about 35 per cent of total precipitation.
2. The position of depressions with respect to the grid determines the accuracy of the Q.P.F. values calculated by the said model. When the depression is within or near the centroid of the grid, the forecast amounts were in close agreement with the actuals.
3. The vertical velocities computed in some cases of moderate to heavy rainfall were of

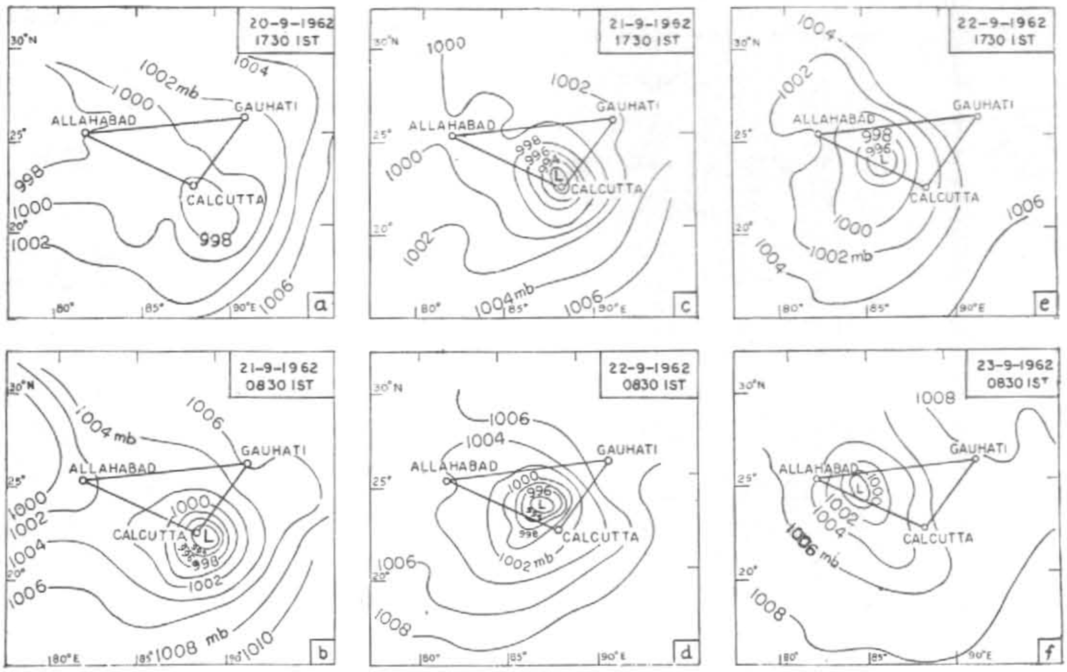

Fig. 5. Sea level pressure distribution

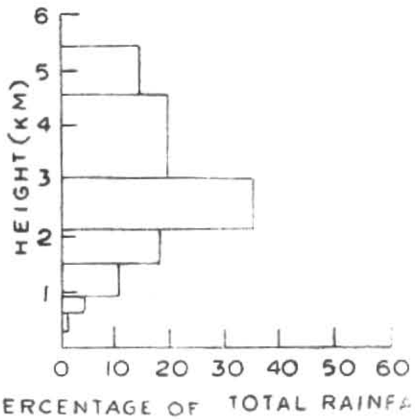

Fig. 6. Layerwise percentage contribution to total precipitation

(Based on computation of July and September 1962)

the order of 15 cm sec^{-1} at a height of 6 km.

It has become apparent from this study that for attaining greater accuracy in application of this model, work on proper grid size as well as an objective prognostication of the pressure systems which have far reaching effects on the computation is necessary. Preliminary study in this direction has yielded interesting results which are expected to be given out in another paper.

Partial divergence table for the particular grid used to test the model and nomogram for computing precipitation rates have been constructed to reduce computational time and make the model operationally useful.

5. Acknowledgement

The authors are grateful to the Director General of Observatories for his interest in the present study. The authors are also thankful to Shri R. S. Reen for computational work.

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