A statistical analysis of gravity data

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Gravity data is usually presented in the form of certain anomalies giving the difference between observed values of gravity (after applying corrections to a certain extent) and the theoretical values. The sign of the anomaly has been associated with the areas of excess or smallness of the particular correction used for that anomaly. However this association is not always correct. Sometimes the computed correction is of the opposite sign to the one that is required. This fact is not clearly brought forth by the anomaly values as they stand. In order to study the relative importance of various types of isostatic corrections or to see if the earth's crust in a particular area is rigidly supporting the topography, in fact for any statistical handling of the data, it is seen that the ratios of an anomaly to the next correction is more convenient.

2. Let us define the two ratios r_1 and r_2, " the rank of rigid support " and the "degree of isostatic adjustment " respectively as follows—

$$r_1 = \frac{\textit{Free Air Anomaly}}{-(\textit{Topographic}+\textit{Landscape})\ \textit{correction}}$$

$$= \frac{\textit{Free Air Anomaly}}{\textit{Free Air Anomaly}-\textit{Bouguer Anomaly}}$$

$$\text{and } r_2 = \frac{\textit{Bouguer Anomaly}}{-\textit{Isostatic correction}}$$

$$= \frac{\textit{Bouguer Anomaly}}{\textit{Bouguer Anomaly}-\textit{Isostatic Anomaly}}$$

3. These ratios have simple significance. If r_1 is not equal to 1 for most of the stations in an area, the crust in that area then is not rigidly supporting the topography. If r_1 is negative for a large number of stations, then effect of topography is being masked by some unknown cause. In case of r_2, its value is unity for full compensation. r_2 is less than 1 for under compensation and it is greater than 1 for over compensation. For stations having r_2 negative, the particular method of computing isostatic correction fails, or we have to look for errors in measurement.

4. As an example of the use of these ratios, the data for 916 stations in the USA and 457 stations in India published by Heiskanen in his Catalogue of the Isostatically Reduced Gravity Stations (Helsinki—1939) was analysed. The frequency distribution of stations for different values of the ratios and altitudes above sea level is given in the Tables 1-4.

5. Also tables (not given here) for frequency distribution of r_2 were constructed from the same catalogue for 297 stations in USA for different methods of isostatic reduction namely Hayford, Airy-Heiskanen and Modified Heiskanen. A plot of these distributions showed that Modified Heiskanen gave the narrowest distribution around $r_2 = 1$. The median for this distribution is at 1·00, and the inter-quartile range is 1·61—0·48=1·13. In case of r_2 using Airy-Heiskanen reductions the inter-quartile range is 1·70—·36 =1·34 and for Hayford reductions it is 1·82—0·42 = 1·40. The number of stations

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TABLE 1

Number of U.S. stations at different altitudes for various values of r_1

(Total number of stations 916)

r_1	Altitude in metres: 150 (0–300)	450	750	1050	1350	1650	1950	2250	2550	2850	3150	3450	3750
—∞ to —30	44												
—30 to —10	33												
—10 to — 4	38												
— 4 to — 2	66												
— 2 to — 1	60	5											
— 1 to — 0·5	42	10	3										
— 0·5 to 0	38	11	15	27	38	13	3						
0 to 0·5	45	27	27	15	20	10	5	9	0	0	0	0	1
0·5 to 1·0	23	11	22	2	2	4	2	0	5	3	0	1	
1 to 2	53	5											
2 to 4	56												
4 to 10	36												
10 to 30	22												
30 to ∞	64												
	620	69	67	44	60	27	10	9	5	3	0	1	1

TABLE 2

Number of U.S. stations at different altitudes for various values of r_2

(Total number of stations 916)

r_2 —Hayford	Altitude in metres: 150 (0–300)	450	750	1050	1350	1650	1950	2250	2550	2850	3150	3450	3750
—∞ to —10	9												
—10 to — 5	23												
— 5 to — 2	58	2											
— 2 to — 1	37												
— 1 to — 0·5	63	1											
— 0·5 to 0	50	3											
0 to 0·5	60	4	1	3	3								
0·5 to 1	50	20	42	19	29	12	5	6	5	3	0	1	1
1 to 1·5	60	13	20	21	28	15	5	3					
1·5 to 2	29	11	3	1									
2 to 5	79	12	1										
5 to 10	44	3											
10 to ∞	58												
	620	69	67	44	60	27	10	9	5	3	0	1	1

