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Mountain waves over northeast India and neighbouring regions

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ABSTRACT. Theoretical computations of the wave lengths of mountain waves have been made from the available wind and temperature data over northeast India and neighbouring regions. The results have been compared with the observational data and have been found to agree with them.

1. Introduction

The influence of orography on the flow of air, cloud patterns and rainfall are well known. Formation of typical wave clouds on the lee of mountains is one of such phenomena. It has been theoretically established that under suitable conditions of thermal stability and wind flow, waves are formed on the lee of mountains. These are also supported by observational studies made in different parts of the world with the help of gliders, light airplanes and radar tracked no-lift balloons etc.

In recent years cloud pictures have been made use of by Doos (1962), Fritz (1965), Cohen *et al.* (1966, 1967) to identify mountain waves. De (1970) reported the occurrence of mountain waves in the Assam-Burma hills with the use of APT pictures from ESSA-3. He worked out the wave lengths of these waves and also discussed their formation in relation to the atmospheric wind and thermal structure. In the cases studied by him, he found that the conditions of static stability and the direction of wind flow with respect to the mountain ranges as required by theory were satisfied. In the present study an attempt has been made to work out from theory the wave lengths of mountain waves and compare them with observed wavelengths. The data used in this study are the same as used earlier and taken from the cloud pictures from ESSA-3 during the period 1966-68.

2. Theory of computation

In order to verify the observed values of wave lengths of mountain waves, a two-dimensional steady state linear model (Sarker 1965, Sawyer 1960) has been considered.

The equation for vertical velocity for such a model in x-z plane is given by,

$$\frac{\partial^2 w_1}{\partial x^2}+\frac{\partial^2 w_1}{\partial z^2}+f(z)\,w_1=0 \qquad (1)$$

where, $w=w_1\exp\left(\frac{g-R_v}{2RT}z\right)$

$$f(z)=g\left(\frac{r^*-r}{U^2T}\right)-\frac{1}{U}\frac{d^2U}{dz^2}+\left(\frac{r^*-r}{T}-\right.$$
$$\left.-\frac{g}{\chi RT}\right)\frac{1}{U}\frac{dU}{dz}-\frac{2}{\chi RT}\left(\frac{dU}{dz}\right)^2-\left(\frac{g-R_v}{2RT}\right)^2 \qquad (2)$$

U and T represent wind speed and temperature in the undisturbed airflow which is a function of z only,

r^* is the dry adiabatic lapse rate,
r is the actual lapse rate, and
$\chi = C_p / C_v = 1.4$

The values of $f(z)$ have been computed by using the radiosonde-rawin data of Gauhati (26° 06′N, and 91° 35′E). The computed profiles of $f(z)$ with height showed that the required conditions for the formation of lee waves were fulfilled. Two typical cases are shown in Figs. 1 and 2.

(a) *Analytical Method*

To solve equation (1) analytically the values of $f(z)$ are approximated by an exponential function of the type—

$$f(z)=f_0\,e^{-\lambda z} \qquad (3)$$

where, f_0 and λ are constants chosen to represent $f(z)$ in the entire atmosphere. Further, the ground profile is assumed to be sinusoidal. For this we write,

$$w_1 = W\,e^{ikx} \qquad (4)$$

Substituting (3) and (4) in (1) we get,

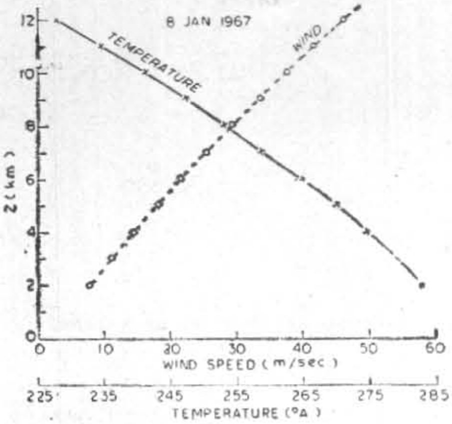

Fig. 1(a)

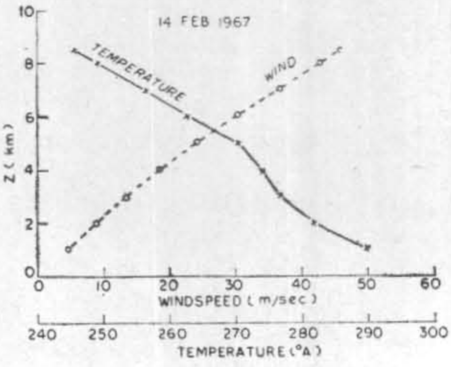

Fig. 2(a)

