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Intercomparison of evaluation of low-flow characteristics of streams using statistical modelling approach

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सार – जल आपूर्ति की योजना और डिजाइन बनाने, पर्यावरणीय और आर्थिक दुष्प्रभावों का विश्लेषण करने, जलधारा के पानी की गुणता का मॉडुलन करने, जलधारा के उपयोगों को नियमित करने तथा प्राकृतिक और नियमित जलधारा तंत्रों की जानकारी के सामान्य स्तर में सुधार लाने के लिए जलधारा के निम्न प्रवाह लक्षणों का उपयोग किया गया है। तीन भिन्न नदी बेसिनों नामतः महानदी, गोदावरी और नर्मदा के विभिन्न प्रत्यागमन काल के निम्न प्रवाह लक्षणों का पता लगाने के लिए सांख्यिकीय मॉडुलन पद्धति का उपयोग किया गया है जिसमें बॉक्स–कॉक्स रूपांतरण के मानक संभाव्यता वितरण, लॉग नॉर्मल, लॉग पीअरसन टाइप III और पीअरसन टाइप III तथा वीबुल शामिल हैं। विभिन्न जलधाराओं के निम्न प्रवाह लक्षणों की तुलना करने के लिए काई वर्ग (χ^2) जाँच का उपयोग किया गया है। इस शोध पत्र के अनुसार लॉग नॉर्मल, वीबुल और पीअरसन टाइप III वितरण क्रमशः नर्मदा, महानदी और गोदावरी नदी के निम्न प्रवाह लक्षणों के लिए उचित पाए गए हैं। इसमें निम्न दाब आवृति वक्रों का भी विकास किया गया है और उन्हें प्रस्तुत किया गया है।

ABSTRACT. Low-flow characteristics of streams are used in planning and design of water supplies, analysing environmental and economic impacts, modelling stream water quality, regulating instream uses, and improving the general level of understanding of natural and regulated stream systems. Statistical modelling approach involving standard probability distributions of Box-Cox Transformation, Lognormal, Log Pearson Type III and Pearson Type III and Weibull are used to determine low-flow characteristics for different return periods for three different river basins, namely, Mahanadi, Godavari and Narmada. Chi-square (χ^2) test is used for comparison of low-flow characteristics of different stream. The paper presents that Lognormal, Weibull and Pearson Type III distributions are found to be suitable for determination of low-flow characteristics for rivers Narmada, Mahanadi and Godavari respectively. Low-flow frequency curves are also developed and presented.

Key words – Box-Cox Transformation, Low-flow, Lognormal, Log pearson, Pearson, Weibull, Chi-square, Frequency curve.

1. Introduction

Low-flow characteristics of streams are used in planning and design of water supplies, analysing environmental and economic impacts, modelling stream water quality, regulating instream uses, and improving the general level of understanding of natural and regulated stream systems. Low-flow at a site is often characterized by an associated, annual event based, low-flow statistic is $Q(d,T)$; defined as the annual minimum *d*-day average flows that is expected to be exceeded in (*T*-1) years out of *T* years (Kernell, 1994). *T* signifies the recurrence interval of the corresponding low-flow event. Theoretical estimation of $Q(d,T)$, the estimate of *d*-day *T*-year event, requires the selection of a probability distribution for modelling the frequency of the annual minimum *d*-day stream flow series.

Two streams of approach are available for evaluation of low-flow characteristics of stream: one based primarily on statistical modelling approach, and other relying on physically based modelling approach. The first approach involves fitting of standard probability distributions to the annual minimum *d*-day average flow and the second approach exemplifies that the catchments with contrasting hydrogeology and physical characteristics and modelled using several deterministic models. In this paper, an approach based on statistical modelling is used.

The paper presents the methodology adopted in estimating the parameters of Box-Cox Transformation, Lognormal, Log Pearson Type III, Pearson Type III and Weibull distributions for determination of low-flow characteristics in a stream using the statistic $Q(d,T)$. The methodology can be used to evaluate the frequency and magnitude of the annual minimum *d*-day average flows at a gauged site for different return periods. The paper also presents the methodology adopted in Chi-square (χ^2) test and the results obtained on the study.

2. Methodology & data used

2.1. *Low-flow modelling*

Analysis of low-flows of a stream pre-supposes that: (*i*) the river is perennial, (*ii*) no significant withdrawals and diversions are in operation, and (*iii*) the flows can reasonably be considered to be natural. The data on daily flows, for the entire period of record, are divided into yearly intervals. The annual minimum average flows, for different values of *d* such as 7, 10, 14 and 30 days are subsequently obtained. Consequently, the parameters of selected distributions are computed by using annual minimum *d*-day average flows. Number of parameter estimation methods such as method of moments (MOM), maximum likelihood method (MLM) and probability weighted moments (PWM), etc. are commonly used for estimating the parameters of the distributions. The procedures involved in estimating the parameters of Box-Cox by Transformation method, Lognormal by MLM, Log Pearson Type III and Pearson Type III by MOM and Weibull by PWM and determining the low-flow characteristics of stream is briefly described in the subsequent sections.

