# A theoretical study of progressive developments in raindrop size distribution and other characteristics in rain showers from 'warm' convective type clouds

### A. K. ROY and R. C. SRIVASTAVA

Rain and Cloud Physics Research Unit, C.S.I.R., New Delhi

### (Received 14 November 1957)

ABSTRACT. A study, on a theoretical basis, of the expected size distribution of raindrops and rainfall<br>intensity at various phases of rain showers from an overhead large cumulus cloud of 'warm' type has been<br>made, assuming growth beyond a certain size is due mainly to coalescence following collisions between cloud droplets. The study, based on assumptions of plausible physical conditions in a cloud in regard to (a) concentration and size distribution of giant sea-salt nuclei in air,  $(b)$  mean cloud liquid water content and  $(c)$  updraft rate and its variations with time, brings out features which are in general conformity with what are observed in rain from warm convective type clouds, and provides some corroboration of the present accepted theory of coalescence growth of raindrops, and of the salt-nucleus hypothesis of rain formation in such clouds,

#### 1. Introduction

It is by now well known that moderate to heavy rain often falls from 'warm' clouds, *i.e.*, clouds confined wholly below the freezing level. The earlier theory of rain formation, namely, ice-crystal hypothesis of Bergeron and Findeisen being clearly inapplicable to such cases, an alternative mechanism of precipitation growth by collision-cum-coalescence process has been considered in recent years. According to this theory, a few of the relatively larger droplets in a cloud, with a fall speed appreciably higher than that of average cloud particles, overtake the latter and by collisions and coalescence with them grow finally to a raindrop. This conception, developed further and treated mathematically by Langmuir  $(1948)$  and Bowen  $(1950)$ , enables us to estimate readily final sizes attained by raindrops by the above process, depending upon  $(a)$  initial size of giant cloud droplet, (b) average size of cloud particles,  $(c)$  mean cloud liquid water content and  $(d)$ mean rate of updraft inside a cloud.

As is obvious, the essential requirement of droplet growth by this mechanism is a certain degree of heterogeneity of cloud particle sizes, introducing a relative motion in the vertical direction amongst droplets belonging to different size groups. From Langmuir's calculations of collection efficiency of two colliding droplets in various size ranges it is further seen that a certain minimum width of particle size spectrum is necessary to initiate appreciable coalescence growth within a cloud, for, there is a certain critical radius  $(r_c)$  of the capturing droplet below which the collection efficiency on collision with particles of a smaller radius  $(r)$  becomes nearly negligible. This condition of minimum limit of spectrum width becomes more stringent as one considers the depth of the cloud in a given situation to see that the entire trajectory of the growing giant cloud droplet lies well within the cloud. This aspect was considered in detail by Ludlam (1951), who concluded that the initial size of the giant cloud particle must be large enough to grow to a radius of  $150 \mu$  or more on reaching a height of some 300 metres below the cloud top, so that by escaping evaporation at that level it may fall back quickly again in the main body of the cloud, and then growing further during its downward journey, attain the size of a raindrop before emerging below the cloud base. From consideration of observed sizes of giant sea-salt nuclei in the lower layers of the atmosphere over certain areas and their rate of growth by condensation,

Ludlam further concluded that on activation, these nuclei would give rise to giant cloud droplets of required sizes. The view that raindrops falling from a warm cloud are the results of growth of cloud droplets formed on sea-salt nuclei has also been substantiated by observations made by Woodcock (1952). Based on measurements of concentration and size-distribution of sea-salt nuclei and of raindrops, Woodcock and Blanchard (1955) have established a one-to-one correspondence between sea-salt nuclei and raindrops and, on the basis of this conclusion, explained Turner's (1955) observations on variation of salinity of raindrops with size.

Thus, assuming that each raindrop is the result of growth on a giant hygroscopic nucleus, one can calculate the resulting dropsize distribution at different phases of a rain shower from knowledge of concentrations and size-distributions of such nuclei in air, and some of the important cloud parameters determining precipitation growth, such as, cloud depth, liquid water content, updraft rate and its variations with height and time. The present paper attempts such a theoretical deduction, based on assumptions as detailed in a subsequent section.

