Eastern and North Eastern sub-divisions of India : An analysis of trend and chaotic behaviour of rainfall in different seasons

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सार – प्रस्तुत अध्ययन का उद्देश्य पूर्वी तथा उत्तर-पूर्वी उपखंडों, नामतः ओडिशा, गांगेय पश्चिम बंगाल, उप हिमालयी पश्चिम बंगाल, असम और मेघालय तथा नागालैंड, मणिपुर, मिज़ोरम और त्रिपुरा की 143 वर्षों (1871-2013) की वर्षा के आंकड़ों के आधार पर अव्यवस्थित व्यवहार के साथ प्रवृत्ति या अरैखिकता को समझना है। यह विश्लेषण प्रत्येक ऋतु जैसे मॉनसून-पूर्व, दक्षिण-पश्चिम मॉनसून, उत्तर-पूर्व मॉनसून और मासिक आंकड़ों से प्राप्त वार्षिक वर्षा की प्रकृति का पता लगाने के लिए किया गया है। इस उद्देश्य से, हर्स्ट एक्सपोनेंट के साथ एक प्रवृत्ति विश्लेषण और लायपुरनोव एक्सपोनेंट के साथ अरैखिक विश्लेषण किया गया। विश्लेषण से ज्ञात हुआ कि ओडिशा में सभी ऋतुओं में वर्षा होती है जबकि गांगेय पश्चिम बंगाल में मॉनसून पूर्व में तथा उत्तर-पूर्व मॉनसून ऋतु में और नागालैंड, मणिपुर, मिज़ोरम तथा त्रिपुरा सहित असम तथा मेघालय में दक्षिण-पश्चिम मॉनसून ऋतु और वार्षिक वर्षा की प्रकृति दर्शाता है। उपहिमालयी पश्चिम बंगाल में केवल वार्षिक वर्षा की स्थिरता दिखाई देती है। अनेक मामलों में निम्न परिमाण में अव्यवस्थित प्रवृत्ति दिखाई देती है जबकि मुख्य रूप से मॉनसून पूर्व ऋतु में जब वर्षा होती है तब व्यवस्थित प्रवृत्ति देखी गई। इसके अलावा हर्स्ट और लायपुरनोव एक्सपोनेंट से उपखंडों के दो समूहों में हर तरह से बिल्कुल समान क्षेत्र की पहचान दिखाई दी। दो समूह सैकड़ों कि.मी. दूरी पर (*i*) ओडिशा तथा असम और मेघालय उपखंड तथा (*ii*) उपहिमालयी पश्चिम बंगाल और नागालैंड, मिज़ोरम, मणिपुर और त्रिपुरा उपखंड थे। एक समूह में इन उपखंडों का सभी प्रकार से समान व्यवहार होता है।

ABSTRACT. The aim of the study is to understand trend or non-linearity along with a chaotic behaviour, if any, of Eastern and North Eastern sub-divisional rainfall, namely Orissa, Gangetic West Bengal, Sub Himalayan West Bengal, Assam and Meghalaya and also Nagaland, Manipur, Mizoram and Tripura based on rainfall data of 143 years (1871-2013). The analysis is performed for examining behaviour of rainfall in each of the seasons, namely, Pre monsoon, South West monsoon, North East monsoon and also Annual rainfall extracted from the monthly data. For that purpose, a trend analysis with Hurst Exponent and non-linearity analysis with Lyapunov Exponent are employed. The analysis revealed that rainfall of Orissa is persistent for all the seasons whilst the rainfall is persistent in Gangetic West Bengal in Pre monsoon and North East monsoon and Assam and Meghalaya along with Nagaland, Manipur, Mizoram and Tripura exhibit persistent behaviour in South West Monsoon and annually. Sub Himalayan West Bengal exhibit persistence in annual rainfall only. Chaotic tendency in low magnitude is located in many cases whilst non-chaotic situation has occurred when the persistence is found, mainly in pre-monsoon season. Moreover, the analysis of Hurst and Lyapunov Exponent revealed to identify two groups of sub-divisions with exactly similar region of every respect. Those two groups contain (*i*) sub-divisions Orissa and Assam and Meghalaya and also (*ii*) sub-divisions Sub Himalayan West Bengal and Nagaland, Mizoram, Manipur and Tripura although those are at distances of hundreds of kilometers away. The behaviour of those subdivisions in a group has similar behaviour in all respects.

Key words – Pre-monsoon rainfall, Hurst Exponent, Lyapunov Exponent, Southwest monsoon, Northeast monsoon.

1. Introduction

The country, India, is considered to be consisting of 33 sub-divisions covering a wide area with varied character of rainfall and also differs over long period of time. The monthly rainfall data for the period 143 years (1871-2013) is reported on IMD website. Whilst it is reasonable to study and analyse many of the sub-divisions with varied character of rainfall, we are concentrated, for simplicity, on the analysis over the Eastern and North-eastern sub-divisions of India. Those meteorological sub-divisions are Orissa (IMD sub-division No.7, Orissa), Gangetic West Bengal (IMD sub-division No. 6, GWB), Sub Himalayan West Bengal (IMD sub-division No. 5, SHWB), Assam and Meghalaya (IMD sub-division No. 3, AM) and also Nagaland, Manipur, Mizoram and Tripura (IMD sub-division No. 4, NMMT). The agriculture, economy and development of the sub-divisions are vastly

dependent on the monsoonal rainfall. The rainfall of the regions fluctuate temporarily, say month to month and also spatially, say station to station over a wide range. The variability may be considered on different time scales such as day, month and season and diverse spatial scales such as station, district, state and country. The country as a whole like India, rainfall is classified as three main seasons, namely, Pre Monsoon rainfall (PEM; March to May), South West Monsoon rainfall (SWM; June to September) and North East Monsoon rainfall (NEM; October to December). It is because of the dynamics of monsoonal convection with role of Indian Ocean, Bay of Bengal and local topography (Chowdhury *et al.*, 2018; Kripalini *et al.*, 2003; Gadgil *et al.*, 2002). Different sub-divisions throughout the country showcase variable rainfall patterns along with variety of temporal rainfall patterns during the mentioned rainfall seasons.