2.2. *Box-Cox Transformation*

Tasker (1987) suggested that the Box-Cox transformation (BCT) method is one of the most widely appreciable methods in modelling low-flow. The generalised power transformation of Box-Cox for low-flow is :

$$q_i = \begin{cases} \dfrac{x_i^{\lambda} - 1}{\lambda}, \lambda \neq 0 \\ \mathrm{Ln}(x_i), \lambda = 0 \end{cases}, \tag{1}$$

where, λ is chosen so as to make q an approximately normal random variable with mean ($\bar{q}$) and standard deviation (S_q), and x_i is the annual minimum *d*-day average flow. The method of estimating λ is to choose λ that makes the coefficient of skewness (C_s) of sample of q to zero. This is accomplished by a trial and error method. The value of $Q(d,T)$ may be estimated by substituting sample estimates of λ, $\bar{q}$ and S_q into the following equation :

$$Q(d,T) = [Z_p * \lambda + 1]^{1/\lambda} \tag{2}$$

where, Z_p is the standard normal deviate for probability *p*, and is given by :

$$Z_p = \bar{q} + Y_p S_q \tag{3}$$

where, $\bar{q}$ and S_q are the mean and standard deviation of the transformed series of the annual minimum *d*-day average flow and Y_p is the frequency factor corresponding to probability of exceedance (with reference to return period) and C_s (= 0.0).

2.3. *Lognormal distribution*

Following Vogel and Kroll (1990), MLM is used to estimate the parameters of the Lognormal (LN) distribution. The MLM estimates of the LN distribution are given by :

$$Q(d,T) = \mathrm{Exp}\left[\mu'_y(d) + Z_T \sigma'_y(d)\right] \tag{4}$$

where, $\mu'_y(d) = (1/n)\sum_{i=1}^{n} y_i(d)$ (5)

$$\sigma'_y(d) = \sqrt{[1/(n-1)]\sum_{i=1}^{n}\left[y_i(d) - \mu'_y(d)\right]^2} \tag{6}$$

$$y_i(d) = \mathrm{Ln}[q_i(d)] \tag{7}$$

Here, $q_i(d)$ is the minimum *d*-day average flow in year *i*, *n* is the number of days of record, and Z_T is the standard normal random variable corresponding to the *T*-year event. Stedinger (1980) suggested that Turkey proposed the following formula for computing Z_T for values of different return periods.

$$Z_T = 4.91\left[(1/T)^{0.14} - \{1 - (1/T)\}^{0.14}\right] \tag{8}$$

2.4. *Log Pearson type III Distribution*

The Log Pearson Type III (LP III) distribution is also widely used for evaluation of low-flow events for different return periods. Loganathan *et al*. (1985) expressed that the probability distribution function of LP III distribution is in the form of :

$$f(x) = (1/\mathrm{a}x\Gamma(\mathrm{b}))\left(\tfrac{\ln x - \mathrm{c}}{\mathrm{a}}\right)^{\mathrm{b}-1}\exp\left[-\frac{(\ln x - \mathrm{c})}{\mathrm{a}}\right],$$

c ≤ ln*x* ≤ ∞ for a > 0, -∞ ≤ ln*x* ≤ c for a < 0 (9)

where, a, b and c are scale, shape and location parameters of LP III distribution respectively, and Γ(b) is a gamma distribution The parameters a, b and c of LP III can be computed from the recorded data by replacing population statistics with sample statistics such as mean ($\bar{y}$), standard deviation (S_y) and coefficient of skewness ($C_s^{'}$) of log transformed series of the annual minimum *d*-day average flow. The parameters are further used for evaluation of $Q(d,T)$ for different return periods through the Eqn. (10) and is given by :

$$Q(d,T) = \mathrm{Exp}(\mathrm{a} + \mathrm{b}K_T) \tag{10}$$

where, K_T is the frequency factor corresponding to probability of exceedance and $C_s^{'}$.

2.5. *Pearson type III Distribution*

Bulu (1997) stated that the Pearson Type III (P III) distribution is a three-parameter gamma distribution and is widely used in low-flow studies. The probability distribution function of P III is defined by :

$$f(x) = \left(1/\mathrm{a}^*\Gamma(\mathrm{b}^*)\right)\left(\tfrac{x-m}{\mathrm{a}^*}\right)^{\mathrm{b}^*-1} \exp\left[-\frac{(x-m)}{\mathrm{a}^*}\right]$$
$$\mathrm{m} \le x \text{ if } \mathrm{a}^* > 0 \text{ and } x \le \mathrm{m} \text{ if } \mathrm{a}^* < 0 \tag{11}$$

where, a^*, b^* and m are scale, shape and location parameters of P III distribution respectively, and $\Gamma(\mathrm{b}^*)$ is a gamma distribution. By using the recorded values of annual minimum *d*-day average flows, the mean ($\bar{x}$), standard deviation (S_x) and co-efficient of skewness (C_s^*) are calculated as :

$$\bar{x} = (1/n)\sum_{i=1}^{n} x_i \; ; \; S = \left(\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \Big/ n-1\right)^{1/2};$$
$$C_S = \left(n\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^3 \Big/ (n-1)(n-2)S^3\right) \tag{12}$$

The parameters (a and b) of P III can be computed from the recorded data by replacing population statistics with sample statistics such as mean ($\bar{x}$) and standard deviation (S_x) of original series of the annual minimum *d*-day average flow. By using $\bar{x}$ and S_x, the low-flow statistic $Q(d,T)$ for different return periods are determined by using the Eqn. (13) and is given by :

$$Q(d,T) = \mathrm{Exp}(\mathrm{a} + \mathrm{b}K_T) \tag{13}$$

where, K_T is the frequency factor corresponding to probability of exceedance and C_s^*.