#### 2. Certain facts of observation on rainfall intensity, dropsize distribution and salinity of rain water at different stages of a shower process

The use in recent years of intensity raingauges, providing information of rainfall rates during short intervals of time, has helped to bring out many of the distinguishing characteristics of rain showers in different situations. Additional valuable information in this regard is obtained by carrying out systematic measurements of raindrop size in a regular sequence of time through an entire spell of rainfall.

 $2.1.$  Intensity of rainfall—An analysis of intensity raingauge records relating to rain showers shows that, superposed on the broad features as are indicated by eye observation. namely, a progressive increase in rainfall rate at the beginning, a rather abrupt rise to a peak value at some intermediate stage, follo-

wed by an almost equally sudden fall in the precipitation rate, and a gradual decay in shower activity thereafter, quite well-marked fluctuations in rain intensity occur during short intervals of time at various epochs of rainfall. Such fluctuations are more clearly brought out by an examination of intensity values calculated on the basis of dropsize records.

 $2-2.$  Size distribution of raindrops-Much useful data on raindrop size distribution in different types of rainfall have been collected in recent years at various centres by measuring stains on filter paper or other suitable techniques. By plotting the number of drops in different size groups against corresponding diameters, Marshall and Palmer (1948) have deduced a theoretical exponential relationship for size distribution of drops corresponding to different rates of rainfall. While this relationship represents approximately the average conditions as given by a large number of samples, curves based on individual samples are seen to deviate considerably from the mean theoretical curve and often show well-marked peaks and troughs, particularly towards the large size end. Further, it is seen that the mean dropsize and also the nature and extent of the size spectrum at different phases of a rain cycle reveal a wide range of variations from one situation to another, the characteristic features observed in different instances being broadly classified as follows -

 $(i)$  Continuous rain type

(a) Precipitation commences with drops of very small size, restricted to two or three size groups, each covering  $0.2 \text{ mm}$ diameter, the mean size and also the spectrum range increasing progressively with time until the epoch of maximum rainfall rate is reached.

 $(b)$  Time sequence of variation is similar to that under  $(a)$ , but the initial size spectrum is somewhat wider.

(*ii*) Showery rain type

(a) Precipitation begins with nearly uniform drops, but of appreciable size. Spatial

distribution of drops at this stage is very sparse, as shown by only one or two drops of moderate size being caught on a Whatman filter paper on exposure to rain for a few seconds. The next phase shows continued tendency of the majority of drops crowding round the initial dropsize, together with a few drops of slightly smaller size. Gradually drops of the next higher size group start arriving, but the number of drops smaller than the initial ones continue to be limited. Later, as the intensity of rainfall increases progressively, the spectrum begins to extend both ways, the extension being most marked and rapid at the stage of sudden increase in precipitation rate.

(b) Dropsize characteristics at the start of shower process are similar to those in  $(a)$ . although the spectrum is not quite so narrow. For a time, the extension of the spectrum is confined mainly towards larger size drops, smaller drops arriving only at a relatively late stage.

While type  $(i)$  is typical of continuous type rain falling from layer clouds, this may also represent in a general way the features accompanying rain showers from a drifting cumuliform cloud. Type (ii) characterises rain showers from a well developed convective cloud developing more or less locally over the observation site. The detailed features, in a particular case, of progressive changes in dropsize characteristics with time, however, are determined principally by the time variation of updraft rate within cloud, and also partly by the position of the observer vis-a-vis the core of the precipitation cell.

2.3. Variation of salinity of bulk rain water samples with time-Turner (1955) has made measurements of the salinity of bulk rain water samples, collected at the base of cumulus clouds in a regular sequence of time. He finds from his measurements that the salinity at first is high and decreases progressively as later samples during the phase of increasing rainfall rate are taken. Thereafter, as the intensiy falls off after attaining a peak value, salinity increases until the end of the shower.