The rainfall pattern in PEM (March-May) is important as it helps in determining weather conditions and crop related activities in many of the sub-divisions. It is the change of climate with warming of atmosphere in the tropic (Glecker *et al.*, 2016; Sinha *et al.*, 2018). In the Eastern and North Eastern part of the country, the rainfall occurs in the form of thunderstorm (Norwester's) having symptoms like destructive nature accompanied by heavy rainfall, lightening and squalls. Few efficient efforts are made to investigate mechanism of forcing the air mass and occurrence of such thunderstorm (Ghosh *et al.,* 1999; Chaudhuri, 2010; Narayan *et al.*, 2016). In this phase, Barak valley and adjoining regions receive more rainfall than the Brahmaputra valley indicating greater impact of local activity, local lows and depression from Bay of Bengal - A reason for early floods in the rainy season are common in Barak and its tributaries, thus distribution of monsoon rainfall in the region is erratic and varies over a wide range. The effect is also observed in the rainfall pattern of West Bengal and Assam (Gouda *et al.*, 2017; Nandargi and Barman, 2018).

The SWM rainfall is vastly important in the sense of agriculture, economy and industrialisation in the critical season in the sub-divisions. The medium monsoonal convection while crossing the Bay of Bengal accumulate moisture and proceed towards heated land mass of the eastern and north eastern part of India including GWB and Orissa resulting in adequate SWM monsoon rainfall (June-September) usually. The SWM is studied extensively by the researchers throughout India to understand its dynamics, predictability, cloud physics and also its tele-connection aspects. The mechanism and analysis has been examined in extensive literatures for all India undertaken by different investigators (Chanda and Dhar, 1975; Hastenrath and Rosen, 1983; Iyenger and Basak, 1994; Thapliyal, 2001; Basu, 2001: Basu *et al.*,

2004). The study is also focussed for different sub-divisions. For different regions, study has been undertaken by different researchers, namely, North East region (Mahapatra *et al.*, 2011), Karnataka (Iyenger, 1991), West Bengal and Assam [Basak (2014, 2017)].

During the autumn (October-December), the zone of maximum rainfall migrates to southern India, Sri Lanka and neighbouring seas. The season of rainfall over the southern parts of India is the NEM (October-December). It is a major period of rainfall in southern peninsular India; in particular the south eastern parts (Indira *et al.*, 2013). It determines the agriculture production in the concerned area.

During October, the monsoon trough is positioned over northern parts of India in a North West-South East orientation at the surface and lower troposphere starts a rapid shift southwards (Khole and De, 2003). During the withdrawal process, substantial rainfall occurs in the rain shadow belts in the Siang-Dibang valley in the northern corner of Arunachal Pradesh and adjoining valley area of the sub-divisions covering Assam, Meghalaya and West Bengal. This rainfall occurs probably due to topographic effect, western disturbances through jet stream and wind shearing at the interface of the surface north easterlies and upper westerlies, although in lesser extent (Singh and Sontakke, 1999; Kripalini and Kumar, 2004).

In the present paper, the presence of trend and non-linearity in PEM, SWM, NEM and Annual rainfalls over the subdivisions of Orissa, GWB, SHWB, AM and NMMT are examined utilizing the mathematical tools of Hurst Exponent and Lyapunov Exponent.

2.   Data and classification

The data for sub-division of Orissa, GWB, SHWB, AM and NMMT has been procured from National Data Centre ADGM (R), Pune for the period 1871-2013. The monthly data is suitably categorised into PEM, SWM, NEM and Annual rainfall. The Hurst Exponent (H) is executed for each of the categories. Through the H value, the given time series data is identified as persistent, anti-persistent or chaotic. When the time series data is identified as persistent behaviour, then auto-regressive or moving average methods are applied for their analysis. If chaos is present is in the system, the next step is to check the magnitude of chaos. The Lyapunov Exponent is employed to analysis weather the data has low dimensional chaos. The basic characteristic of rainfall in different sub-divisions and different seasons are presented in Table 1. Primarily, it may be realised that SWM rainfall contributes 70-80% of the annual rainfall in all sub-divisions.

TABLE 1

Basic characteristic of data in different sub-divisions and seasons

Sub-division No. & Name	Season	Average Rainfall (mm)	Percentage of annual rainfall
7 Orissa	PEM	1888.23	12.8
	SWM	11694.72	79.25
	NEM	1635.72	11.08
	ANN	14757.37	100.0
6 Gangetic West Bengal (GWB)	PEM	1888.23	12.25
	SWM	11565.84	75.01
	NEM	1569.71	10.18
	ANN	15418.1	100.0
5 Sub Himalayan West Bengal (SHWB)	PEM	3343.95	13.37
	SWM	19960.13	79.81
	NEM	1429.08	5.71
	ANN	25010.13	100.0
3 Assam and Meghalaya (AM)	PEM	5905.22	25.4
	SWM	14971.5	64.39
	NEM	1880.61	8.08
	ANN	23250.18	100.0
4 Nagaland, Manipur, Mizoram and Tripura (NMMT)	PEM	4235.81	21.47
	SWM	13185.28	56.71
	NEM	1876.53	9.51
	ANN	19726.85	100.0

PEM : Pre-monsoon rainfall (March-May); SWM : South West monsoon rainfall (June-September); NEM : North East monsoon (October-December); ANN : Annual rainfall.

3. Hurst rescaled range analysis

For the analysis, Hurst's rescaled range method is applied and the method is presented in detail. To calculate the Hurst Exponent (H), one must estimate the dependence of the rescaled range on the time span of n observation (Mandelbrot & Wallis, 1969; Lopez Lambrano *et al.*, 2017).

For a time series, $X = X_1, X_2, …, Xn$, R/S is computed as follows.

(*i*) Mean value is calculated

$$m = (1/n) . \sum_{i=1}^{n} X_i$$

(*ii*) Mean of the adjusted series Y is computed as

$$Y_t = X_t - m, \quad t = 1, 2, …, n$$

(*iii*) Cumulative derivative series Z is computed as

$$Z_t = (1/n) . \sum_{i=1}^{t} Y_i, \quad t = 1, 2, …, n$$

(*iv*) Range series R is calculated as

$$R_t = \max (Z_1, Z_2, …, Z_t) - \min (Z_1, Z_2, …, Z_t),$$
$$t = 1, 2, …, n$$

(*v*) Standard deviation S of the series is

$$S_t = \sqrt{(1/t) . \sum_{i=1}^{n} (X_i - u)^2} \quad t = 1, 2, …, n$$

In this case, u is the mean value from X_1 to X_t

(*vi*) Rescaled Range series (R/S) is now calculated as

$$(R/S)_t = R_t/S_t$$
$$t = 1, 2, …, n$$

Hurst found that (R/S) increments follows power law as time increases that indicates $(R/S)_t = c.t^H$. H can be estimated as the slope of log-log plot of $(R/S)_t$ versus t (Mandelbrot and Wallis, 1969).