2.6. *Weibull distribution*

Weibull distribution, as described by D'Agostino and Stephens (1986), is also employed for modelling the low-flow data. The method based on PWM is used to estimate the parameters of the distribution. The cumulative distribution function of Weibull (WB) distribution is given by:

$$F(x) = P(X \le x) = Exp\left[-(x/\beta)^{\alpha}\right] \tag{14}$$

where, α and β are the parameters and $x > 0$; α, $\beta > 0$. Following Nathan and McMahon (1990), the PWM is used to estimate the parameters of the Weibull distribution. PWM provide an alternative approach to that of conventional moments for summarizing the characteristics (location, scale, skewness, kurtosis) of an observed data set or theoretical probability distribution. The theory of PWMs parallels that of conventional moments, but, being linear functions of the data, PWM suffer less from the effects of sampling variability and data outliers. PWMs are defined for a distribution function $F = F(x)$ as :

$$M_{p,r,s} = \int_0^1 \left\{x(F)^p F^r (1-F)^s\right\} \mathrm{d}F \tag{15}$$

where, *p*, *r*, *s* are integers and $x(F)$ is the quantile function, or inverse cumulative distributive function, of *x*. The quantities $M_{p,r,s}$ may be used to describe and characterize probability distributions; for example, $M_{p,0,0}$, $p = 1,2,3\ldots$, are just the conventional moments of *X*. As described by Greenwood *et al*. (1979), for a random sample of size *n* from the distribution F, the quantity $M_{1,r,0}$ may be estimated as b_r from the following equation :

$$\mathrm{b}_r = \frac{1}{n}\sum_{j=1}^{n} p_j^r x_j \tag{16}$$

where, p_j^r is a plotting position of $F(x_j)$. He also expressed that the simulation experiment based on Monte Carlo trials could be found the best overall estimate of plotting position and is given by $p_j^r = (j\text{-}0.35/n)$ and accordingly this was adopted in this study.

The inverse distribution function of the Weibull distribution is given by :

$$x(F) = \beta(-\log F)^{1/\alpha}$$

for which the PWMs may be written as :

$$M_{1,r,0} = \int_0^1 \beta(-\log F)^{1/\alpha} F^r \mathrm{d}F$$

Substituting u = -log$F$ yields

$$M_{1,r,0} = \beta\int_0^\infty u^{1/\alpha} e^{-(r+1)u} \mathrm{d}u = \frac{\beta\Gamma[1+(1/\alpha)]}{(r+1)^{1+(1/\alpha)}} \quad (17)$$

Using the Eqn. (17), the parameters α and β may be found explicitly by solving the following system of simultaneous equations.

$$\mathrm{b}_0 = \beta\Gamma(1+(1/\alpha))\,;\, b_1 = 2^{1+(1/\alpha)} \quad (18)$$

where, the term b_r (r = 0,1) is used to denote the sample estimate of $M_{1,r,0}$, which can be computed by using the Eqn. (18). The following equation is used to compute the low-flow statistic $Q(d,T)$ for different return periods and is given by:

$$Q(d,T) = \beta\,[Y_T]^{1/\alpha} \quad (19)$$

where, Y_T is the reduced variate and is defined by Y_T = -Ln [1-(1/T)].

2.7. *Chi-square test*

Number of goodness of fit tests like Chi-square (χ^2) and Kolmogorov-Smirnov are commonly used to judge the applicability of the selected distribution for determining low-flow characteristics for different return periods for a stream. By considering the convergence of the series of data, χ^2-test is used in the study. The χ^2-test statistic is defined as :

$$\chi_C^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \quad (20)$$

where, O_i is the frequency of observed value of annual minimum d-day average flows, E_i is the frequency of expected value of annual minimum d-day average flows and χ_C^2 is the computed value of χ^2-test statistic of the selected distribution. Pearson and Hartley (1966) expressed that the rejection region of χ^2-test statistic at the desired significance level η is $\chi_C^2 > \chi^2_{n\text{-}m\text{-}1,\ 1\text{-}\eta}$. Table 1 gives the selected theoretical values of χ^2-test statistic.

TABLE 1

Selected theoretical values of χ^2-test statistic

Degrees of freedom	$p = P\,(\chi^2 \le \chi_p^2)$				
	0.950	0.975	0.990	0.995	0.999
2	5.99	7.38	9.21	10.6	13.8

If the computed value of χ^2 of the selected distribution is less than of its theoretical value at the desired significance level η with n-m-1 degrees of freedom, then the selected distribution is more appropriate than any other distributions for evaluation of low-flow characteristics. Here, n is the number of classes and m is the number of parameters of the selected distribution.