One important purpose of calculations as in this paper, is to see how the theoretically estimated dropsize distribution at different phases of showers occurring from the type of cumulus cloud under study is in conformity with some of the observed facts as mentioned above.

### 3. Conditions prescribed for purposes of computation

The following assumptions have been made-

- (i) The cloud is a large  $Cu$  with a fairly low base and has its top at a height of about  $4.5$  km above base.
- (*ii*) The cloud is restricted to below the 0°C isotherm and the rain process from it is of warm type, arising out of collision and coalescence.
- (iii) Raindrops form as a result of growth of giant cloud droplets colliding and coalescing with smaller ordinary cloud particles.
- (iv) Mean size (radius) of ordinary cloud particles =  $10 \mu$ .
- $(v)$  Liquid water content is uniform and is  $1 \text{ gm/m}^3$ .
- (vi) Updraft within cloud is uniform in all layers, 2 m/s initially, decreasing at later stages in the manner shown below-



Decrease in updraft starts from about the stage when the first amongst the raindrops, formed on the largest giant cloud particle, reaches cloud base after having

Radius $\circ$ f giant cloud <i>droplets</i> $(\mu)$	No. of droplets moving upward every 3 minutes through an area of 100 sq. cm under varying conditions of updraft $(U)$ and during different intervals of time $(T)$						
	$U = 2 \cdot 0$ m/s $T = 0.36$ min	$U$ 1.5 $T_{36.45}$	U100 $T_{45-54}$	$U\ \n0 \cdot 5$ $T\;54\;63$			
25	800	600	400	200			
30	300	225	150	75			
35	160	120	80	40			
40	90	67	45	22			
45	50	37	25	13			
50	30	22	15	s			

TABLE 1

gone through its life cycle of growth by coalescence.

(vii) Rate at which giant cloud droplets of different sizes, formed at a short height above cloud base, drift upwards, depending upon, concenand size distribution of tration giant sea-salt nuclei and the prevailing rate of updraft at various epochs of cloud and precipitation development, are as shown in Table 1.

Total number of giant cloud droplets of different sizes drifting upwards under an updraft rate of 2 m/s, as in column 2 of the above table, comes to 1430. This number corresponds to a concentration of the order of 400 giant hygroscopic nuclei per cubic metre of air, with mass ranging from 630 to 10,000  $\mu\mu q$ , and is consistent with determinations made by Woodcock (1952) of sea-salt nuclei under mean conditions of moderate to strong winds. The relative concentration of giant cloud particles of different size groups is also in acceordance with what would result by condensational growth of hygroscopic nuclei belonging to different size groups during their rise from near the ground to the cloud base. The discontinuous variations of updraft rate, as under (vi) above, and of giant cloud

droplets in steps of  $5 \mu$  have been assumed to simplify calculations, and may not strictly represent conditions obtaining in nature. However, treating the results arrived at as a necessary first approximation, one could visualise what would result if the variations were of a more continuous nature.

### 4. Basic data as obtained by calculation

Computations of final sizes attained by rain drops at different stages of rainfall and of the time taken by each such drop in reaching the cloud base have been made by applying the usual formulae of droplet growth by collision and coalescence, namely,

$$
\frac{dr}{dt} = \frac{E w v}{4}
$$
  
and 
$$
\frac{dz}{dt} = U - v
$$

where the symbols have their usual meanings. The value of liquid water content  $w$ , as mentioned already, has been taken to be uniform and constant at 1  $\text{gm/m}^3$ . For E, the collection efficiency corresponding to different values of  $r$ , values as computed by Langmuir (1948) have been used as a first approximation. Terminal velocity v, for different sizes has been obtained by interpolation on the basis of values determined by Gunn and Kinzer (1949).

