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Numerical prediction of the movement of Bay depressions

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ABSTRACT. The 24-hour displacement of two monsoon depressions in the Bay of Bengal was computed on the basis of the non-divergent barotropic model of Charney and Estoque's baroclinic model. Relaxation methods were used for the barotropic model, and Fjortoft's graphical technique was employed for Estoque's model. The difficulties experienced with the data available, and effect of errors caused by inadequate data are discussed. Finally, the predicted contours of the 1000 and 500-mb surfaces are compared with the observed patterns and suggested lines for improvement are indicated.

1. Introduction

The numerical prediction of large scale pressure systems may be said to have its beginning in the classical work of Richardson (1922). The subject was revived in more recent years by Charney (1949) and collaborators (Blackburn and Gates 1956, Thompson and Gates 1956); but perhaps the biggest factor contributing to this renewed interest was the advent of electronic computing devices. A rough estimate revealed that these computers were about 10^5 times faster than conventional desk computers, and could perform the couple of million multiplications and divisions needed for a 24-hour forecast in an hour.

Progress in this branch of Meteorology in the tropics has been understandably slow because there has been no opportunity to use high-speed computers for numerical prediction, and graphical methods (Fjortoft 1952) are to a large extent subjective because of the sparse network of stations reporting upper winds and temperatures. Despite this limitation, the present study was undertaken to gain experience with the different models in use, and to have greater insight into the type of difficulties likely to be encountered when high-speed computers become available.

As a beginning, the displacement of two monsoon depressions in the Bay of Bengal was computed with the non-divergent barotropic model of Charney (1949), and with Estoque's model (1956, 1957), based on a variation of Fjortoft's earlier work. We used relaxation methods (Southwell 1946), and Fjortoft's graphical technique to obtain the predicted contours of the 1000 and 500-mb surfaces for a period 24 hours ahead. The theoretical implications of both these models are by now fairly well known; we shall, therefore, only briefly indicate the theory for continuity and then proceed to discuss the results.

2. Theoretical aspects

(*a*) *The non-divergent barotropic model*— This model is built round a simplified form of the equation for conservation of vertical vorticity (hereafter referred to as vorticity for simplicity). The main assumptions in deriving the model are as follows —

(1) The wind field is quasi-geostrophic,

(2) There exists a level in the mid-troposphere, generally associated with 500 mb, where there is no divergence,

(3) The generation of vorticity by such factors as the rotation or stretching of vortex tubes, horizontal gradients of the vertical velocity or the existence of isobaric—isosteric solenoids is negligible and

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(4) The horizontal or isobaric divergence at any level is small compared to the vorticity.

With the above assumptions, the equation for conservation of vorticity for frictionless flow assumes the following simple form —

$$\frac{\partial \zeta_p}{\partial t} + \boldsymbol{V}_h \cdot \nabla_p (\zeta + \lambda) = 0 \qquad (2.1)$$

where ζ_p is the vertical component of vorticity, $\boldsymbol{V}_h$ is the horizontal wind vector and λ is the Coriolis parameter. Introducing the geostrophic assumption, equation (2·1) is further reduced to the following form of Poisson's equation—

$$\nabla^2 \left(\frac{\partial z}{\partial t} \right) = J \left(\frac{g}{\lambda} \cdot \nabla^2 z + \lambda, z \right) \qquad (2.2)$$

where z is the height of the isobaric surface corresponding to the level of no divergence, and

$$J(\alpha, \beta) = \frac{\partial \alpha}{\partial x} \cdot \frac{\partial \beta}{\partial y} - \frac{\partial \beta}{\partial x} \cdot \frac{\partial \alpha}{\partial y} \qquad (2 \cdot 3)$$

is the Jacobian operator. Equation (2·2) may be solved as a boundary value problem in $\partial z / \partial t$ either by analytical methods, numerical iteration or graphically as indicated by Fjortoft (1952).

One of the main difficulties in using this model in the tropics lies in the assumption of quasi-geostrophic motion. As indicated by Charney (1949) this assumption is necessary to filter out fast moving, but meteorologically insignificant, perturbations as solutions of the equations of motion, and although the barotrophic model in the above form has been used with varying success in low latitudes (Jordan 1956), it is generally agreed that the geostrophic assumption breaks down seriously as we go towards the equator beyond 20°N. Preliminary investigations in India also indicate that deviations from the geostrophic wind could often exceed 25 per cent of the observed wind in regions between 10–20°N. However, as the pressure systems chosen for study lay to some extent north of 20°N, and as our main purpose was to gather experience in numerical prediction, it was decided to use this model in spite of this limitation.