2.8. *Probability plot*

Graphical methods are commonly used at various stages of hydrologic analysis from the beginning exploratory plots through various stages of analysis. Such methods come to the aid in a various number of ways such as estimating the magnitude of hydrologic events and their corresponding probabilities of occurrence, detecting outliers, and in evaluating the adequacy of fit. A qualitative assessment of the goodness of fit can advantageously be carried out by probability plot of the low-flow estimates. An empirical probability plot is obtained by plotting the return period and probability of exceedance along the horizontal axis and the low-flow estimates of BCT, LN, LP III, P III and WB along the vertical axis.

The low-flow characteristics in three different river basins have been evaluated using the low-flow statistic $Q(d,T)$ for different return periods from 2 years to 50 years by adopting BCT, LN, LP III, P III and WB distributions. Data in respect of river Mahanadi at Basantpur site for the years 1965-94, river Godavari at Pathagudam site for the years 1964-93, river Narmada at Mandleshwar site for the years 1969-98 are used in this study.

3. Results and discussions

3.1. *Estimation of $Q(d,T)$ using BCT, LN, LP III and P III and WB Distributions*

By adopting the procedures of BCT, as described earlier, a computer program was developed and used to determine the transformation constant (λ) for different values of d such as 7, 10 14 and 30 days from the computed annual minimum d-day average flows. By

TABLE 2

Parameters of BCT for different values of *d*

Site	Parameters ($\bar{q}$ and S_q in m^3/s) of BCT for :							
	d = 7		*d* = 10		*d* = 14		*d* = 30	
	$\bar{q}$	S_q	$\bar{q}$	S_q	$\bar{q}$	S_q	$\bar{q}$	S_q
Basantpur	15.302	7.867	13.962	7.080	13.491	6.768	12.321	6.002
Pathagudam	2.270	0.371	2.759	0.497	4.870	1.208	5.456	1.344
Mandleshwar	12.321	3.515	13.007	3.661	13.734	3.831	14.309	3.930

TABLE 3

Parameters of LN distribution for different values of *d*

Site	Parameters (μ_y' and σ_y' in m^3/s) of LN distribution for :							
	d = 7		*d* = 10		*d* = 14		*d* = 30	
	μ_y'	σ_y'	μ_y'	σ_y'	μ_y'	σ_y'	μ_y'	σ_y'
Basantpur	1.732	0.411	1.767	0.425	1.803	0.439	1.825	0.431
Pathagudam	2.232	0.359	2.278	0.345	2.351	0.345	2.496	0.349
Mandleshwar	3.479	0.453	3.505	0.439	3.534	0.428	3.585	0.419

TABLE 4

Parameters of P III distribution for different values of *d*

Site	Parameters ($\bar{x}$ and S_x in m^3/s) of P III distribution for :							
	d = 7		*d* = 10		*d* = 14		*d* = 30	
	$\bar{x}$	S_x	$\bar{x}$	S_x	$\bar{x}$	S_x	$\bar{x}$	S_x
Basantpur	6.040	1.893	6.280	2.023	6.540	2.152	6.670	2.188
Pathagudam	9.910	3.606	10.333	3.574	11.083	3.598	12.833	4.235
Mandleshwar	35.456	14.305	36.190	14.187	37.190	14.263	38.963	14.780

TABLE 5

Parameters of WB distribution for different values of *d*

Site	Parameters (α and β in m^3/s) of WB distribution for :							
	d = 7		*d* = 10		*d* = 14		*d* = 30	
	α	β	α	β	α	β	α	β
Basantpur	3.969	6.040	3.827	6.280	3.725	6.540	3.734	6.670
Pathagudam	3.348	9.910	3.558	10.333	3.770	11.083	3.665	12.833
Mandleshwar	3.033	35.456	3.138	36.190	3.215	37.190	3.284	38.963

TABLE 6

Estimates of annual minimum *d*-day average low-flow events for different return periods using Box-Cox Transformation, Lognormal, Log Pearson Type III, Pearson Type III and Weibull distributions for river Mahanadi at Basantpur site

Return period T (yrs)	Q (d, T) [m^3/s]																			
	d = 7					d = 10					d = 14					d = 30				
	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB
2	6.27	5.65	6.31	6.24	5.51	6.51	5.85	6.55	6.48	5.71	6.77	6.07	6.83	6.75	5.93	6.87	6.20	6.94	6.86	6.05
5	4.67	4.00	4.30	4.54	4.14	4.79	4.09	4.41	4.66	4.24	4.94	4.20	4.54	4.82	4.37	5.01	4.32	4.65	4.91	4.46
10	3.62	3.33	3.28	3.52	3.43	3.68	3.39	3.33	3.59	3.49	3.79	3.46	3.39	3.68	3.57	3.86	3.57	3.50	3.77	3.65
25	2.07	2.74	2.30	2.34	2.70	2.14	2.78	2.32	2.36	2.72	2.22	2.81	2.33	2.38	2.77	2.37	2.91	2.44	2.48	2.83
50	1.68	2.42	1.77	1.51	2.26	1.76	2.44	1.78	1.53	2.27	1.90	2.46	1.82	1.56	2.29	2.19	2.56	1.86	1.59	2.35