4. Significance of Hurst Exponent

H describes the correlation between the past and future in the time series. For independent random process with finite variances, the value of H is 0.5.

When H > 0.5, the time series is persistent meaning that an increasing trend in the past is indicative of an increasing trend in the future. Conversely, as the reverse rule, a decreasing trend in the past signifies a persistent decrease in the future.

On the other hand, when H < 0.5, the time series is anti-persistent meaning that an increasing trend in the past implies decreasing trend in the future and *vice versa*.

Lastly, if H is almost equal to 0.5, it indicates that the time series concerned is random.

5. Lyapunov Exponent

We examine whether the PEM, SWM, NEM and Annual rainfall of all the sub-divisions would possess any non-linear tendency and chaotic behaviour.

The null hypothesis that the rainfall time series is a linear process and the goal is to reject hypothesis that the original data have come from a linear process. The rejection of the null hypothesis and existence can be made based on some discriminating statistics such as the correlation dimension, Lyapunov Exponent, Kolmogorov entropy etc. In this paper, Lyapunov Exponent (L) is chosen for identification of chaotic behaviour of the rainfall series with the works of direct estimation from short time series (Lopez *et al.*, 2002; Zeng *et al.*, 1991).

The method consists of the following steps:

(*i*)   Let the time series is defined as (Y_k), $k = 1, 2,…,T$ and d_{max} as the maximum distance between two points of the series under consideration as 'infinitesimally' close. If the series behaves chaotically for points Y_i and Y_j with a distance $d_0 < d_{max}$, where d_0 and d_{max} are the initial and maximum distances between two close points of discrete time series. The divergence of close trajectories would be shown in the sequence of differences

$$d_0 = | Y_j - Y_i |$$

$$d_1 = | Y_{j+1} - Y_{i+1} |$$

$$d_2 = | Y_{j+2} - Y_{i+2} |$$
....
$$d_T = | Y_{j+T} - Y_{i+T} |$$

(*ii*)   This will show an exponential increase, of at least the mean, *T*. With this method for calculating L, two close trajectories in the state space are found and we calculate the series of distances that derive from these two initial conditions.

(*iii*)   For a given time series, it is necessary to demonstrate this assumption that a separation rate is exponential approximation between two close trajectories (Lopez *et al.*, 2002). One way of doing this by plotting the natural logarithm of the differences (ln d_T) is exponential

as a function of T; the points (ln d_T, T) would approximate a line given by the following expression

$$\ln d_T = \ln d_0 + L.T$$

(*iv*) The slope of the fitted line, generally by curve fitting is the value of Lyapunov Exponent.

(*v*)   The initial separation required between two points so that they can be considered as initial conditions of two different trajectories (T_{sep}), must be greater than lag 1 auto-correlation time (T_{sep}) is given as per expression (Hilboen, 1994; Theiler, 1986)

$$T_{sep} = [1/\ln(1/r)]$$

which is approximated as

$$T_{sep} = [1/(1 - |r|)]$$

where, r is the auto-correlation of lag 1 of the series.

The Lyapunov Exponent gives the quantitative value for a non-linear dynamical system (evolutes from time series). A positive largest L indicates chaos. It is thus useful to study the mean exponential rate of divergence of two initially close orbits using the formula (Dechert and Genacy, 1992).

6. Significance of Lyapunov Exponent

The uni-dimensional discrete time series is unstable and has chaotic trajectories, if L is positive,

If L is equal to zero, the time series is in steady state mode.

If L is less than zero, there is no such chaotic tendency in the series and the series is stable.

