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Distribution of horizontal wind components in the Surface Boundary Layer (SBL) over Sriharikota

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सार – इस शोध-पत्र में क्षैतिज पवन अवयव वितरण के लिए द्विचर सामान्य वितरण (बी. एन. डी.) का उपयोग सैद्धांतिक निदर्श के रूप में किया गया है। इस निदर्श को प्रारंभ करने के लिए पाँच प्राचलों अर्थात पवन अवयवों के औसत मानों (U, V) अवयवों के मानक अपसरणों (σ_u, σ_v) और पवन अवयवों (u एवं v) के मध्य परस्पर सह संबंध और अंतः स्तरीय परस्पर सह संबंध (ρ_{uv}) की आवश्यकता होती है। उपर्युक्त आँकड़ों का उपयोग करके पवन वेगों का संभावित वितरण तैयार किया जा सकता है जो पवन–पथ के लिए सैद्धांतिक निदर्श प्रदान करता हैं। बी. एन. डी. निदर्श के द्वारा तैयार किए गए पवन-पथ के संभावित वितरण से पवन की संभाव्य स्थितियों में बिना अधिमूल्यांकन अथवा अधोमूल्यांकन के किसी भी तरह के समन्वयों (क्षेत्रीय और याम्योत्तर अथवा किसी तरह के परिवर्तित समन्वय अवयवों) में क्षैतिज पवन अवयवों के वितरण का मूल्यांकन किया जाता है। पवन के चलने के स्थान से इसके पथ के किसी विशेष चतुर्थांश से समय की प्रतिशतता का पता चलता है इसके लिए कुछ भाग का या उस स्थान का बी. एन. डी. से निगमन किया जा सकता है। इस शोध–पत्र में द्विचर सामान्य संभाविता दीर्घवृत संरचना प्रक्रियाओं और एक उष्णकटिबंधीय तटीय स्थान के लिए भिन्न-भिन्न महीनों के लिए एस. बी. एल में विभिन्न स्तरों पर तैयार किए गए दीर्घवृत्तों के लक्षणों पर विचार-विमर्श किया गया है।

ABSTRACT. This paper brings out the use of Bivariate Normal Distribution (BND) as theoretical model for horizontal wind component distribution. The generation of this model requires five parameters *viz*., mean values of wind components (U, V), standard deviations of components (σ_u,σ_v) and correlation coefficient between wind components (u and v), the intra-level correlation coefficient (ρ_{uv}). By the use of the above statistics, probability distribution of wind vectors can be constructed which gives a theoretical model for wind vectors. This probability distribution of wind vectors constructed through BND model can be able to assess distribution of horizontal wind components in any coordinates (zonal and meridional or any transformed coordinate components) without overestimation or underestimation in the probable wind conditions. The percentage of time that the wind vectors come from a specific quadrant, some portion or from a spot can be deduced from BND. Bivariate normal probability ellipses construction procedures, the features of constructed ellipses for different levels in the SBL for different months for a tropical coastal station are discussed in this paper.

Key words – Bivariate normal distribution, Gaussian distribution, Intra-level correlation coefficient, Meridional wind, Univariate normal distribution, Zonal wind, Wind rose.

1. Introduction

The use of wind modelling studies to industry is essential since wind is of vital importance to all phases of commerce-like wind energy studies, runway constructions, structural design works and air pollution impact assessments. Winds are three dimensional but for almost all direct application purposes, especially in engineering needs, they are considered only in their two dimensional form along horizontal surfaces. Crutcher (1962) described the estimation of frequency of winds from any given point, sector or area is only possible with the use of estimates of statistical parameters of a wind distribution. Crutcher (1957) and Scott (1956) provided the general bivariate normal elliptical distribution in order to represent sets of data. Smith *et al*., (1990) made wind models for aerospace vehicle ascent trajectory biasing for wind load alleviation through bivariate normal probability distribution.