BCT: Box-Cox Transformation; LN: Lognormal; LP III Log Pearson Type III; P III: Pearson Type III; WB: Weibull

TABLE 7

Estimates of annual minimum *d*-day average low-flow events for different return periods using Box-Cox Transformation, Lognormal, Log Pearson Type III, Pearson Type III and Weibull distributions for river Godavari at Pathagudam site

Return period T (yrs)	Q (d, T) [m^3/s]																			
	d = 7					d = 10					d = 14					d = 30				
	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB
2	9.32	9.31	9.31	9.43	8.88	9.85	9.76	9.84	9.88	9.32	10.82	10.50	10.85	10.82	10.06	12.52	12.13	12.45	12.58	11.61
5	6.88	6.89	6.88	6.82	6.33	7.31	7.30	7.31	7.27	6.78	8.00	7.86	7.96	8.00	7.45	9.20	9.05	9.14	9.22	8.52
10	5.87	5.88	5.88	5.71	5.06	6.22	6.26	6.23	6.14	5.49	6.68	6.75	6.64	6.68	6.10	7.65	7.76	7.65	7.59	6.94
25	4.96	4.96	4.97	4.70	3.81	5.21	5.31	5.23	5.10	4.21	5.39	5.73	5.39	5.36	4.74	6.14	6.58	6.26	5.96	5.36
50	4.44	4.45	4.45	4.14	3.09	4.64	4.78	4.66	4.52	3.45	4.63	5.16	4.67	4.57	3.94	5.24	5.92	5.46	4.96	4.43

TABLE 8

Estimates of annual minimum *d*-day average low-flow events for different return periods using Box-Cox Transformation, Lognormal, Log Pearson Type III, Pearson Type III and Weibull distributions for river Narmada at Mandleshwar site

Return period T (yrs)	Q (d, T) [m^3/s]																			
	d = 7					d = 10					d = 14					d = 30				
	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB
2	34.30	32.44	34.34	34.13	31.42	35.12	33.28	35.15	34.97	32.20	36.18	34.32	36.18	36.04	33.18	38.72	36.06	37.86	37.64	34.85
5	23.12	22.18	22.75	23.20	21.62	23.96	23.02	23.59	24.04	22.44	24.90	23.96	24.52	24.98	23.32	26.73	25.38	25.91	26.30	24.68
10	18.00	18.16	17.72	18.19	16.88	18.80	18.97	18.53	18.98	17.67	19.65	19.83	19.38	19.81	18.47	21.14	21.10	20.64	21.08	19.64
25	13.11	14.67	13.21	13.36	12.35	13.83	15.42	13.95	14.06	13.06	14.56	16.20	14.71	14.74	13.75	15.73	17.31	15.83	16.03	14.71
50	10.28	12.78	10.76	10.50	9.79	10.94	13.50	11.44	11.13	10.44	11.58	14.23	12.14	11.71	11.05	12.56	15.25	13.16	13.03	11.88

BCT: Box-Cox Transformation; LN: Lognormal; LP III Log Pearson Type III; P III: Pearson Type III; WB: Weibull

TABLE 9

Computed values of χ^2 for different values of *d* for BCT, LN, LP III, P III and WB distributions for rivers Mahanadi at Basantpur, Godavari at Pathagudam and Narmada at Mandleshwar sites

Site	Computed values of χ^2 for :																			
	d = 7					d = 10					d = 14					d = 30				
	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB	BCT	LN	LP III	P III	WB
Basantpur	3.472	2.651	3.239	3.909	2.394	3.573	2.880	3.591	4.023	2.625	3.436	2.962	3.734	4.013	2.646	3.006	2.703	3.429	3.698	2.514
Pathagudam	0.561	0.572	0.574	0.343	0.844	0.387	0.499	0.402	0.326	0.856	0.462	0.527	0.410	0.441	1.249	1.074	0.666	0.874	1.475	2.441
Mandleshwar	6.569	3.643	5.633	5.989	9.880	6.234	3.172	5.205	5.789	9.322	6.262	2.917	5.062	5.995	9.290	6.017	2.888	4.984	5.168	9.675