7. Result of Hurst Exponent and Lyapunov Exponent

In the current paper, the Hurst Exponent is extracted separately for PEM, SWM, NEM and also Annual rainfalls to understand the behaviour of rainfall in near future on the basis of 143 years (1871-2013) data for each of the category. The graphs of Hurst Exponent (H) for PEM, SWM, NEM, ANN for the sub-divisions of Orissa, GWB, SHWB, AM and NMMT are presented in the Figs. 1(a-d), 2(a-d), 3(a-d), 4(a-d) and 5(a-d) respectively. In the same way, the figures of Lyapunov Exponents (L) for the seasons PEM, SWM, NEM, ANN for the corresponding sub-divisions are displayed in the Figs. 6(a-d), 7(a-d), 8(a-d), 9(a-d) and 10(a-d) respectively.
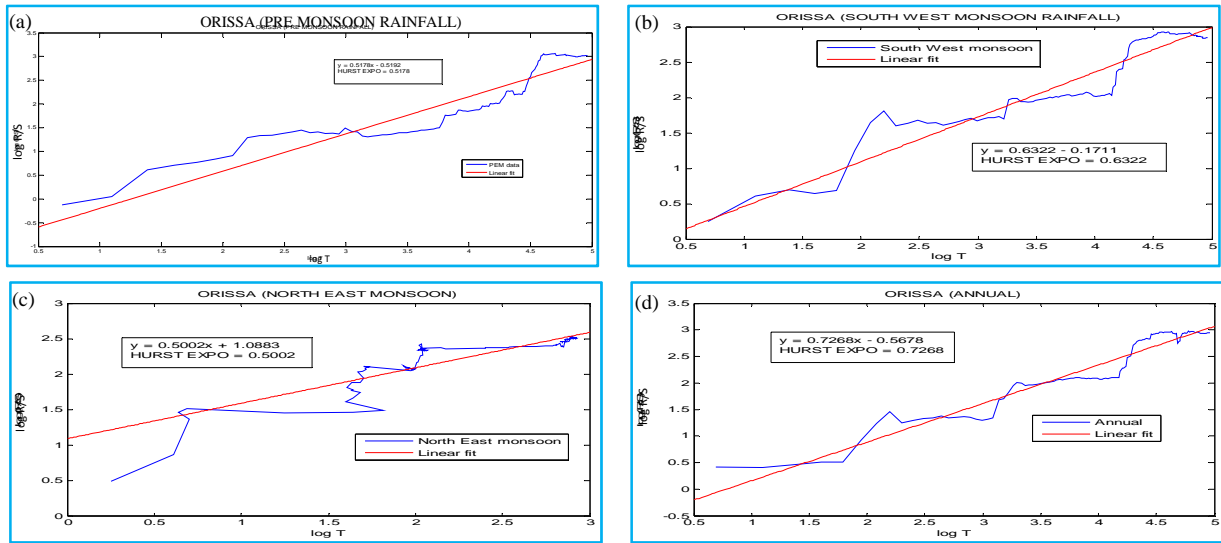
Figs. 1(a-d). The log-log plot of T *vs* R/S of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of Orissa
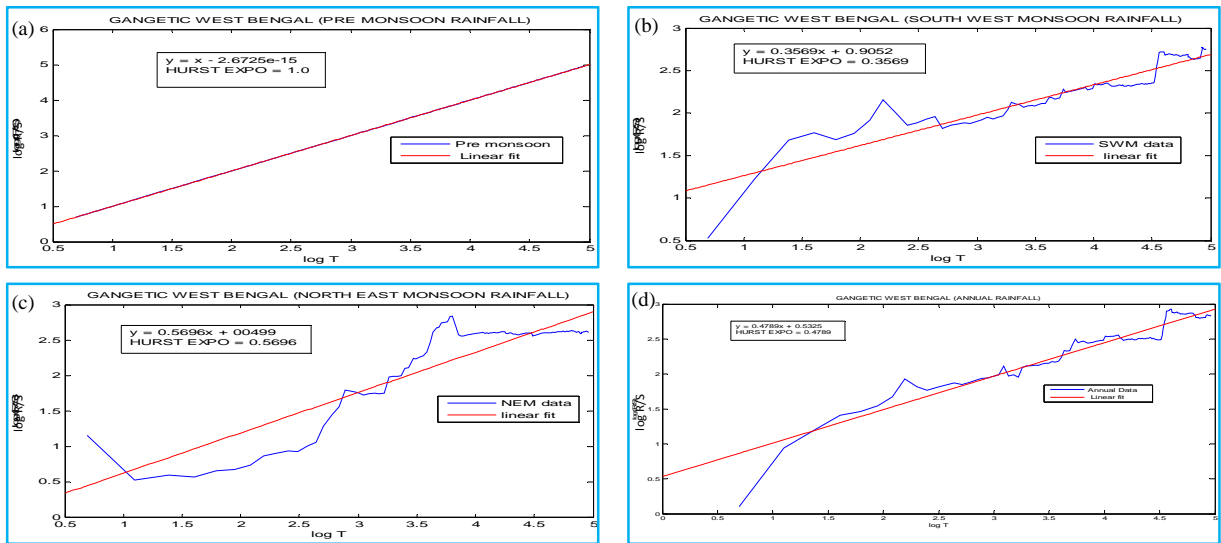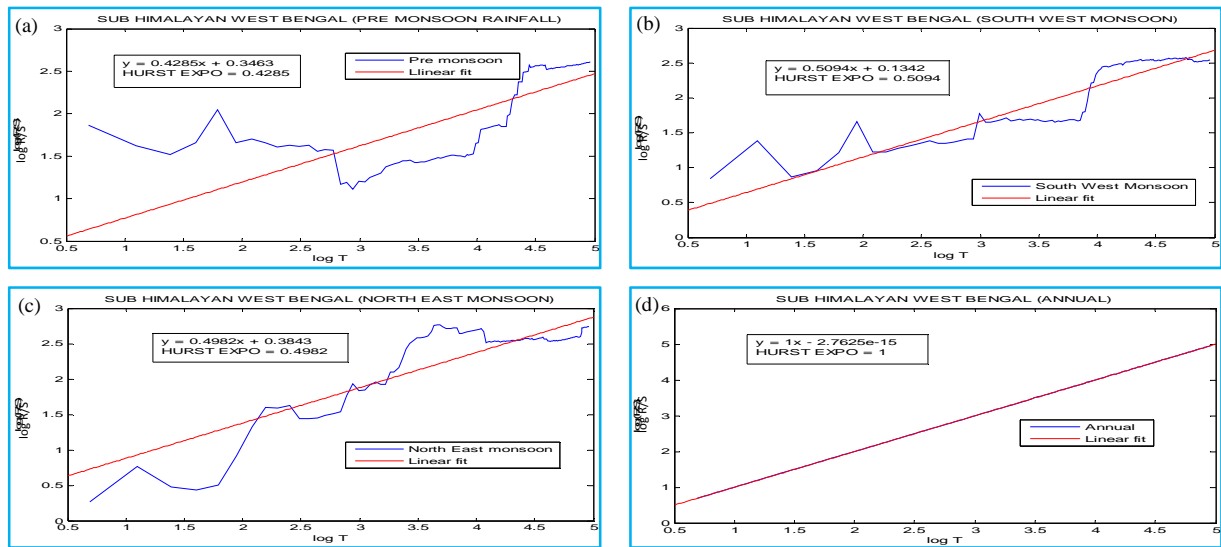

Figs. 2(a-d). The log-log plot of T *vs* R/S of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of GWB

The results Hurst Exponent (H) and Lyapunov Exponent (L) for each of the seasons and each of the sub-divisions mentioned above are presented in Tables 2(a-e) respectively.

It is interesting to note that for the subdivision of Orissa, rainfall is persistent for all seasons (value of H varies between 0.5 to 0.72); meaning that the rainfall in the past follows its own tendency in future, that is increasing/decreasing tendency would retain its increasing/decreasing tendency and *vice versa* and indicates signs of chaotic behaviour (L > 0) in all the seasons except PEM [Table 2(a)]. In connection with this observation, the

works of authors (Mohanty and Mohapatra, 2007) may be mentioned.

The rainfall of GWB is persistent for the PEM and annually (H = 1.0 and 0.57 respectively), that is, increasing/decreasing tendency presently would follow increasing/decreasing tendency in future. The rainfall is anti-persistent in SWM and annually. In particular, for SWM rainfall H = 0.35 (<0.5) and H (=0.47) value being slightly less than 0.5 for Annual rainfall marks a tendency of anti-persistent behaviour. A chaotic tendency is observed in case of SWM and ANN rainfall *i.e.*, L > 0 and no chaotic tendency for PEM and NEM [Table 2(b)].

