551.551 : 533.6.63

Calculation of the Coefficient of Aerodynamic Resistance of pilot balloons from the empirical formula used for computing their rate of ascent

K. L. SINHA

Meteorological Office, Jodhpur

(*Received 30 October 1956*)

1. Introduction

For pilot balloon flights at night or early morning, when convective air currents are generally absent, the empirical formula, $V = C.F^{\frac{1}{2}} / (W+F)^{\frac{1}{3}}$, where V is the uniform rate of ascent, F the free lift weight, W the weight of the balloon and C is the constant, is daily used by the India Meteorological Department for computation of the uniform rate of ascent of the balloon. When the balloon moves up, it experiences resistance of the air and the vertical velocity of balloon depends upon the coefficient of the aerodynamic resistance. It is intended to work out here the value of the coefficient of the aerodynamic resistance, as obtainable from the application of the above empirical formula, used for the pilot balloons.

2. Aerodynamic resistance

The resistance offered to a body moving through air is expressible in the following form (Lamb 1932, Shaw 1942)—

$$R = \rho L^2 V^2 f(LV/\nu) \tag{1}$$

where ρ is the air density, L the linear dimension of the body, V the speed of the moving body, ν the coefficient of kinematic viscosity and f a function of the Reynolds number LV/ν, and $f(LV/\nu)$ the coefficient of aerodynamic resistance generally denoted by k. In the case of a spherical body, L is equal to d, the diameter of the sphere. Thus, the resistance R in the case of a spherical balloon of diameter d is representable as—

$$R = \rho d^2 V^2 f(dV/\nu) = k \rho d^2 V^2 \tag{2}$$

3. Coefficient of aerodynamic resistance at different Reynolds Numbers

Saha (1956) determined the coefficient of the aerodynamic resistance at different Reynolds numbers by using tethered balloons of different sizes. Cave and Dines (1919), Brazier (1921), Horiguti (1923) and others measured the effect of air resistance on the rate of ascent of the gas filled pilot balloons rising freely in still or open air. The curves showing the coefficient of aerodynamic resistance at different Reynolds numbers obtained for tethered balloons and pilot balloons are reproduced in Fig. 1.

The coefficient of the aerodynamic resistance decreases with increasing Reynolds numbers upto a certain limit. It remains constant after certain Reynolds number which was found to be 3.0×10^5 in the case of the curve obtained from data of pilot balloon flights, and $4 \cdot 0 \times 10^5$ in the case of the curve obtained from the data of tethered balloons. The values of k indicated by the curve from the pilot balloons are generally higher than those from the tethered balloons.

4. Resistance when the balloon uniformly rises up

When a gas-filled balloon rises in the air, the force acting on it upwards is the freelift of the balloon and the force acting downwards is the resistance offered by the air to the rising balloon. Just at the initial stage of the release of the balloon, the upward velocity of the balloon is small and thus the corresponding resistance of the air is also small. The upward velocity of the balloon increases and so

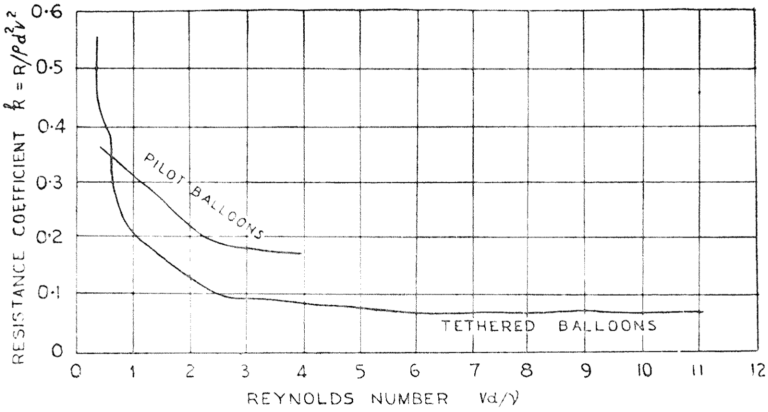

Fig. 1. Coefficient of aerodynamic resistance at different Reynolds numbers

also the air resistance till, within a short while, the resistance becomes equal to the free lift force. The balloon then moves up with more or less uniform velocity and the following relation holds —

$$R = Fg \qquad (3)$$

where R is the resistance, F the free lift weight and g the acceleration due to gravity.

5. Expression for coefficient of aerodynamic resistance in terms of free lift

Substituting the value of R from equation (3) in equation (2) we have,

$$\rho\, d^2 V^2 f(dV/\nu) = Fg \qquad (4)$$

The coefficient of aerodynamic resistance k, which is $f\ (dV/\nu)$, is, therefore, given by—

$$k \text{ or } f(dV/\nu) = Fg/\rho\, d^2 V^2 \qquad (5)$$

Thus, if corresponding to the free life F, the value of the uniform vertical velocity of the balloon is known and the diameter of the inflated balloon is measured and the density of the air is worked out, the coefficient of the aerodynamic resistance can be found out from equation (5).

6. Evaluation of V from empirical formula

As indicated earlier, in the case of small balloons used for pilot balloon observations, V can be calculated from the empirical formula

$$V = C\ F^{\frac{1}{2}}/(W+F)^{\frac{1}{3}} \qquad (6)$$

where V is the uniform rate of vertical ascent or vertical velocity, F the free lift weight and W the weight of the balloon and C is a constant.