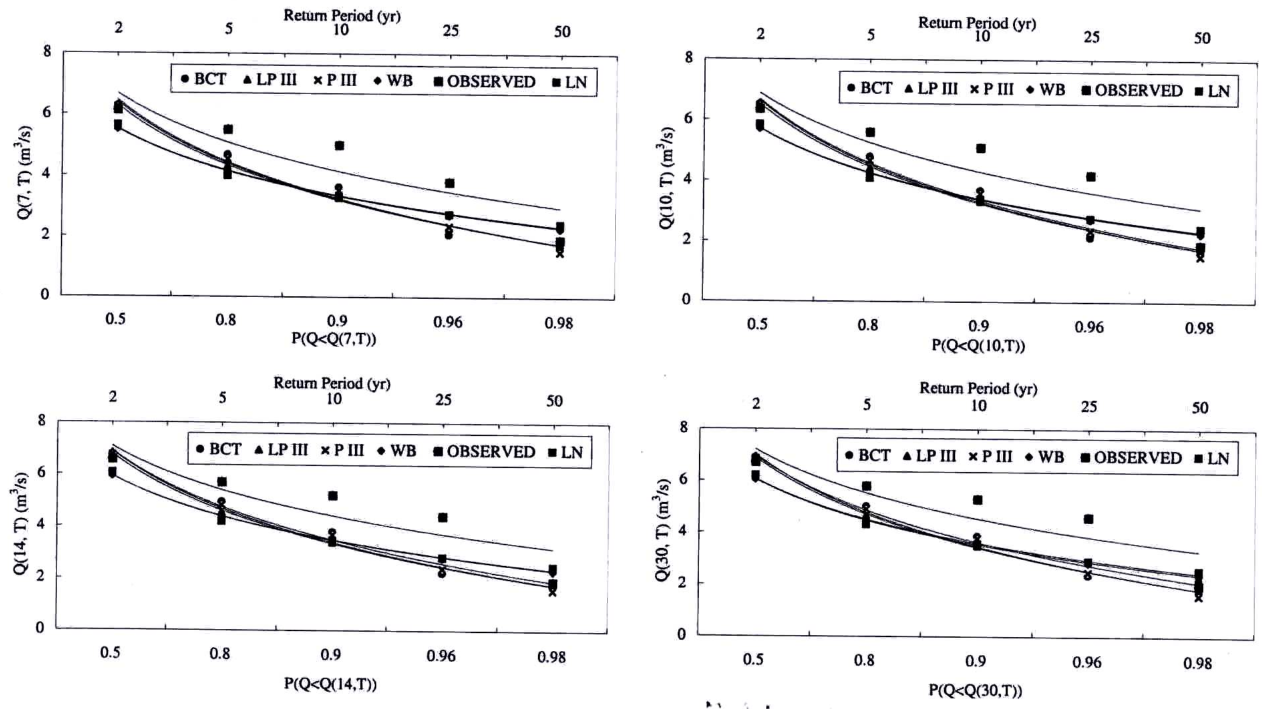

Fig. 1. Low-flow frequency curves using BCT, LN, LP III, P III and WB estimates for different return periods for river Mahanadi at Basantpur site

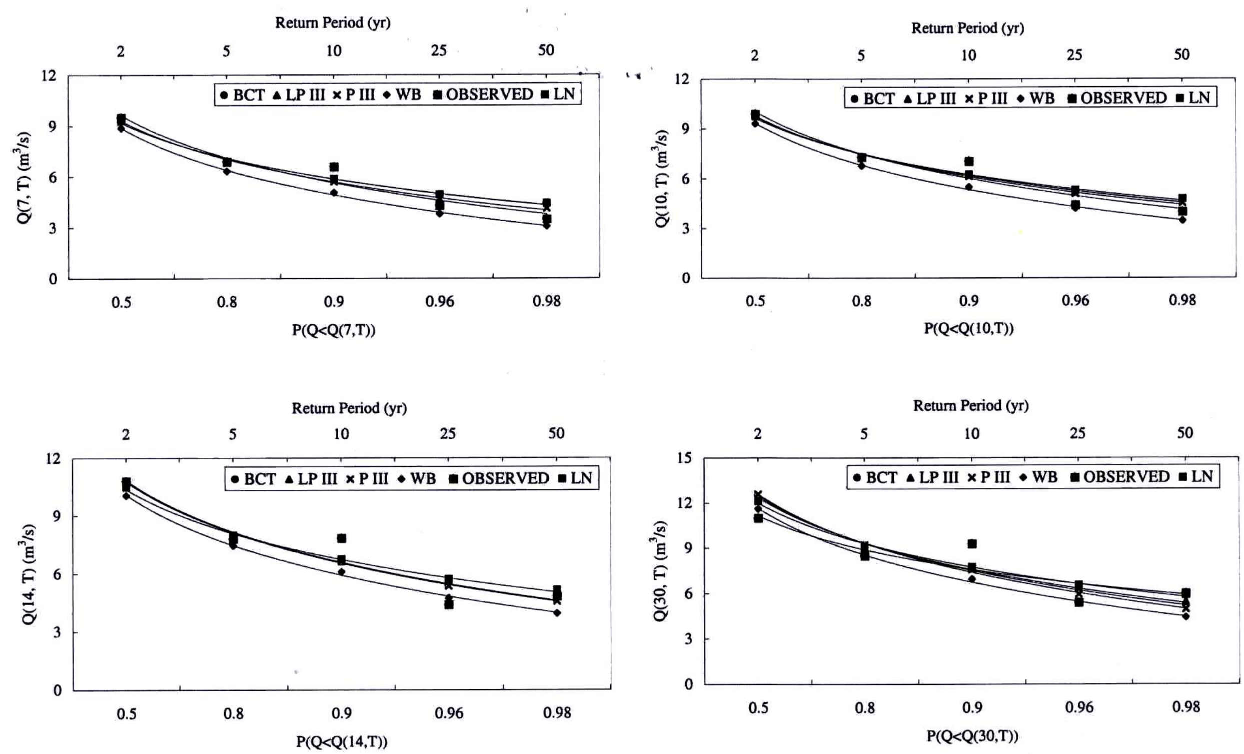

Fig. 2. Low-flow frequency curves using BCT, LN, LP III, P III and WB estimates for different return periods for river Godavari at Pathagudam site