Figs. 3(a-d). The log-log plot of T *vs* R/S of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of SHWB
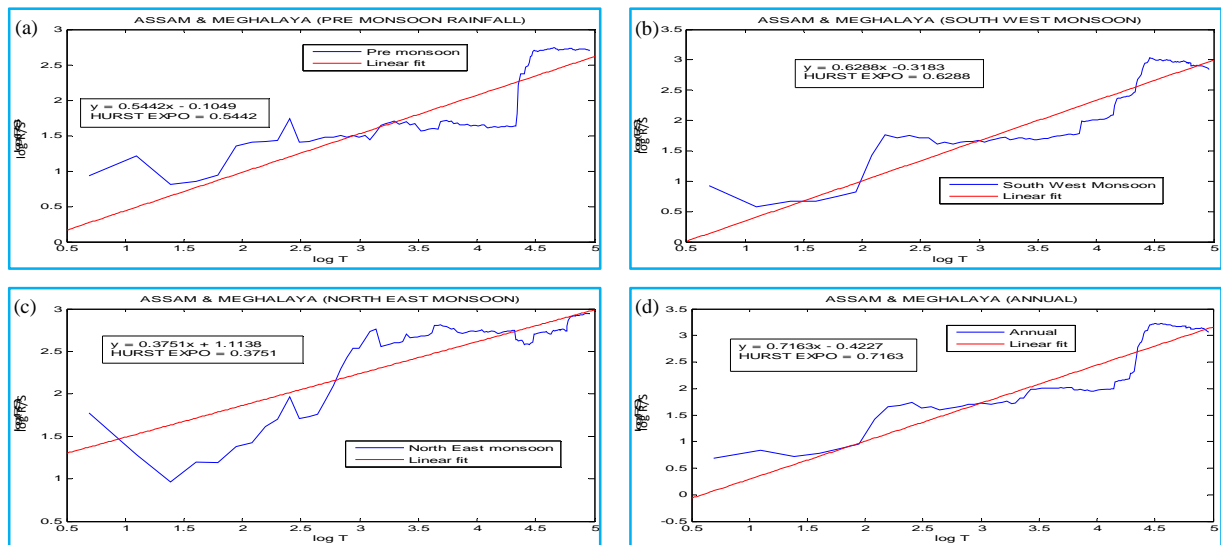
Figs. 4(a-d). The log-log plot of T *vs* R/S of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of AM

The result analysis is quite justified of the GWB region as the ANN of GWB is dominated by SWM rainfall that mainly controls the agriculture and economy of GWB. In GWB, several attempts have made to predict and analyse the behaviour (Basu *et al.*, 2004: Basak, 2014).

An opposite kind of rainfall is observed for SHWB sub-division. The rainfall in SWM and Annual season is persistent (H = 0.51 and 1.0 respectively), very strongly for Annual rainfall but anti-persistent for PEM and NEM seasons. This feature may be considered complement of the

rainfall behaviour in GWB [Table 2(c)]. The rainfall in SHWB is chaotic all the seasons except NEM [Table 2(c)].

The rainfall of AM sub-division of North East India is persistent for PEM, SWM and Annual seasons; however, shows non-persistent behaviour in NEM rainfall, a typical pattern of rainfall. Surprisingly, the feature may be considered similar as in Orissa sub-division except non-persistent behaviour in NEM season and opposite to the behaviour of GWB sub-division except PEM season. The rainfall in AM is chaotic in all the seasons except PEM. These features may be assumed to be balancing character of rainfall
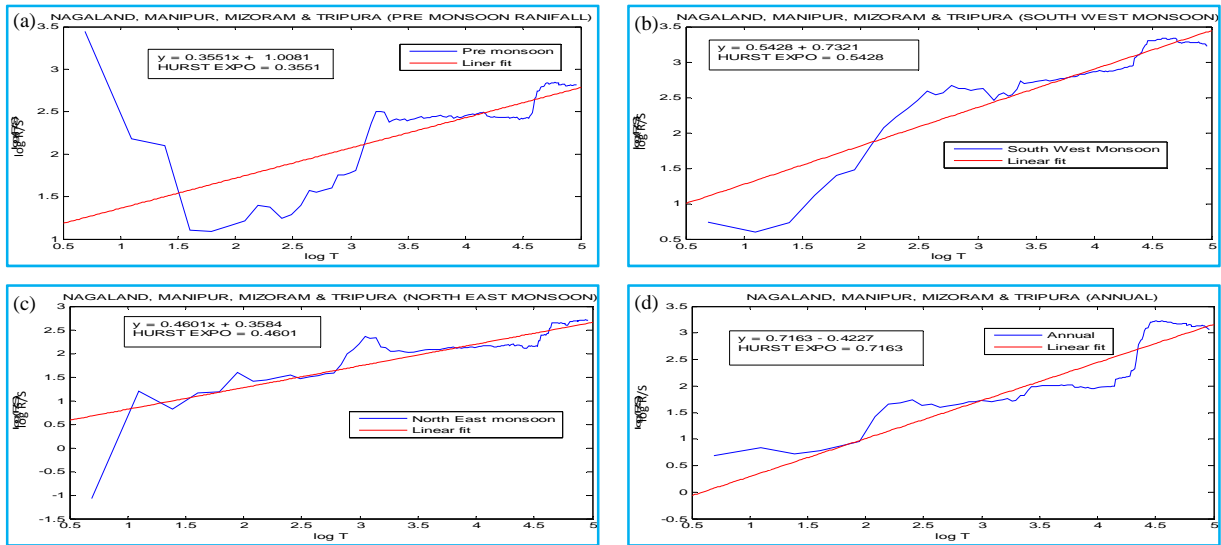
Figs. 5(a-d). The log-log plot of T *vs* R/S of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of NMMT
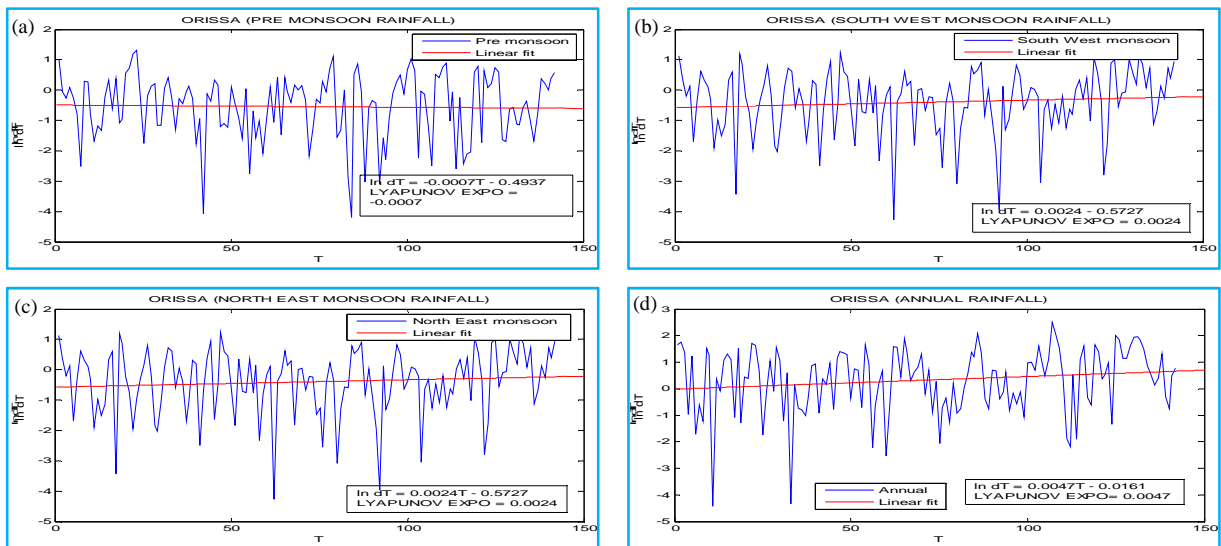
Figs. 6(a-d). The plot of T *vs* ln dT of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of Orissa

behaviour of GWB sub-division. It indicates that an increase/decrease balance of the rainfall in the concerned seasons with reference to neighbouring sub-divisions [Table 2(d)].