Various problems in dealing with vector quantities require the integral of elliptical Bivariate Normal Distribution (BND). The estimation of the frequency of winds from any given point, sector or area is possible by use of estimates of the statistical parameters of a wind distribution. This paper presents the representation of wind component modelling through BND as a theoretical model for wind vector distribution. It is known that the wind components, zonal and meridional can be separately modelled through Univariate Normal Distribution (UND) or Gaussian Distribution and scalar wind speed through Weibull Distribution (especially for peak wind speed). But through Gaussian Distribution, the relation connecting zonal and meridional wind components is not considered, but this relation can be established through intra-level correlation coefficient. Crutcher (1962) mentioned in the SBL that BND may not be suitable, but investigations carried out in the present study with voluminous data shows BND ellipses can be satisfactorily fitted for wind components, *i.e.*, u and v distribution together. The objective of this paper is to present the horizontal wind components distribution through BND model in the SBL which is considered superior to Conventional Wind Rose (CWR) and UND. Properties of BND ellipses modelled in SBL over the tropical Indian coastal station Sriharikota (13.7° N / 80.2° E) are also discussed.

2. UND and its limitations in wind vector distribution

In meteorological coordinate system, wind vector (raw data) can be resolved into zonal (u) and meridional (v) components. The component distribution can be modelled by UND separately. The probability density function (*pdf*) of UND for zonal can be

$$P_g(u) = 1/\sqrt{2\Pi}\,\sigma_u \exp[-(u-U)^2 / 2\sigma_u^2]\,; \quad -\infty \le u \le \infty \tag{1}$$

similarly for meridional $P_g(v)$

$$P_g(v) = 1/\sqrt{2\Pi}\,\sigma_v \exp[-(v-V)^2 / 2\sigma_v^2]\,; \quad -\infty \le v \le \infty$$

where U, V, σ_u, σ_v are mean and standard deviation of zonal and meridional components respectively. Numerical values of probability that u or v falls in the interval (u_1 and u_2 or v_1 and v_2);

$$P_g\,[u_1 = U - t\,\sigma_u \le u \le u_2 = U + t\sigma_u] \text{ and}$$

$$P_g\,[v_1 = V - t\,\sigma_v \le v \le v_2 = V + t\sigma_v] \tag{2}$$

where t = 1.6449 for 90% value probability and t = 1.9602 for 95% value probability based on theory of normal distribution. In this UND, the correlation between u and v, the intralevel correlation coefficient (ρ_{uv}) is not taken into account. This results in some improbable values of wind vectors for a given percentile.

3. Bivariate normal distribution model

The philosophy behind the combined distribution holds good for u and v components which results in BND. When the two variables u and v are correlated, it is a necessary but not a sufficient condition for the marginal distribution $f(u)$ and $f(v)$ to be normal and the joint distribution to be BND.

The probability density function for BND is

$$P_b\,(u, v) = 1 / (2\Pi\sigma_u\sigma_v)\sqrt{(1-\rho_{uv}^2)} \exp[-1/2\,(1 - \rho_{uv})^2 \{(u - U)^2 / \sigma_u^2 - 2\rho_{uv}\,(u-U)\,(v-V) / \sigma_u\sigma_v + (v - V)^2 / \sigma_v^2\}] \quad -\infty \le u \le \infty,\ -\infty \le v \le \infty \tag{3}$$

The probability distribution function can be derived by setting the terms of the exponent of Equation (3) to a constant λ as

$$\{(u - U)^2/\sigma_u^2 - 2\rho_{uv}\,(u-U)\,(v-V)/\sigma_u\sigma_v + (v - V)^2/\sigma_v^2\} = \lambda^2 \tag{4}$$

The Equation (4) is recognised as a family of ellipses depending on the value of λ^2. The density function has constant values on these ellipses. So the ellipses generated out of Equation (4) are referred to as ellipses of equal probability. The percentile ellipses can be generated based on λ value variations as

$$\lambda = \sqrt{2}\,\{\sqrt{-\ln\,(1-P)}\} \tag{5}$$

where P is the probability. For 90% probability ellipse λ = 2.146 and 95% probability ellipse λ = 2.447.