(*b*) *Estoque's model*—Of the other drawbacks of the previous model, perhaps the most serious was the assumption of a barotropic atmosphere. For it is known the atmosphere could never be barotropic, because such a state did not provide for conversion of potential energy into the kinetic energy of the system. Earlier work (Das 1957) had shown that even a broad air current like the Indian S.W. Monsoon, which was assumed to be homogeneous regarding its moisture content, was far removed from an ideal barotropic atmosphere.

More realistic models (Sawyer and Bushby 1953, Charney and Phillips 1953, Thompson 1953) have, therefore, been devised to incorporate some of the baroclinic features of the atmosphere. These models have many common features, and are of varying degrees of complexity depending upon the computing facilities available. We chose a comparatively simpler model due to Estoque (1956), primarily because it required no high-speed computer and could be used to test Fjortoft's graphical methods in India.

This model, in common with other two dimensional models, is based on the conservation of vorticity and the thermodynamic energy equation. The conservation of vorticity is expressed in formal terms by the equation

$$\frac{\partial \zeta_p}{\partial t} + \boldsymbol{V}_h . \nabla_p (\lambda + \zeta) - \lambda . \frac{\partial w}{\partial p} = 0 \qquad (2.4)$$

where the subscript p refers to differentiation along an isobaric surface, and w is the vertical component of velocity. The thermal field is brought in by assuming adiabatic motion, which enables us to express the height tendency of an isobaric surface in terms of the following equation—

$$\frac{\partial}{\partial p} \left(\frac{\partial z}{\partial t} \right) + \sigma . w = \boldsymbol{V}_h \nabla \left(\frac{\partial z}{\partial p} \right) \qquad (2.5)$$

where $\sigma = \alpha / g\theta \times \partial \theta / \partial p$; θ and α refer to the potential temperature and specific volume, and z is the height of an isobaric surface. From a study of observed pressure changes on the earth's surface, which reflect the sensitive balance between positive and negative divergence in a vertical column through the atmosphere, it could be inferred that the variation of the vertical velocity with height would be parabolic. This was, therefore, expressed by assigning to w the following form—

$$w = (2)^{\frac{1}{2}}.\, w_c \cdot \sin\left[\frac{\pi}{2} \cdot \frac{p_0 - p}{p_0 - p_L}\right] \quad (2\cdot6)$$

where the subscripts 0 and L refer to two isobaric surfaces corresponding to $w = 0$ and a level in the mid-troposphere where w attains its maximum value ($2^{\frac{1}{2}}.\, w_c$).

Substituting (2·6) in (2·4) and (2·5), and making a few minor adjustments, Estoque's final prediction equations were—

$$\frac{\partial}{\partial t}\left(z_L - \bar{z}_L - G\right) = -\frac{g}{\lambda} \times J\left(z_L,\, z_L - \bar{z}_L - G\right) \quad (2\cdot7)$$

$$\frac{\partial}{\partial t}(h - B\bar{h}) = -\frac{g}{\lambda} . J\left(z_L,\, h - B\bar{h}\right) \quad (2\cdot8)$$

The symbols have the following meanings—

$$G = \int_0^{\phi} \frac{\lambda^2 . d^2}{4m^2 g} \cdot \cot\phi \, d\phi$$

$$B = \left(1 + 0.56\, \lambda^2 d^2 / m^2 g h \,.\, \ln.\, \frac{\theta_L}{\theta_0}\right)^{-1}$$

m = the magnification factor,

d = size of the unit square grid,

$h = z_L - z_0$, *i.e.*, the thickness between the isobaric surfaces corresponding to z_0 and z_L and

$\bar{z}$ = the mean value of z obtained by averaging values at four corners of a square grid, *i.e.*, $\bar{z} = \frac{1}{4}(z_1 + z_2 + z_3 + z_4)$ in Fig. 1.