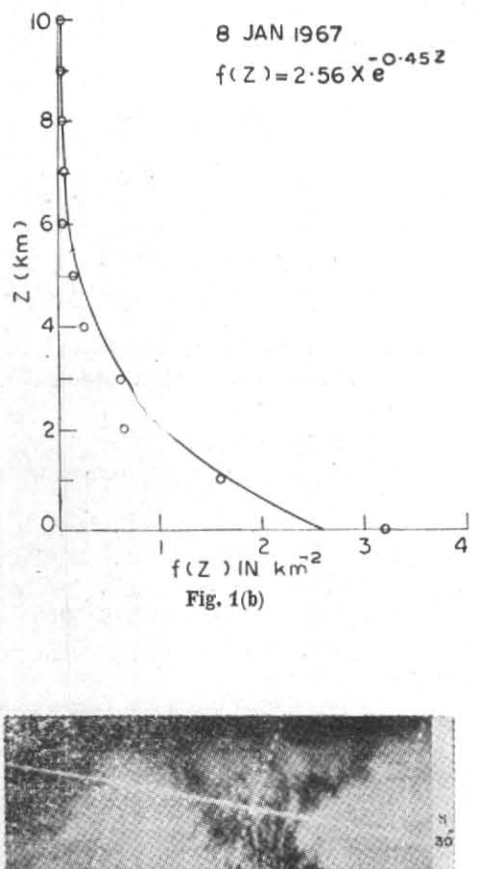

Fig. 1(b)

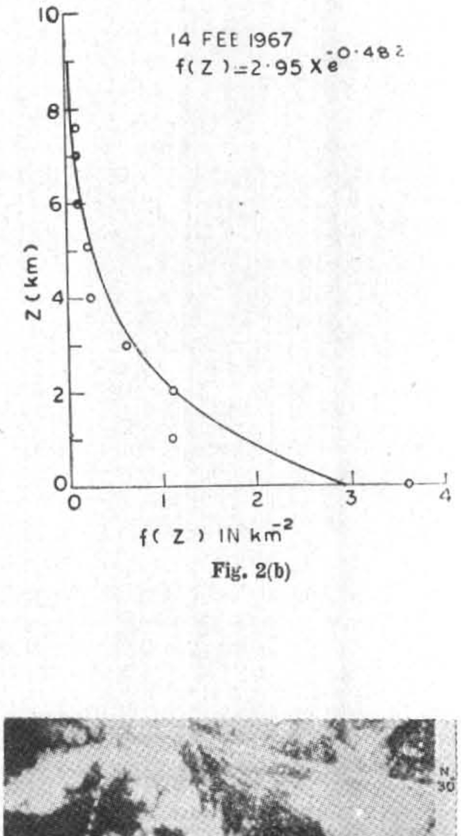

Fig. 2(b)

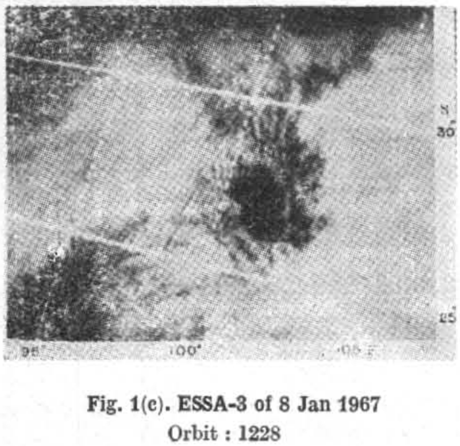

Fig. 1(c). ESSA-3 of 8 Jan 1967
Orbit : 1228

Fig. 2(c). ESSA-3 of 14 February 1967
Orbit : 1693

$$\eta^2 \frac{d^2W}{d\eta^2} + \eta \frac{dW}{d\eta} + \left(\eta^2 - \frac{4}{\lambda^2} x^2\right) W = 0 \quad (5)$$

where, $\eta = \beta e^{-\lambda z/2}$

and $\beta = \frac{2}{\lambda} (f_0)^{\frac{1}{2}}$

The wave length is then given by the roots of the equation,

$$Jm(\beta) = 0 \quad (6)$$

where $m = 2k/\lambda$

and $Jm(\beta)$ is a Bessel function of real order m and real argument β.

Knowing the value of β we determine the value of m such that equation (6) is satisfied. From wave number k the wavelength can be determined.

(b) *Numerical Method*

Using a two-dimensional mountain wave model, Sawyer (1960) gave an expression for computing vertical velocity numerically. He solved the equation of the type of the original Scorer's (1949) problem,

$$\frac{\partial^2 \psi}{\partial z^2} + (l^2 - k^2)\psi = 0$$

where ψ is the stream function and l^2 is Scorer's parameter.

In equation (1) if we substitute,

$$w_1(x,z) = \int_0^\infty w'(x, k) \exp(ikx)\, dk \quad (7)$$

it reduces to,

$$\frac{\partial^2 w'}{\partial z^2}(z, k) + [f(z) - k^2]\, w'(z, k) = 0 \quad (8)$$

Then the expression for the vertical velocity becomes,

$$w'(z,k) = U(0)\, ab \int_0^\infty \frac{\psi(z,k)}{\psi(0,k)} \exp(-ka)\, dk \quad (9)$$

where, $\psi(z, k)$ is a function similar to $w'(z, k)$ and satisfies the equation (8). Equation (9) becomes indeterminate when $\psi(0, k) = 0$. These singularities of the integrand correspond to the occurrence of lee waves as discussed by Scorer (1949).