TABLE 1

Value of C calculated from data of departmental free lift table

Weight of balloon (gm)	Free lift (gm)	Rate of ascent (km/hr)	Value of C from Eq.(6)
24	95	10	5·0
75	142	10	5·0
10	78	10	5·0
50	57	8	5·0
33	47	8	5·0

7. Value of C in the case of pilot balloon

The value of C may be found out from the data available in the free lift tables supplied by the India Meteorological Department for pilot balloon ascents. Some data to work out C are given in Table 1.

Table 1 shows that when weight of the balloon and free lift are expressed in grammes and rate of ascent in km/hr, the value of C in the case of the pilot balloon comes to 5·0.

8. Applicability of the formula at a particular Reynolds number

It is seen from earlier consideration that the uniform vertical velocity is attained when the resistance corresponding to the vertical velocity is equal to the free lift. In other words, the vertical velocity of the balloon depends upon the resistance and hence the coefficient of the resistance. The coefficient of aerodynamic resistance again varies with the Reynolds number when the Reynolds number is less than 3 or 4×10^5, as seen from the curves in Fig. 1. Hence the vertical velocity will also depend upon the Reynolds number. Therefore, any formula giving V, which should be valid for different Reynolds number, should contain terms varying with the Reynolds number, otherwise, it will be valid only for a particular Reynolds number. As the empirical formula as such does not include such a term, the formula with a particular value of C, say 5, as in the case of pilot balloon mentioned above will be valid for obtaining correct V only at the corresponding Reynolds number or approximately within the small range of the Reynolds number.

9. Reynolds number corresponding to pilot balloon ascent

The balloons used for the pilot balloon ascents assuming the constant rate of ascent, are mostly the NR 32, and the rate of ascent used are of the order of 10 km/hr. Corresponding to the dimension of the NR 32 inflated for rate of ascent of the order of 10 km/hr the Reynolds number is near about 1×10^5. So making some measurements of the balloons corresponding to near about 1×10^5 Reynolds number and using the empirical formula for the evaluation of V, the coefficient of aerodynamic resistance corresponding to near about 1×10^5 Reynolds number, may be found out.

10. Measurements

Some measurements were made in the case of NR 32 balloon. A few measurements in the neighbourhood of Reynolds number 1×10^5 were also made in the case of NR 27 and NR 15 (ceiling balloons). The balloon was first weighed and then inflated with hydrogen gas so that the rate of ascent might be near about the rate employed for the observations. The free lift was noted, and the corresponding circumference of the balloon was measured for obtaining the diameter. The atmospheric pressure at the station at the time of the above measurements and the prevailing temperature at the place where the balloons were filled up were also noted. Ignoring the small effect of the vapour pressure, the density of the air was calculated from the formula $\rho = 348{\cdot}4\ P/T$, where P and T are air pressure (mb) and air temperatue (°A) respectively. The kinematic viscosity which is required for finding out the Reynolds number and is obtained by dividing the coefficient of dynamic viscosity by the air density, has been based on the values of the coefficient of dynamic viscosity given by Montgomery (1947).

11. Data

The weight of the balloons, the free lifts and the corresponding diameters of the balloons as measured and also the temperature and the pressure at the station at the time of measurement are shown in Table 2. The uniform vertical velocity of the balloon computed from equation (6), the Reynolds number, and the coefficient of the aerodynamic resistance computed from equation (5), are also included in the table.

12. Results

It is observed from Table 2 that the value of the coefficient of aerodynamic resistance near about the Reynolds number 1×10^5 as derived from the application of the empirical formula of the rate of ascent of the pilot balloons is about 0·29. The value of the coefficient of aerodynamic resistance at this Reynolds number is 0·23 according to the curve in Fig. 1 obtained from tethered balloons and 0·33 according to the curve obtained by rising balloons. The value of the

TABLE 2

Data for evaluation of the coefficient of aerodynamic resistance

Temperature at place of filling balloons (°F)	Pressure at station level (mb)	Balloon used	Weight of balloon (gm)	Free lift (gm)	Diameter of balloon (cm)	Calculated V (cm/sec)	Kinematic viscosity (cm^2/sec)	Reynolds number $\times 10^5$	Coefficient of aerodynamic resistance
74	989	NR 32	29	126	66	291	·162	1·2	0·29
74	989	,, 32	29	125	66	290	·162	1·2	0·29
74	989	,, 32	29	111	63	284	·162	1·1	0·28
74	989	,, 32	29	100	61	276	·162	1·1	0·30
74	989	,, 32	29	91	60	269	·162	1·1	0·30
89	986	,, 32	29	140	68	298	·168	1·2	0·29
89	986	,, 32	29	118	65	286	·168	1·1	0·29
89	986	,, 32	29	102	63	276	·168	1·0	0·28
89	986	,, 32	29	84	60	263	·168	0·9	0·29
81	987	,, 32	24	100	62	279	·163	1·1	0·29
81	987	,, 32	24	130	67	296	·163	1·2	0·29
87	986	,, 27	21	101	61	282	·167	1·0	0·29
87	986	,, 27	21	90	59	274	·167	1·0	0·29
83	987	,, 27	22	80	58	272	·167	0·9	0·28
83	987	,, 27	22	130	66	297	·167	1·2	0·29
87	986	,, 15	15	84	57	275	·167	0·9	0·29
87	986	,, 15	15	77	56	270	·167	0·9	0·29

coefficient of aerodynamic resistance obtained by the empirical formula is thus slightly lower than that obtained by rising balloons and higher than that of the tethered balloons.

13. Acknowledgement

I wish to express my thanks to Shri L. C. Sharma for his help in computation of the data.

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