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BAROMETRIC TENDENCY EQUATION FOR AN ATMOSPHERE OF FINITE EXTENT

An atmosphere of infinite vertical extent is assumed, as a rule, for calculations from theoretical considerations of barometric pressure in meteorological text books. From knowledge of the upper atmosphere gained in recent times the non-infinite character of the terrestrial atmosphere has been brought out with some definiteness. Beyond the so-called "fringe" region (Mitra 1952) 800—900 km above the earth's surface the atmospheric gases escape into space. Considerable knowledge regarding tidal and other oscillations of the different ionospheric layers has been gained through recent investigations (Mitra 1952, Mitra 1950).

Let us assume that the mean sea level is the base level which can be uniquely defined with respect to the earth's centre. Let H be the height of the "free surface" of the atmosphere vertically above any point A whose height above the base level is h. For our purposes H is a variable dependent only upon time t, while h is a constant.

The pressure at the point A is

$$p_A = -\int_h^H g\rho dz \qquad \ldots\ldots \text{(i)},$$

where z is assumed to increase vertically upwards. Then the barometric tendency at the point A is given by

$$\frac{\partial p_A}{\partial t} = -\frac{\partial}{\partial t}\int_h^H g\rho dz \qquad \ldots\ldots \text{(ii)}.$$

The integral on the right-hand side of equation (*ii*) is differentiable since the integrand is continuous in the interval (h, H).

$$\therefore \frac{\partial p_A}{\partial t} = -\int_h^H g\frac{\partial \rho}{\partial t}dz - (g\rho)_H\frac{\partial H}{\partial t} \quad \ldots\text{(iii)}$$

Using the general equation of continuity

$$\frac{\partial \rho}{\partial t} + \text{div}\,(\rho \mathbf{V}) = 0 \qquad \ldots\ldots \text{(iv)}$$

for eliminating $\frac{\partial \rho}{\partial t}$ from equation (*iii*), we have

$$\frac{\partial p_A}{\partial t} = +\int_h^H g\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dz + \int_h^H g\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right)dz + \int_h^H g\frac{\partial}{\partial z}(\rho w)\,dz - (g\rho)_H\frac{\partial H}{\partial t} \quad \ldots\text{(v)}$$

which is a generalised form of the barometric tendency equation.

The tendency equation has been expressed in different forms by Godson (1948), Matthewman (1946) and Houghton and Austin (1946), starting from different premises. Austin (1951) has discussed the implications of some of these forms.

It is evident that by putting H to be an infinite constant, equation (v) reduces to the standard tendency equation due to Bjerknes (1937). Haurwitz and his collaborators (1945) have analysed the Bjerknes equation to show that the horizontal advective integral (the second term on the right hand side of equation (v) above) is of the same order of magnitude as the total tendency. The vertical advection integral (third term) and the horizontal divergence integral (first term) are of opposite signs and generally have the same order of values. Thus the joint contribution to the tendency from these two terms is generally small.

It may be emphasised that apart from readily estimable factors contributing to the different terms of the tendency equation there are many other parameters (Mazumdar 1953) governing barometric fluctuations. To evaluate the individual contributions of these parameters is a matter of considerable difficulty.

The fourth term on the right hand side of equation (v) contains the factor $\partial H/\partial t$ and can, therefore, be called the " oscillational " or " tidal " term. The evaluation of this term is to be done by considering the vertical column as built up of specific layers regarding the oscillational characteristics of which reliable information is available, and estimating the integrated contributions of these strata. It appears possible that the oscillational term assumes importance on occasions of large scale pressure changes of the same sign which Humphreys (1940) has described as pressure " surge ".

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REFERENCES

Austin, J. M. (1951). *Comp. Met.*, pp. 630-638.

Bjerknes, J. (1937). *Met. Z.*, **54**, pp. 462-466.

Godson, W. L. (1948). *J. Met.*, **5**, 5, pp. 227-235.

Haurwitz, B. and Collaborators (1945). *J. Met.*, **2**, 2, pp. 83-93.

Haughton, H. G. and Austin, J. M. (1946). *J. Met.*, **3**, 3, pp. 57-77.

Humphreys, W. J. (1940). *Physics of the Air*, p. 238.

Matthewman, A. G. (1946). *Phil. Mag.*, **37**, pp. 706-716.

Mazumdar, S. (1953). On Parameters Associated with Barometric Fluctuations (Read at Calcutta Meteorological Office Colloquium).

Mitra, A. P. (1950). *Ind. J. Phys.*, **24**, pp. 387-404.

Mitra, S. K. (1952). *The Upper Atmosphere* (Asiatic Society, Calcutta), pp. 3-5.

Ibid., pp. 336-344.