Again, the kind of rainfall in NMMT sub-division is the reflexion of the pattern in SHWB sub-division showing anti-persistent behaviour in PEM & NEM seasons; however, persistent in SWM and annual seasons and indicates the chaotic tendency in all the seasons except NEM [Table 2(e)].

It may be considered that, as an effect of AM, NMMT and GWB, SHWB, shows persistent behaviour only in case of Annual rainfall and posses' anti-persistent behaviour in PEM, SWM and NEM rainfall. It may be thought to be adjustment of rainfall pattern between GWB, AM and NMMT, as per geographical position of the sub-divisions.

The results of Hurst Exponent and Lyapunov Exponent are already computed and the graphs of the exponents for the seasons are presented in Figs. 1(a-d),…, 5(a-d) for Hurst Exponent and Figs. 6(a-d),…, 10(a-d) for Lyapunov Exponent respectively. Those are presented systematically for sub-divisions Orissa, GWB, SHWB, AM and NMMT in Tables 2(a-e) respectively.
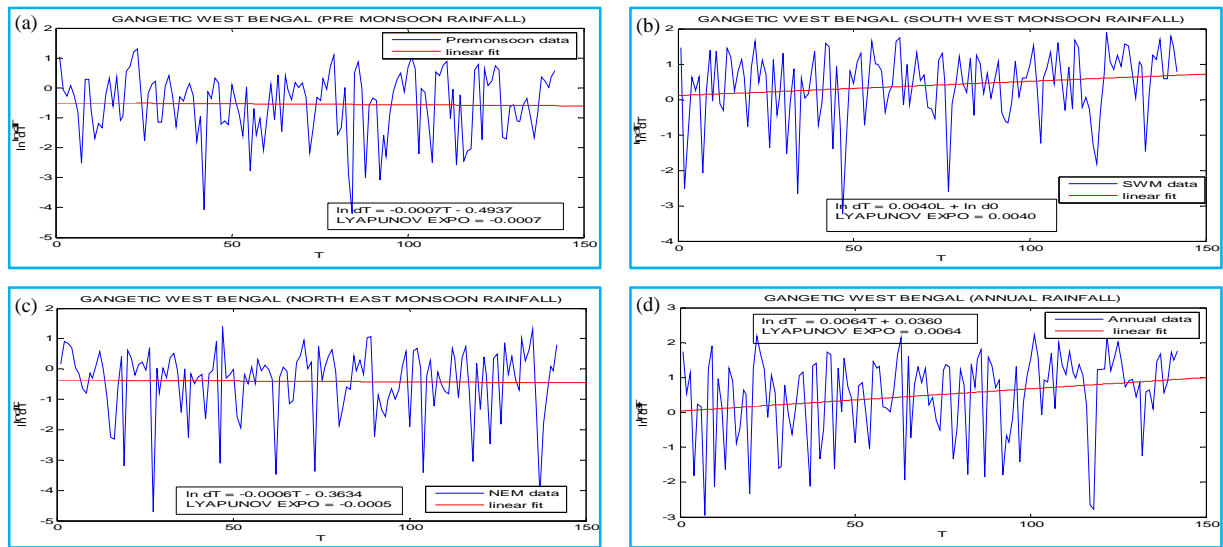
Figs. 7(a-d). The plot of T *vs* ln dT of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of GWB
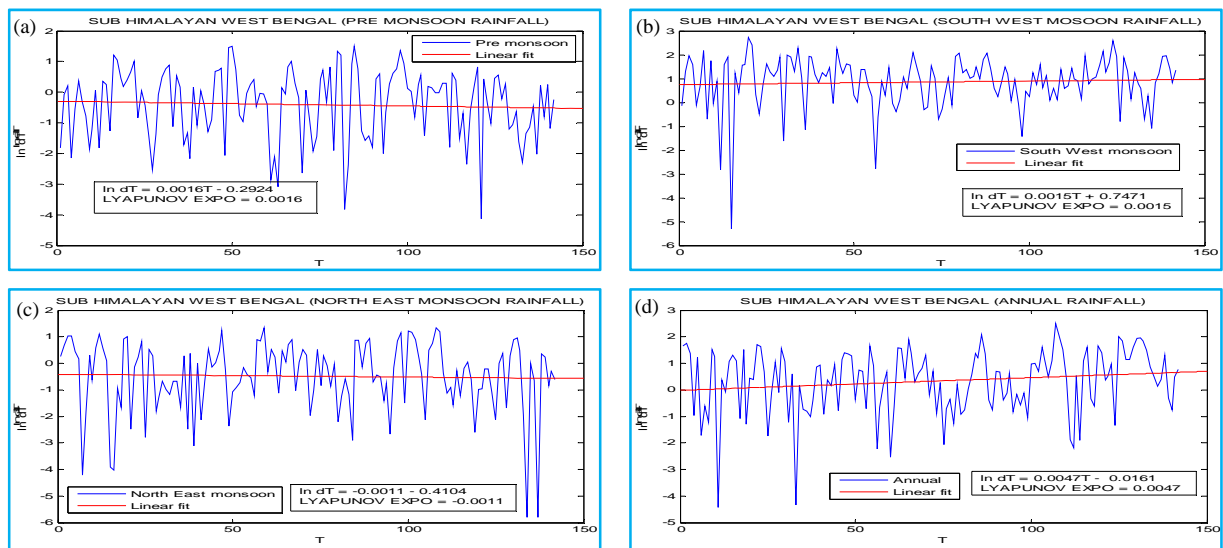
Figs. 8(a-d). The plot of T *vs* ln dT of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of SHWB

It may be mentioned that Lyapunov Exponent (L) is negative signalling chaotic behaviour in the follows cases: Orissa (PEM); GWB (PEM, SWM), SHWB (NEM), AM (PEM). It indicates that chaotic behaviour of rainfall persists in the above mentioned cases.