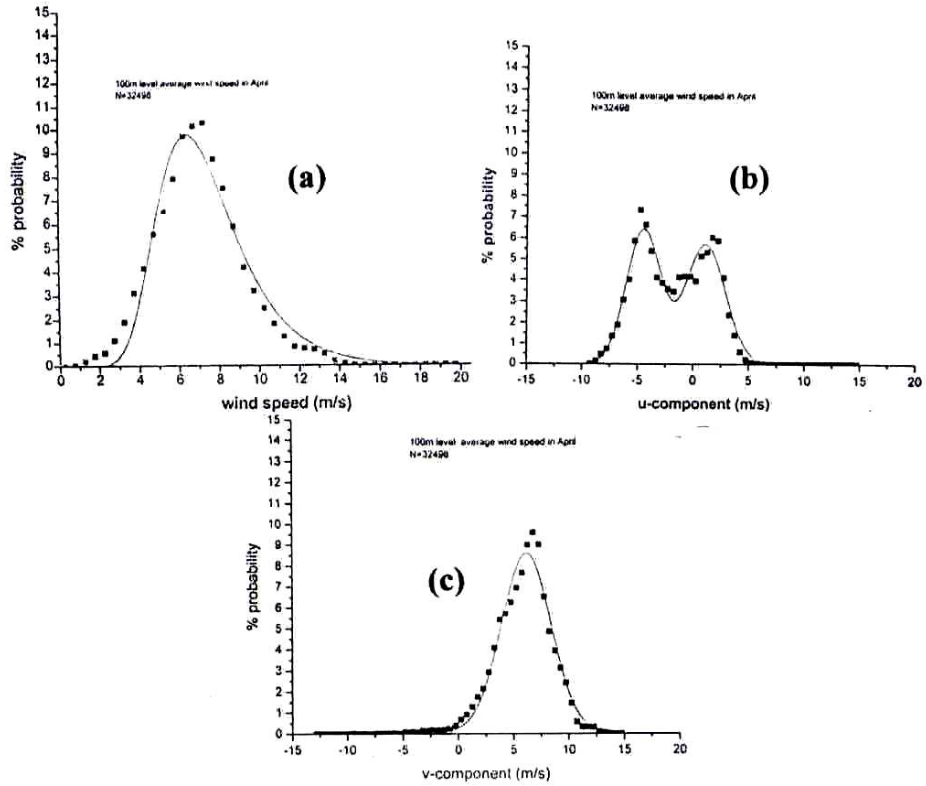

Fig. 1. (a) Weibull distribution of peak wind speed, (b&c) Gaussian distribution of wind component (u, v) in April at 100m level

4. Methodology for the generation of BND ellipses

In order to handle the Eqn. (4) for the generation of BND ellipses the following steps are adopted. The Eqn. (4) is rewritten as

$$AX^2 + BXY + CY^2 + DX + EY + F = 0 \tag{6}$$

where,

$A = \sigma_v^2$

$B = -2\,\rho_{uv}\,\sigma_u\,\sigma_v$

$C = \sigma_u^2$

$D = BV + 2\,AU$

$E = -(BU + 2\,CV)$

$F = AU^2 + CV^2 + BUV - AC\lambda^2$

For constructing the ellipse, the range of variables *i.e*., the smallest and largest values of X and Y for a given probability ellipse P are given by $X_{L\rightarrow S} = U \pm \sigma_u\,\lambda$ and $Y_{L\rightarrow S} = V \pm \sigma_v\,\lambda$. So once the statistical parameters such as mean, standard deviation and intra-level correlation coefficient (U, V, σ_u, σ_v, ρ_{uv}) are obtained for a given set of wind samples, the desired probability ellipses can be constructed by the following steps.