The computed data are summarised in Tables 2 and 3. Table 2 consists of six sections, each pertaining to a giant cloud droplet of a given initial radius and furnishes figures relating to the history of development of each such droplet at different stages. For example, with reference to entries in line 10 under section (a) of Table 2 the second and fourth columns show that a cloud droplet of radius  $25 \mu$ , starting its upward journey from near the cloud base at time  $t = 1620$  sec. leaves the cloud base with a radius of 535  $\mu$ . The time  $(t_1)$  spent by it in the cloud, *viz.*, 3260 sec is shown in column 3, while the time  $(t_2=t_1+t)$ , viz., 4880 sec, after which its product in the shape of a

raindrop finally leaves the cloud base, counting from the instant when the updraft over the region in question led to the formation of the first giant cloud particle, is given in column 5.

In Table 3(a), figures given in column 1 indicate the time, counting from the instant the first group of giant cloud particles start their life history of development, while those in column 2 give the time as reckoned from the moment of arrival of the first raindrop. The final sizes of raindrops formed on giant cloud droplets of various sizes, as they emerge below the cloud base during successive periods of 3 minutes, beginning from the time when the first raindrop leaves the base, are shown in various sub-columns of column 3. The number of drops, shown in brackets, corresponding to each indicated drop size have been determined with reference to figures in appropriate columns of the table under assumption (vii), discussed in the preceding The rainfall intensity during sucsection. cessive epochs of 3 minutes, calculated on the basis of size and number of drops, is entered in column  $4$  of Table 3(a).

Table 3(b) provides data similar to those in Table 3(a) under slightly different condition of rate of decay of updraft, namely, by  $0.5$ m/s every 3 minutes instead of every 9 minutes as in the case of computations relating to Table 3(a).

#### 5. Results of calculations and general conclusions

The following are some of the features as are brought out by examination and further analysis of data in Tables 2 and 3.

- in dropsize A. Progressive development distribution and rainfall rates during successive 3-minute periods
	- (i) The first raindrop originating on the largest amongst the giant cloud particles  $(50 \mu \text{ radius})$  reaches cloud base after 36 minutes with a radius of 930  $\mu$ , while that formed on the smallest among the giant cloud particles (25  $\mu$  radius) emerges from the cloud base after 63 minutes, *i. e.*,



Data relating to progressive developments of precipitation growth on giant cloud particles of various sizes



217

# A. K. ROY AND R. C. SRIVASTAVA

TABLE 2 (contd)

TABLE 2 (contd)



# RAINDROP SIZE DISTRIBUTION IN RAIN SHOWERS



Final radius as a function of initial radius and time of arrival at cloud base, and resulting<br>rainfall rates during different epochs



 $\dot{2}1\ddot{9}$ 

## A. K. ROY AND R. C. SRIVASTAVA

Total		Initial radius										
time $(\sec)$	Time (min.)	$25~\mu$	$30~\mu$	$35 \mu$	$40~\mu$	$45~\mu$	$50 \mu$	rate $(\text{mm/hr})$				
4860 to 5040	$45 - 48$	535 (800) 450 (800)						1.6				
5040 to 5220	$48 - 51$	375 (800) 305(800) 260 (600) < 100				i,		0.43				
5220 to 5400	$51 - 54$	210 (600) 150 (600) 120 (400)						0.05				
(b) Updraft decreasing by 0.5 m s every 3 minutes after start of rain												
2160 to 2340	$0 - 3$						930,900	0.59				
2340 to 2520	$3 - 6$					985,940	860	0.70				
2520 to 2700	$6-9$				1020,950	890	800	1.87				
2700 to 2880	$9 - 12$			1020	875	805,725	735,655	2.48				
2880 to 3060	12-15			950,870	790,715	645	575,490, 100	2.87				
3060 to 3240	$15-18$		1020	785	635,100	565,485, 100	415,345	3.66				
3240 to 3420	18-21		940,840	715,625, 100	555,475	405,265	275,210	4.65				
$3420$ to 3600	$21 - 24$		780	550	385,325, 260,195, 125	205		1:50				
3600 to 3780	24-27	1015,935	700,620, 100	470,385, 320,120				14.2				
3780 to 3960	27-30	855	540,435. 115	250,185				4.82				
3960 to 4140	30-33	775,695, 100	380,315, 250,175					5.64				
4140 to 4320	33-36	615						1.56				
4320 to 4500	36-39	535,455, 105						$1 - 66$				
4500 to 4680	$39 - 42$	375,305, 245,170						0.68				

TABLE 3 (contd)

# RAINDROP SIZE DISTRIBUTION IN RAIN SHOWERS



## Fig. 1. Variation of rainfall rates with time

after an interval of nearly 27 minutes, with a radius of 1250  $\mu$ .