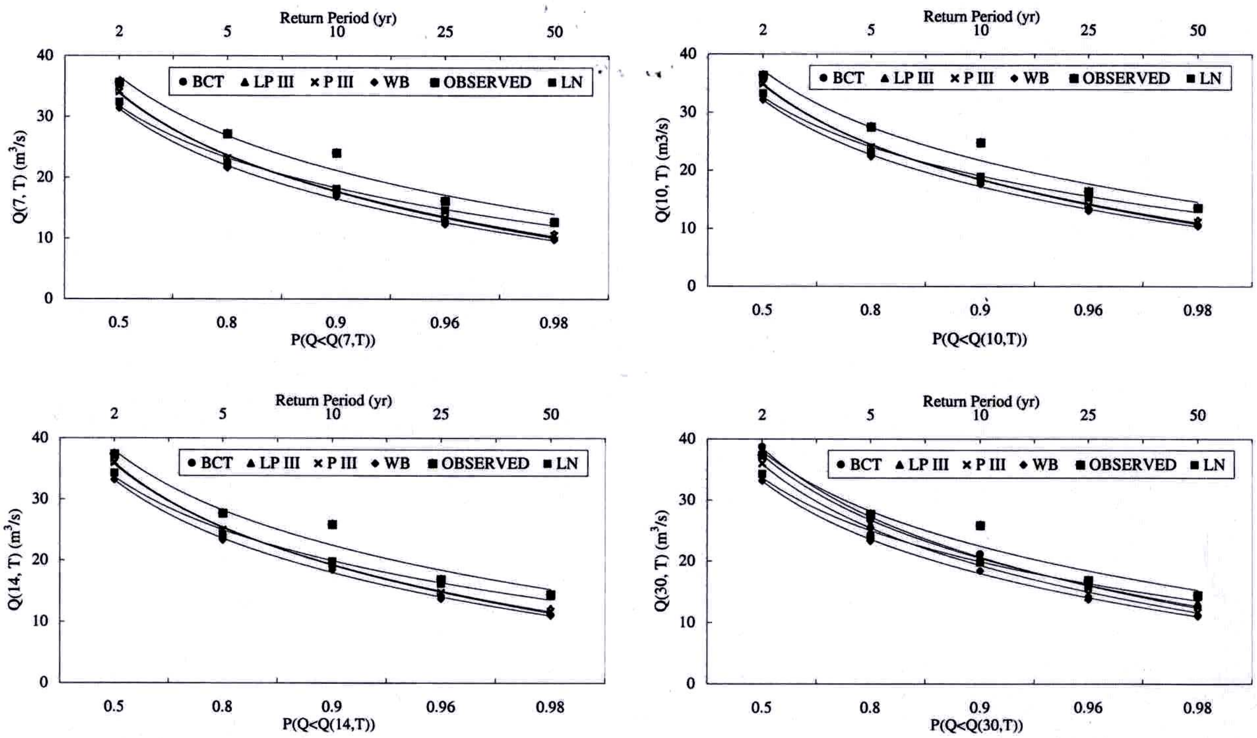

Fig. 3. Low-flow frequency curves using BCT, LN, LP III, P III and WB estimates for different return periods for river Narmada at Mandleshwar site

using the Eqn. (1), λ was determined in such a way that the coefficient of skewness of the transformed series of the annual minimum *d*-day average flow becomes zero. Table 2 gives the parameters of BCT for different values of *d*.

For BCT, the annual minimum *d*-day average flows for different values of *d* such as 7, 10, 14 and 30 days for river Mahanadi at Basantpur site were transformed by using transformation constant (λ) of 1.835, 1.718, 1.644 and 1.560 respectively. Likewise, the different values of *d* computed from annual minimum *d*-day average flows for river Godavari at Pathagudam sites were transformed by 0.015, 0.159, 0.543 and 0.547 respectively. For river Narmada at Mandleshwar site, the values of λ were found to be 0.603, 0.619, 0.633 and 0.635 respectively and the same was used to transform the annual minimum *d*-day average flows.

A computer program was developed and used to determine the annual minimum *d*-day average flows for periods of time of : 7, 10, 14 and 30 days. The parameters (μ_y' and σ_y') of LN distribution were deter-mined by using the set of Eqns. (5-7) for different values of *d* such as : 7, 10, 14 and 30 days from the computed annual minimum *d*-day average flows. Table 3 gives the parameters of LN distribution for different values of *d*.