(*i*) Increment the values of X within the limit (*i.e*., $X_{L\rightarrow S} = U \pm \sigma_u\,\lambda$).

(*ii*) Form a quadratic equation in Y from Eqn. (6) as $CY^2 + (BX + E)\,Y + (AX^2 + DX + F) = 0$ and can be in reduced form as $CY^2 + ZY + W = 0$ (7)

where $Z = BX + E$ and $W = AX^2 + DX + F$

(*iii*) Put each increment value of X in this quadratic equation and find out the roots of Y as

$$Y = [\,-Z \pm \sqrt{Z^2 - 4\,C\,W}\,] / 2\,C \tag{8}$$

(*iv*) The two roots obtained for Y represent two points in the ellipse. Locus of roots generated for different values of X from X_L to X_S together with X values will lead to the completed elliptical BND model for wind component distribution.

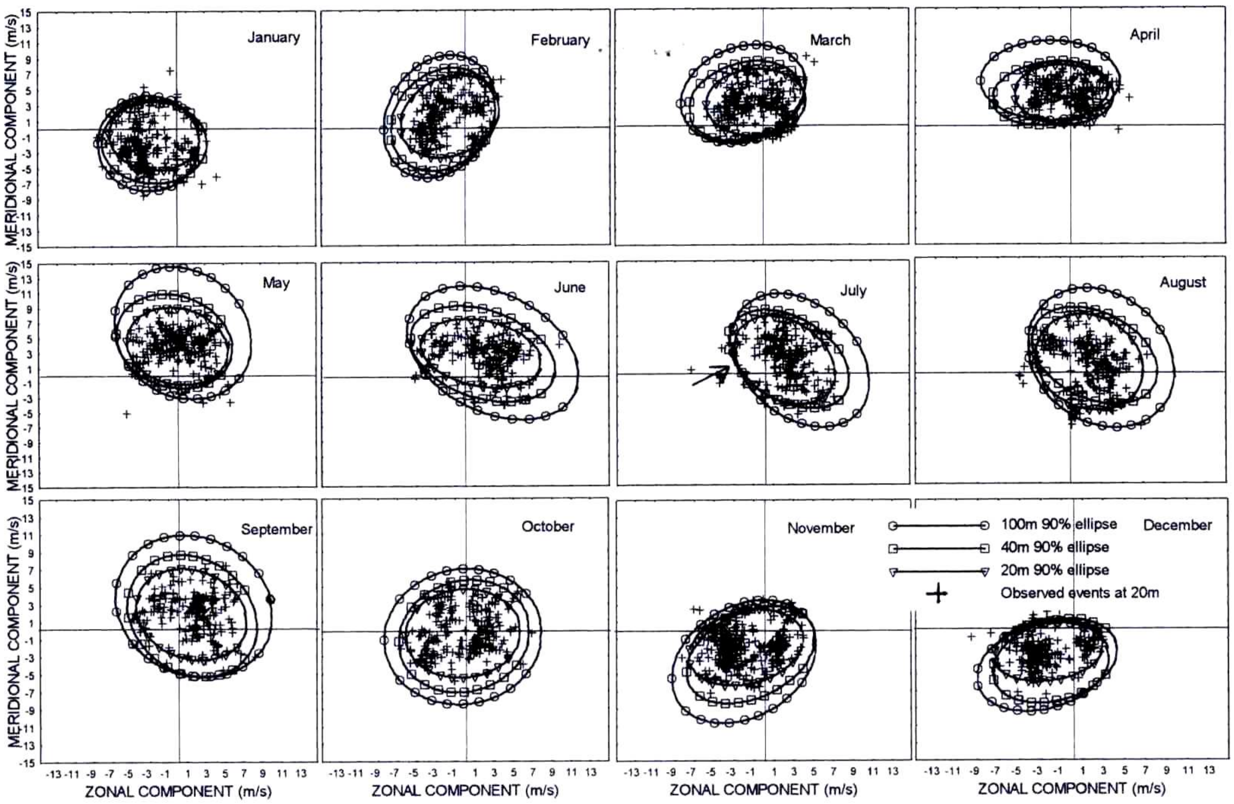

Fig. 2. Bivariate normal distribution ellipses for different months

5. Features of 90% BND ellipses in the SBL over Sriharikota

5.1. *Data*

Speed and direction data from three levels *viz*., 20, 40, and 100 m for every five minutes from May 1993 to April 1996 is used as the input. The voluminous raw data, in speed and direction is resolved into components. For each month from all the available data on zonal and meridional wind components, the five statistical parameters (U, V, σ_u, σ_v, ρ_{uv}) are obtained for different levels in the SBL. The five parameters are used to construct BND ellipse for a level pertaining to a particular month. The ellipses are generated for 90% probability values by the methodology explained above. Also in order to assess the Gaussian distribution of wind components, the above said data is used and for Weibull distribution model on peak wind events, peak occurred in every 5 min. is used.

5.2. *Results on the properties of ellipses*