- (ii) Under the assumed condition of decreasing updraft, the final size of raindrop formed on giant cloud droplet of a given size decreases progressively and eventually falls below radius  $100 \mu$ . In the case of drops originating from 25  $\mu$  cloud particles size, however, a slight departure from the usual trend occurs at later stages, as it is seen that drops of radii less than 100  $\mu$ fall out in the interval 48-51 minutes, while those of somewhat larger size in the range of 120-210  $\mu$ reach the base during the following 3-minute period.
- $(iii)$  Dropsize distribution picture of rainfall during each successive 3minute period, as indicated by figures in column 3 of Table 3 (a), shows the following features-
- (a) Raindrops are of nearly uniform size during the first two epochsa feature which will not be materially altered even if we consider continuous variation in sizes of giant cloud particles and in the rate of updraft.
- (b) Characteristic changes in the nature of dropsize spectrum from the



Fig. 2. Space distribution of raindrops belonging to various size groups

start of shower to the intermediate stage of rainfall are similar to those under class  $(ii)$   $(a)$  of dropsize variations, discussed in an earlier section  $(2\cdot 2)$ .

- (iv) From Fig. 1, showing variation in rainfall rate against time, it is seen that, superposed on a general trend of progressive increase in rainfall rate until the peak intensity is reached, fluctuations in intensity occur during each 9-minute epoch when updraft rate is constant. This feature is in accord with the observed short-period fluctuations in rainfall rates, as shown by intensity raingauge records of rain showers.
- $(v)$  Fig. 2 shows number of raindrops per cubic metre of air, plotted against various size groups on a semilogarithmic scale, and represents overall situation in regard to size distribution of drops during the entire shower process. The main feature of the curve, which follows the general trend of the empirical

221

relationship suggested by Marshall and Palmer (1948), is the occurrence of two peaks at drop diameters  $1 \cdot 3$  mm and  $2 \cdot 1$  mm. This tendency of peaks and troughs appearing on the otherwise broad- $\overline{w}$  exponential nature of  $N_D \delta_D$ , D curve is shown frequently by short period sampling of raindrop sizes, and is particularly marked in showers falling from well developed convective type clouds. It is felt that the implications of this theoretical study provide a useful pointer to one of the possible causes leading to developments of peaks and troughs in dropsize distribution curves, based on sampling of rainfall during short intervals of time, and that is the chance irregularity in the size distribution of giant cloud particles in a limited volume of cloud air. Size spectrum of cloud particles based on actual measurements very often shows such irregularities with definite gaps at certain points. especially towards the large size end of the spectrum. It is, however, believed that a more important factor influencing occurrence of such peaks and troughs, especially in relation to showers from warm clouds, is a certain type of variation with height of vertical currents within cloud and is perhaps linked chiefly with a sudden increased rate of updraft in a limited layer at some height above cloud base. This aspect is under further detailed examination and will be dealt with in a separate note.

(vi) From figures of final sizes reached by raindrops as given in Table 2, it is seen that the size range of drops forming on  $25-30$   $\mu$  size cloud particles is widest, decreasing propressively to that corresponding to  $45-50$   $\mu$  group. This is more readily seen from Table 4,



giving the final size attained by a giant cloud particle with radius as indicated under condition of steady and uniform updraft at 2 m/s and 1 m/s respectively, but with uniform liquid water content of 1  $\sigma$ m/m<sup>3</sup> in both cases.