8. Comparison of results of Hurst Exponent and Lyapunov Exponent for different seasons and different sub-divisions

For proper comparison and identification of signals of monsoons different monsoonal rainfall in different sub-divisions from Hurst Exponent and Lyapunov Exponent presented in Tables 2(a-e), the signals looks very much complicated and it is almost impossible to identify the signals of the regions present in different subdivisions and different seasons. For convenience for identification, new tables identical to Tables 2(a-e) containing only the skeleton structure as H (> 0.5 or < 0.5) and also L (< 0 or >0) are prepared and presented in Tables 3(a-e). For PEM, SWM, NEM and ANN for Orissa, GWB, SHWB, AM and NMMT are prepared analogous to the previous Tables Tables 2(a-e). After preparing the Tables 3(a-e) and studying very carefully the behaviour of seasons in the sub-divisions presented in the following sections.

Figs. 9(a-d). The plot of T *vs* ln dT of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of AM
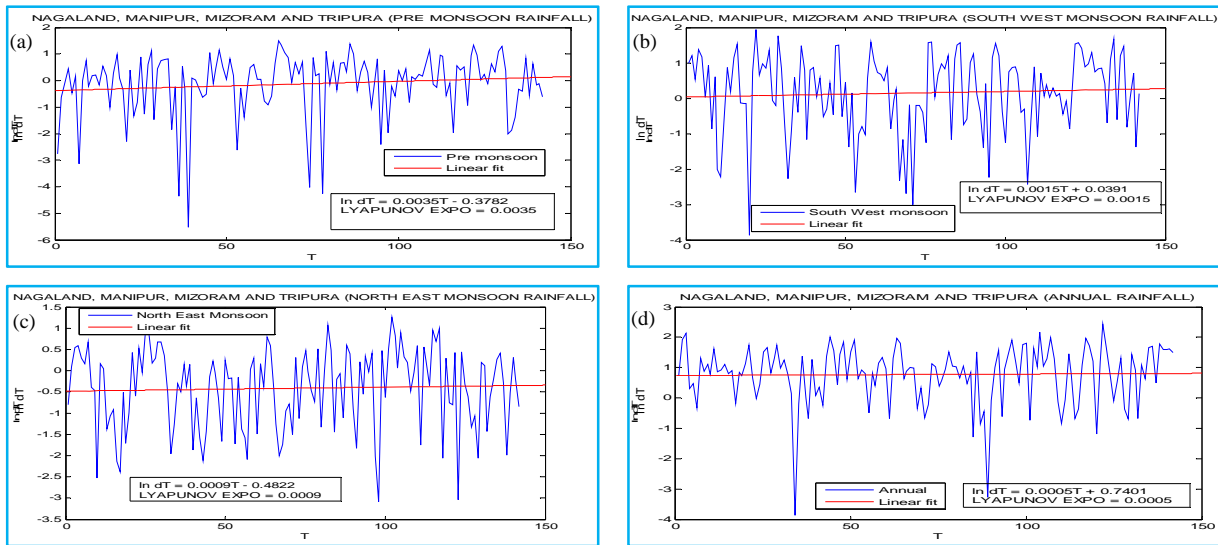
Figs. 10(a-d). The plot of T *vs* ln dT of (a) Pre Monsoon Rainfall (PEM), (b) South West Monsoon Rainfall (SWM), (c) North East Monsoon Rainfall (NEM) and (d) Annual Rainfall (ANN) of NMMT
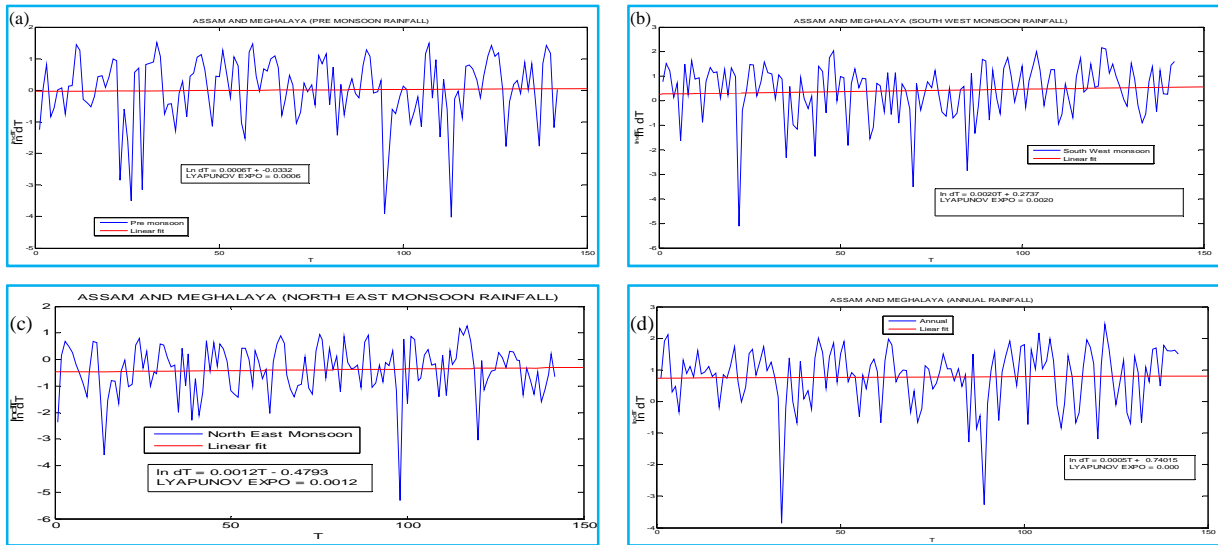
8.1. *Behaviour of rainfall of GWB*

In GWB, data series both for PEM and NEM is H > 0.5, L < 0 [Table 3(b)] indicating that rainfall is persistent (increasing trend is followed by increasing trend and deceasing trend is followed by decreasing trend) and marginal chaotic tend is observed in PEM and NEM seasons.

The rainfall of GWB is persistent for the PEM (H = 1.0) and NEM seasons *i.e.*, the rainfall in the past follows its own tendency in future, but anti persistent (H < 0.5) in SWM season and annually, that is,

increasing/decreasing tendency presently would follow decreasing/increasing tendency in future. It is interesting to note that for PEM season, rainfall H is 1.0 (well above 0.5) indicates that the series is strongly persistent in future. In particular, for SWM rainfall H = 0 .35 (< 0.5) indicating that rainfall series is anti-persistent; For NEM rainfall, H = 0.57 indicating a tendency of persistent behaviour. In the same way H (= 0.47) value being slightly less than 0.5 for Annual rainfall marks a tendency of anti-persistent behaviour. A chaotic tendency is observed in case of SWM and PEM rainfall *i.e.*, L > 0 and no chaotic tendency for PEM and NEM [Table 3(b)].