To have a check on the theoretical understanding as distribution of wind component through Gaussian and peak wind scalar speed occurrence as Weibull, analyses are carried out for the month April for the level 100 m and is shown in Fig. 1. Both scalar and component distributions are shown excellent fitting of their respective theoretical distributions. Fig. 2 shows generated ellipses in the SBL level over Sriharikota for different months. 90% ellipses alone are drawn for levels 20, 40 and 100 m. Some scattered points of 20 m level in the observed events of u and v components are incorporated in the BND ellipses to verify the validity of the elliptical distribution model constructed.

In all months, the distribution is elliptical, except in January where it is almost circular. But circular distribution is also a form of elliptical BND. In January the intra-level correlation coefficient (ρ_{uv}) is nearly 0.0.

Ellipse size increases with height and the orientation is based on the prevailing wind direction of the season. In all levels, the ellipse quadrant having more area possesses more number of occurrence of events. The scatter of observed points at 20 m level fall within the 20 m ellipse in an elongated fashion in conjunction with the orientation of the major axis of the ellipse.

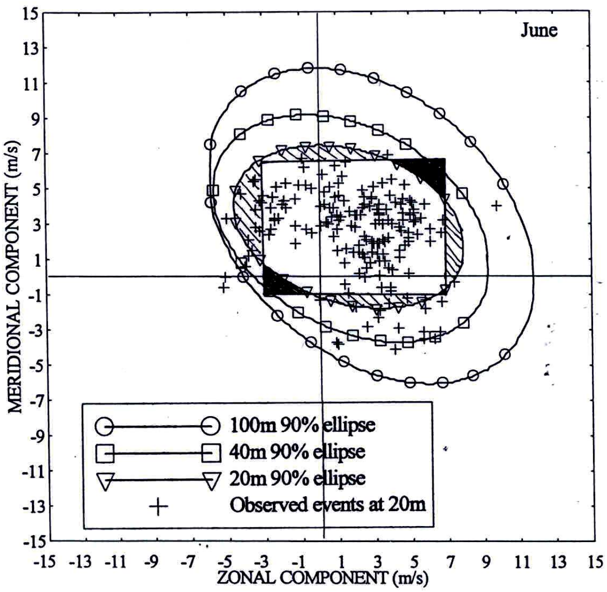

Fig. 3. A comparison of 90% BND ellipse and 90% rectangle for 20 m level

One of the striking feature observed, is the joining of all level ellipses or coming closer to each other in a point as marked in Fig. 2 (for July) with an arrow. The point may be treated as a point of least probable occurrence of events. This least probable occurrence of events, whenever it occurs is uniformly experienced or distributed at all levels. That is, if any improbable wind feature is noticed in any level of SBL, same feature will occur in all levels. This feature is not seen in October, which may be interpreted as the non occurrence of least probable events. In all respects, the analysis summarised the well organised bivariate normal elliptical distribution models in SBL for any level and any month.