It is seen from Table 4 that whereas, under undraft of 2 m/s, raindrops originating on giant cloud particles of radius lying between 45  $\mu$  and 50  $\mu$ are all packed in a 80  $\mu$  radius interval, those corresponding to giant cloud particle group  $25-30$   $\mu$  will have a much wider range of radius from  $1440 \mu$  to  $1750 \mu$ . Thus one sees that under the condition of updraft as above, in the event of chance absence during a short interval of time and in a small sample of rising cloud air of giant cloud particles of sizes, say, 28  $\mu$ and 29  $\mu$ , with relative preponderance of sizes 26  $\mu$  and 27  $\mu$ , the dropsize distribution curve will have a peak corresponding to diameter  $3 \cdot 2$  mm and a drop trough at 3.4 mm. Such irregularities in giant cloud particle sizes will have less influence on the general trend of the drop distribution curve for rainfall of low intensity, in which mean updraft is relatively weak. A study of dropsize distribu-

TABLE 4



Fig. 3. Average salinity of rain water during successive 9-minute periods

tions based on actual measurements show that curves relating to low intensity rainfall are often in better agreement with the empirical curve of Marshall and Palmer, and that peaks and troughs appear more frequently and more prominently on curves representing rain showers of moderate to high intensity.

 $(vii)$  To simplify calculations, liquid water content in cloud has, for purposes of the present study, been assumed to be constant and uniform at all heights. Both theory and actual measurements (Warner and Newnham 1952), however, show that usually  $w$  first increases with height, reaches a maximum at a certain layer, and then falls off rather quickly towards zero at the cloud top. Considering qualitatively the effect of such a variation in liquid water content with height, but with a mean value of 1 gm/m<sup>3</sup> as assumed in the present study, the time lag between arrival of raindrops formed on  $25 \mu$  particles and of those formed on 50  $\mu$  droplets will be shorter than 27 minutes, with the result that there will be more rapid



TABLE 5

variations with time of both dropdistribution characteristics size as well as intensity of rainfall. Further, if instead of the assumed rate of decay of updraft, namely by  $0.5$  m/s every 9 minutes, the decrease is more rapid, the peak intensity of rainfall will be reached earlier than in the case under discussion. Table 3(b), showing dropsize distribution and rainfall intensity during successive epochs of 3minute periods, based on assumption of updraft decreasing in steps of  $0.5$  m/s every 3 minutes, shows this tendency quite clearly.

### B. Variation of salinity of rain water with time

Fig. 3 shows the computed values of salinity of rain water collected during successive 6-minute intervals on the basis of dropsize figures as in Table 3(a). For purposes of calculating salinity of different drops, the values as in Table 5 have been taken as the mass of salt-nucleus in giant cloud droplets of different sizes.

It will be seen that the theoretical curve is broadly similar to the one obtained by Turner (1955) on the basis of measurements of salinity of bulk rain water samples at the base of convective clouds.

### C. Total rain water collected during the period considered and the amount of cloud liquid water replenished by updraft

Calculations show that during the period of 63 minutes of updraft at various rates a  $total of 0.594 cu$ , metre of air moves upwards through an area of 1 sq. cm. With the assumed value of liquid water content in cloud at  $1 \text{ gm/m}^3$ , the total replenishment thus comes to  $0.594$  gm of water. The calculated amount of rain falling on 1 sq. cm area is found to be  $0.483$  gm which shows a fair balance between the liquid water supplied by the rising air and that lost as a result of raindrop growth, considering that the collection efficiency value used for calculation is always less than 1. This near agreement between the amount

of rain falling and the quantity of liquid water replenished in cloud by the prevailing perhaps shows that the updraft also various assumptions made for purposes of the theoretical study are, on the whole, reasonable and not inconsistent with facts.

In conclusion, it is felt that the attempted analysis has been of help in suggesting probable explanations of some of the observed features accompanying rain showers. Further, a broad agreement between the results of the theoretical study and what are observed in actual rainfall in a situation of the type discussed appears to give support to the present accepted theory of coalescence growth of raindrops in warm clouds, and the salt-nucleus hypothesis of rainof formation in such clouds.

#### REFERENCES