TABLE 3

Number of Indian stations at various altitudes for different values of r_1

(Total number of stations 457)

r_1	Altitude in metres													
	150 (0–300)	450	750	1050	1350	1650	1950	2250	2550	2850	3150	3450	3750	4050
—∞ to —30	28													
—30 to —10	51													
—10 to — 4	48	3												
— 4 to — 2	41	6	1											
— 2 to — 1	39	9	1											
— 1 to — 0·5	19	20	5	2	1	4	1							
— 0·5 to 0	17	27	16	4	2	2	1	1	1	2				
0 to 0·5	6	18	4	6	6	6	2	5	1	0	1	1	1	2
0·5 to 1·0	4	4	4	4	1	1	0	1	0	0	0	1		
1·0 to 2	7	2	1											
2 to 4	3													
4 to 10	6													
10 to 30	2													
30 to ∞	5													
	276	89	32	16	10	13	4	7	2	2	1	2	1	2

TABLE 4

Number of Indian stations at various altitudes for different values of r_2

(Total number of stations 457)

r_2	Altitude in metres													
	150 (0–300)	450	750	1050	1350	1650	1950	2250	2550	2850	3150	3450	3750	4050
—∞ to —10	6													
—10 to — 5	10													
— 5 to — 2	6	0	0	1	1									
— 2 to — 1	7	1												
— 1 to — 0·5	21													
— 0·5 to 0	10	2	1											
0 to 0·5	27	4	0	1										
0·5 to 1	36	14	7	4	6	4	2	4	2	2	1	2	1	2
1 to 1·5	57	25	11	4	2	7	2	2						
1·5 to 2·0	35	21	5	2	1	2	0	1						
2 to 5	43	22	6	3										
5 to 10	9	0	1											
10 to ∞	9	0	1	1										
	276	89	32	16	10	13	4	7	2	2	1	2	1	2

for which r_2 is less than zero is respectively as follows—

Modified Heiskanen	..	55
Airy-Heiskanen	..	60
Hayford	..	60

Thus it is evident that modified Heiskanen correction corrects towards isostasy to a greater extent than other methods. It is also clear that just from this data one cannot choose between Hayford and Airy-Heiskanen methods.

6. Considering Tables 1 and 3 we find that for almost half of the stations r_1 is less than zero, *e.g.*, in Table 1 this number is 446/916 and if we restrict to the same height group (0–300 metres) it becomes 321/620. Thus in quite a large number of stations the disturbance in gravity due to topography is masked by an unknown cause. This cause is to be defined as Isostatic Adjustment. We further observe that the median value of r_1 is about zero, which means that the crust in USA or India is not rigidly supporting the topography.

7. There is evidently some correlation between height and r_1 and r_2 values. This has been verified by the statistical χ^2 test. The r_1 correlation is in the sense that the crust provides larger rigid support for stations at higher altitudes.

8. The median value of r_2 from Tables 2 and 4 is 1·0 which shows that isostatic adjustment tends to be complete. Only 246 out of 916 stations in the USA and 66 out of 457 in India have r_2 less than zero. If we restrict to the same altitude (0-300 metres) group, the ratios become 240/620, and 60/276, *i.e.*, 39% and 22% respectively. The situation is likely to improve somewhat if r_2 values were based on modified Heiskanen correction.

9. The inter-quartile range (0–300 metre group) for India is approximately between ·20 and 1·75, while for USA it is between —0·8 and 3·0. This may prove that the plains in India are relatively better compensated than those in the USA, though the isostatic anomaly in India is large on account of much larger disturbance by the Himalayas.

10. The contour maps for r_2 would perhaps point out departures from isostasy more clearly than the anomaly maps.

11. The aim of this paper has not been to derive certain desired geophysical conclusions, but to introduce the ratios r_1 and r_2 and to show how analysis of these can be useful. The results are not stressed as the data especially for India are taken from a 1939 report and it is known that these data are neither adequate nor free from systematic errors. Also a better way would be to consider weighted values for representative elements in a regular mesh than just considering stations by themselves.