Following Sawyer (1960) and Sarker (1967), the function $\psi(z, k)$ was computed numerically using a finite difference form of equation (8) replacing $w'(z,k)$ by $\psi(z,k)$ with appropriate boundary conditions over a large range of wave numbers (k). The roots of $\psi(0,k) / \psi(z,k)$ were determined by iteration. The roots when finally determined give the wave lengths (numbers) for the lee waves.

The computed values by the above two methods together with the observational results are shown in Table 1. The agreement between these values is good.

TABLE 1

Date	Observed wavelength (km)	Computed wave-lengths (km)	
		Analytical	Numerical
23 Nov 1966	23	75·3	25·4
		19·6	—
		10·7	10·9
		6·8	6·6
1 Dec 1966	20	20·9	22·2
		8·1	7·4
8 Jan 1967	22	26·3	—
		7·8	—
9 Jan 1967	22	22·0	19·5
		7·5	6·1
13 Feb 1967	31	—	26·1
10 Feb 1967	17	—	17·1
			6·7
14 Feb 1967	22	23·6	—
		6·9	
5 Mar 1967	21	—	20·1
			7·2
9 Feb 1968	23	—	22·2

3. Conclusions

(*i*) The air stream in the winter months have the required static stability and vertical wind shear to give rise to mountain waves in Assam (India) and neighbouring Burma-China region.

(*ii*) On all the occasions the observed wave lengths of mountain waves were found to vary between 20-30 km approximately.

(*iii*) Theoretical and numerical computations of such cases for which adequate radiosonde and rawin data was available confirm the existence of such waves as deduced from the computed values of wave lengths. In addition shorter wave lengths of the order of 5-10 km were also obtained which are, of course, not important for broad mountains.

Acknowledgements

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REFERENCES

Cohen, A., Doron, E. and Wiener, S.	1966	Meteorological Satellite data of report of studies report of work performed under contract C.W.B. 11055.
Cohen, A. and Doron, E.	1967	*J. appl. Met.*, **6**, 4, pp. 669-673.
De, U. S.	1970	*Indian J. Met. Geophys.*, **21**, 4, p. 637.
Doos, B. R.	1962	*Tellus*, **14**, pp. 301-309.
Fritz, S.	1965	*J. appl. Met.*, **16**, 1, pp. 31-37.
Sarker, R. P.	1965	*Indian J. Met. Geophys.*, **16**, pp. 565-584.
	1967	*Mon. Weath. Rev.*, **95**, 10, pp. 673-684.
Scorer, R. S.	1949	*Quart. J. R. met. Soc.*, **75**, p. 41.
Sawyer, J.	1960	*Ibid.*, **86**, pp. 326-345.

DISCUSSION

DR. K. R. SAHA : How did you compute the $f(z)$ profile shown, analytically or numerically ?

SHRI U. S. DE : The values of $f(z)$ were computed numerically from the observed distributions of wind and temperature.

SHRI G. GURUNADHAM remarked that mountain waves also occur over middle east. But the wave length is less than over the Assam hills. They are also not persistent unlike in Assam.

DR. D. M. PATEL : You have assumed $f(z) = f_0 e^{-\lambda z}$ for working out results of wavelengths. Would you have not got other results if you had assumed any other typical function in your studies ?

SHRI DE : The results were obtained by approximating with an exponential function. Again the values of wavelength were obtained by using the actual values of $f(z)$ numerically ; and the results were not very much different.

SHRI R. K. DATTA : You have got mountain waves of average wavelength of 23 km. Have you checked these with aircraft reports ?

SHRI DE : A few aircraft reports on turbulence are available, but they are too qualitative and do not throw any light on the presence or otherwise of mountain waves.

SHRI JAGAN MOHAN RAO : I find that in addition to the wavelength which agrees roughly with that arrived at by numerical methods, there are four subsequent wavelengths given by the analytical method. What do they represent ?

SHRI DE : An air stream with a particular wind shear and stability may give rise to several waves of different wavelengths. But of these only the wave which matches with the dominant wavelength of the mountain will be significant. It appears that only the waves of length 20-30 km give sufficient amplitude for the mountain concerned and, therefore, these waves could be observed from satellite pictures, even though shorter wavelengths, may be present. Computed wavelengths in this range by analytical and numerical methods agree reasonably well with those observed from satellite pictures.

DR. P.R. PISHAROTY : Have you noticed mountain waves along west coast of India or northern India?

SHRI DE : Although it is theoretically possible, no such waves have been noticed by me along the west coast. During winter, moisture may not be sufficient to produce wave clouds and in the monsoon months wave clouds, if any, may not be discernible in the general overcast.

SHRI G. S. GANESAN : What is the likely error in the measurement of wavelengths from the satellite pictures ?

SHRI DE : The wavelength was measured from a group of waves and the measurements were averaged. The likely error is of the order of 1 km.