TABLE 2(a)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of Orissa sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	0.5178	-0.0007
SWM	0.6322	0.0024
NEM	0.5002	0.0024
Annual	0.7268	0.0047

TABLE 3(a)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of Orissa sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	H > 0.5	L < 0
SWM	H > 0.5	L > 0
NEM	H > 0.5	L > 0
Annual	H > 0.5	L > 0

TABLE 2(b)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of GWB sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	1.0	-0.0007
SWM	0.3569	0.0040
NEM	0.5696	-0.0005
Annual	0.4789	0.0064

TABLE 3(b)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of GWB sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	H > 0.5	L < 0
SWM	H < 0.5	L > 0
NEM	H > 0.5	L < 0
Annual	H < 0.5	L > 0

TABLE 2(c)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of SHWB sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	0.4285	0.0016
SWM	0.5094	0.0015
NEM	0.4982	-0.0011
Annual	1.0	0.0047

TABLE 3(c)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of SHWB sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	H < 0.5	L > 0
SWM	H > 0.5	L > 0
NEM	H < 0.5	L < 0
Annual	H > 0.5	L > 0

TABLE 2(d)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of AM sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	0.5442	-0.0007
SWM	0.6288	0.0020
NEM	0.3751	0.0012
Annual	0.7163	0.0005

TABLE 3(d)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of AM sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	H > 0.5	L < 0
SWM	H > 0.5	L > 0
NEM	H < 0.5	L > 0
Annual	H > 0.5	L > 0

TABLE 2(e)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of NMMT sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	0.3551	0.0035
SWM	0.5426	0.0015
NEM	0.4601	0.0009
Annual	0.7163	0.0005

TABLE 3(e)

Hurst Exponent and Lyapunov Exponent for PEM, SWM, NEM and annual rainfall of NMMT sub-division

Season	Hurst Exponent (H)	Lyapunov Exponent (L)
PEM	H < 0.5	L > 0
SWM	H > 0.5	L > 0
NEM	H < 0.5	L > 0
Annual	H > 0.5	L > 0

8.2. *Similarity of rainfall behaviour characteristic in sub-divisions AM and Orissa*

In both the sub-division, H > 0.5 and L < 0 for PEM indicating rainfall is persistent in the season and no indication of chaotic tendency is observed [Tables 3(a) and 3(c)]. Moreover, for SWM, NEM and ANN seasons, H > 0.5 and L > 0, highlighting persistent behaviour and indicative of chaotic tendency for sub-divisions of AM and Orissa. The only exception is the marginally H < 0 of NEM season for AM and opposite in Orissa respectively. It points out the similarity of behaviour of rainfall pattern in the different seasons of sub-divisions AM and Orissa; although those sub-divisions are situated hundred kilometres away jumping behind the sub-division of GWB which shows a similar pattern.

8.3. *Similarity of rainfall pattern of sub-divisions SHWB and NMMT*

In both the sub-divisions SHWB and NMMT the following similarity of observations in the sub-divisions ate observed. For PEM and NEM, H < 0.5, L > 0, projecting that the rainfall data is anti-persistent (meaning that an increasing trend to follow decreasing trend and *vice versa*) and posses chaotic tendency.

It is also observed that for SWM and ANN seasons, H > 0.5, L > 0 indicating the presence of persistent behaviour and existence chaotic tendency. The only exception is the season NEM when L is different in the subdivisions of SHWB and NMMT, located hundreds of kilometres crossing away crossing the sub-division of AM.

It is, however, relevant to note that the phase of similarity of rainfall in different seasons was observed in case of AM and Orissa and also SHWB and NMMT at the distances hundreds of kilometres away.

9. Discussion

An approach for a seasonal and one dimensional data series is extraction of Hurst Exponent (H) and Lyapunov Exponent (L) through R/S analysis (Mandelbrot and Wallis, 1969; Lopez Jiménez *et al.*, 2002; Zeng *et al.*, 1991). Those exponentials can be achieved through application of the methods. However, one faces difficulty to analyse, classify and also to interpret the rainfall pattern spread over the subdivisions of East and North east subdivisions. As sub-divisions are widely spread and maybe correlated among themselves, the straight forward time series analyses are complicated. The data series of area-weighted sub-divisional rainfall absorb those inherent complexities of station wise correlated analysis. The application of the methods mentioned above indicates clearly the inherent persistency and chaotic tendency of the sub-divisions regional data. Although they are found out, the interpretation and identification of similarity in nature of sub-division wise regional data of similarity are much difficult and confusing. To avoid this difficulty from Tables 2(a-e), we have converted a Tables 3(a-e) based on only indication (like H < 0.5 or H > 0.5 and L < 0 or L > 0). After a careful comparison of the sub-divisions wise data in Tables 3(a-e), we introduced in two similar groups or sub-divisions. Each group contains two sub-division having similar patterns of rainfall pattern. Two such groups are (*i*) sub-divisions Orissa and AM and (*ii*) sub-divisions SHWB and NMMT. Sub-division GWB is separate from both groups. In a group, the rainfall pattern of two sub-divisions is exactly or very nearly same. Another thing to note that, in a group of sub-divisions such as Orissa and AM; the sub-divisions are geographically located hundreds of km away subsiding one sub-division GWB. Similar, is the case of sub-divisions AM and NMMT.

10. Summary and conclusions

Extraction of Hurst Exponential and Lyapunov Exponent are introduced in meteorological data analysis due to its usefulness in identifying pattern of data whether (chaotic or persistent) in different subdivisions of Eastern and North Eastern sub-divisions. The study is motivated by the fact that the sub-divisional data of different regions contain much valuable information of inter-annual rainfall variability of different seasons of rainfall pattern. The sub-divisional rainfall data of different seasons are, in general, looks very simple; however, identification of similar patterns of rainfall are comparatively complicated, difficult and are related to the dynamics of the rainfall pattern like similar/weather condition. Moreover, those similar weather condition persists at the gap of a sub-division away is clearly identified in case of sub-divisions Orissa and AM and also sub-divisions SHWB and NMMT. Sub-division GWB has separate characteristic about signals of different sub-divisions. However, further analysis is required in finding the causes of these aspects and further analysis of other sub-divisions.

The contents and views expressed in the research paper are views of the author and do not necessarily reflect the views of the organisation.

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