6. Comparison of BND and UND

A comparison between BND and UND has been carried out. June is considered as a representative month for the comparison study. 90% probability rectangle is constructed for 20 m level for June month with values U + 1.644 σ_u = 6.5 ms^{-1} , U - 1.644 σ_u = -3.1 ms^{-1}, V + 1.644 σ_v = 6.4 ms^{-1} and V- 1.644 σ_v = -0.7 ms^{-1} as vertices of the rectangle in order to represent UND. This rectangle constructed is superimposed with the 90 % BND ellipse constructed at 20 m level. The analysis is shown in Fig. 3, in which improbable values have fallen outside the ellipse which shows the ability of BND to keep all probable events within the probability ellipse assigned. The darkened area in the 90 % rectangle contains some events, but they are outside the 90 % ellipse constructed, which shows overestimation of UND rectangle for probable events. Probable values have fallen inside the 90% ellipse itself, but outside the 90% rectangle (hatched area). This hatched area shows 90 % probable values are fallen inside 90 % ellipse, but there are a number of events which are not fallen in the 90 % rectangle, so that underestimating the probability of occurrences. From this analysis, it can be concluded that modelling with BND gives correct probable winds in wind components distribution, and UND may lead to overestimation or underestimation of wind conditions. Transformation of coordinates from zonal and meridional to lateral and longitudinal components is possible through BND model.

7. Advantages of BND over Conventional Wind Rose (CWR)

CWR has the usual breakdown of winds into direction and speed groups without any mention about the componentwise distribution. BND gives an estimate of percentage of time that the vectors have a speed equal to or less than a selected value (*e.g.*, a zonal wind speed of 9 ms^{-1}). BND gives an estimate of the percentage of time that the wind vectors come from a specific circular or elliptical area within certain speed limits (*e.g.*, the area which comprises 90 % of the data). It also gives a knowledge of the percentage of time that the wind vectors come from a specific quadrant or some portion (*e.g.*, what is the frequency of SW wind or exceeding SW wind of 9 ms^{-1}.). Through BND describes a possibility how the climatological statistics of the local horizontal wind vector, *i.e.*, speed and direction in combination can be expressed by only few parameters with the help of a theoretical statistical model. This has certainly the advantage when comparing different climatic regions without being forced to show big tables or complicated figures like conventional wind roses. BND probability ellipses can be constructed by using any perpendicular coordinates other than zonal and meridional directions, which can find many applications in boundary layer wind modeling (for *e.g.*, parallel and perpendicular wind components distribution for a coastal station-which can provide better understanding about sea-breeze component distribution.)

8. Conclusions

Bivariate normal distribution statistics derived for wind components distribution has provided a proper theoretical model for wind vectors distribution in SBL over Sriharikota. Its capability is assessed as superior to conventional wind rose and univariate normal distribution. Bivariate normal ellipses are formed for all SBL levels. Ellipse sizes are increased from lower levels to upper levels, and they orient according to the seasonal wind direction climatology over the station. Any least probable event occurrence in SBL winds is observed in equal distribution for all levels, depicted by bivariate normal distribution ellipses for different months. Comparison between bivariate normal distribution and univariate normal distribution showed that bivariate normal distribution gives probable winds whereas univariate normal distribution may lead to overestimation or underestimation of wind conditions. In bivariate normal distribution model, transformation of coordinates can be made possible. Bivariate normal distribution has shown its capacity to generate information regarding the percentage of time that the wind vectors come from a specific quadrant or some portion. The applications of the BND model may be suitably applied in many areas of operational meteorology where wind is playing a crucial role, such as in wind energy studies, air pollution dispersion, in the testing of rifles, artillery or missiles (ballistic meteorology) and in aviation meteorology (to know the probability of head wind and cross wind components along a runway or even in the design of a runway.)

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