The parameters of LP III distribution for different values of *d* such as 7, 10, 14 and 30 days were determined by using the sample statistics of mean ($\bar{y}$) and standard deviation (S_y) of the log-transformed series of annual minimum *d*-day average flow and are given in Table 3 as μ_y' and σ_y' respectively. Likewise, the parameters of P III distribution were determined by Eqn. (12) from the recorded value of annual minimum *d*-day average flows. Table 4 gives the parameters of P III distribution for different values of *d*.

By adopting the procedures of PWM, as described earlier, the parameters of the WB distribution were determined. Initially, the first two moments (b_0 and b_1) by using the annual minimum *d*-day average discharges for different values of *d* such as 7, 10, 14 and 30 were determined by using the Eqn. (16) and further used to determine the parameters (α and β) of WB distribution through the Eqn. (18). Table 5 gives the parameters of the WB distribution for different values of *d*.

Further, the parameters of the BCT, LN, LP III, P III and WB distributions for different values of *d* such as 7, 10, 14 and 30 days were used to determine $Q(d,T)$ for different return periods of 2, 5, 10, 25 and 50 years for rivers Mahanadi at Basantpur, Godavari at Pathagudam, and Narmada at Mandleshwar sites by using the corresponding equation of $Q(d,T)$, as defined earlier. The estimates of annual minimum *d*-day average low-flow events for different return periods obtained from Box-Cox Transformation, Lognormal, Log Pearson Type III, Pearson Type III and Weibull distributions for river Mahanadi at Basantpur site are given in Table 6 as BCT, LN, LP III, P III and WB values. Likewise, the estimates of annual minimum *d*-day average low-flow events for different return periods obtained from the above-mentioned distributions for rivers Godavari at Pathagudam and Narmada at Mandleshwar sites are given in Tables 7 and 8 respectively.

3.2. *Low-flow frequency curves*

The low-flow values of $Q(d,T)$ for different values of *d* such as 7, 10, 14 and 30 for different return periods from 2 to 50 years obtained by BCT, LN, LP III, P III and WB distributions were used to develop low-flow frequency curves. Figs. (1-3) give the low-flow frequency curves using BCT, LN, LP III, P III and WB estimates for rivers Mahanadi at Basantpur, Godavari at Pathagudam and Narmada at Mandleshwar sites respectively.

4. Discussions

It may be noticed from the Tables (6-8) that the estimates of annual minimum *d*-day average low-flow events for different return periods obtained from BCT, LN, LP III, P III and WB distributions for rivers Mahanadi at Basantpur, Godavari at Pathagudam and Narmada at Mandleshwar sites are quite distinct from each other. χ^2-test was carried out to judge the applicability of the distribution for estimation of $Q(d,T)$ for different return periods for rivers Mahanadi at Basantpur, Godavari at Pathagudam and Narmada at Mandleshwar sites. Table 9 gives the computed values of χ^2 for different values of *d* such as 7, 10, 14 and 30 days for BCT, LN, LP III, P III and WB distributions for rivers Mahanadi at Basantpur, Godavari at Pathagudam and Narmada at Mandleshwar sites.

From the Table 9, it can be seen that the computed values of χ^2 for different values of *d* such as 7, 10, 14 and 30 days obtained from BCT and WB distributions for river Narmada at Mandleshwar site is greater than the theoretical value of $\chi^2_{2,\ 0.95}$ and are significant at 5 percent level of significance. On the other hand, it may be noted that the computed values of χ^2 for different values of *d* such as 7, 10, 14 and 30 days obtained from BCT and WB distributions for rivers Mahanadi and Godavari at Basantpur and Pathagudam sites respectively are less than the theoretical value of $\chi^2_{2,\ 0.95}$ and are not significant at 5 percent level of significance.

From the results of the data analysis, it may be further noted that the computed values of χ^2 for different values of *d* such as 7, 10, 14 and 30 days obtained from LN, LP III and P III distributions for rivers Mahanadi at Basantpur, Godavari at Pathagudam and Narmada at Mandleshwar sites are less than the theoretical value of $\chi^2_{2,\ 0.95}$ and are not significant at 5 percent level of significance.

By considering the variation in the magnitude of χ^2-test statistic values for different values of *d* such as 7, 10, 14 and 30 days, two-parameter Lognormal distribution is found to be better suited for determination of low-flow characteristics for river Narmada at Mandleshwar site. Likewise, Weibull and Pearson Type III distributions are found to be suitable for determination of low-flow characteristics for rivers Mahanadi and Godavari at Basantpur and Pathagudam sites respectively.

5. Concluding remarks

The paper described a procedure for fitting various frequency distributions such as Box-Cox by transformation method, Lognormal by MLM, Log Pearson Type III and Pearson Type III by MOM and Weibull by PWM distributions for low-flow stream flow data. The paper also described the methodology involved in evaluating the frequency and magnitude of annual minimum *d*-day average low-flows at a gauged site for different recurrence intervals with *d* taking the values 7, 10, 14 and 30 days. From this paper, it is observed that Log Normal, Weibull and Pearson III distributions are found to be suitable for determination of low-flow stream data for rivers Narmada, Mahanadi and Godavari respectively. It is also observed that no uniform frequency distribution fits to all types of stream flow but different distributions fits to various stream flows. The low-flow frequency curves are developed and presented in this paper.

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