Equations (2·7) and (2·8) were used by us to prepare prognostic charts for two standard isobaric surfaces, *viz.*, 1000 and 500 mb. The graphical technique will be described in a subsequent section, but it may be noticed that although we still use the geostrophic assumption, the model is an improvement on the previous one because (*a*) it assigns a variation — albeit hypothetical— to w and (*b*) the thermal field is included through σ. Estoque used a mean value of σ derived from climatological records; we also used his values of σ and B in our computations because the final results were not very sensitive to the value of these factors.

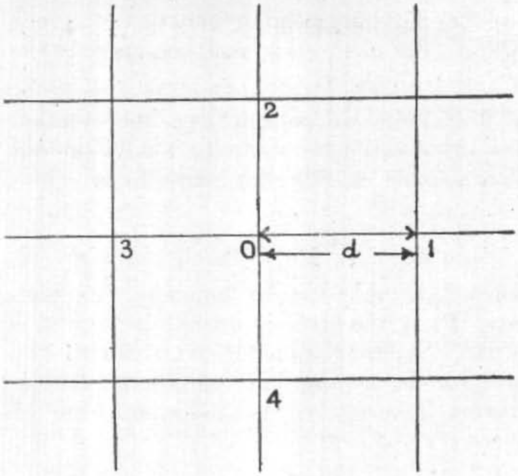

Fig. 1

3. Method of computation

(*a*) *The barotropic model*—The grid over which forecasts were prepared was on a map of scale 1 : 10^7 on Mercator's projection. We, therefore, introduced a magnification factor (m) on the right hand side of eq. (2·2), which for Mercator's projection was equal to sec ϕ, where ϕ is the latitude. To facilitate computation, the factor $m^2 g/\lambda$ was tabulated for each grid point in the forecast area, thereby taking cognizance of the variation of m and λ with latitude. The length of the unit grid (d) was 2° latitude, and the forecast area was made up of about 85 grid points in the two cases chosen for study.

The Laplacian of the z field ($\nabla^2 z$) was obtained from its finite difference analogue,

$$d^2 . \nabla^2 z = \sum_1^4 z - 4 . z_0 \quad (3.1)$$

and the function $J\,(\,m^2 g/\lambda \,.\, \nabla^2 z + \lambda,\, z\,)$ was computed for each grid point using finite differences to evaluate the determinant.

The relaxation process was started by treating each value of this function as the initial residue, and the liquidation of residuals was made as rapid as possible by the judicious use of line, group or block relaxation. Relaxation was stopped when the final residue were considerably less than 5 per cent of the initial values, and care was taken to avoid accumulation of small residues of the same sign over a large area.

We assumed $\partial z/\partial t = 0$ along the edges of the rectangular area chosen for study. The effects of this boundary conditions have been studied by Charney (1950) and others; we chose this boundary condition primarily because it was the simplest to use, but it was realised that there was some scope for improvement if we used the observed height changes during the past 24 hours along the boundary. It is hoped to study this aspect in more detail in a subsequent paper.

Having obtained dz/dt, the new contour field was computed for centred time increments of 3 hours. Thus we have,

$$Z_{t+\delta t} = Z_{t-\delta t} + 2\left(\frac{\partial z}{\partial t}\right)_t . \ \delta t \qquad (3.2)$$

to compute the new height field, except the first interval when uncentred time differences were used.

A time increment of 3 hours may appear too large to be compatible with the requirements of computational instability. Charney, Fjortoft and Von Neumann (1950) derived the criterion,

$$\triangle t \leqslant \frac{\triangle s}{(2)^{\frac{1}{2}}. \ Vg \text{ (max.)}} \qquad (3.3)$$

The maximum geostrophic wind computed over the forecast area in each of the two situations studied by us would indicate a time increment of about 1·5 hours to satisfy (3·3) and in general time increments of 1-2 hours are chosen for numerical prediction. However, as only desk computers were available, it soon became apparent that the labour involved in iteration for steps lesser than 3 hours duration would be prohibitive. The actual observed winds, however, never exceeded 35 knots, and were substantially less than the geostrophic value. In view of this, it was felt that time steps of 3 hours would suffice for our preliminary approach.

(*b*) *Estoque's model*—The different steps necessary for graphical integration have been enumerated in the original papers of Fjortoft (1952) and Estoque (1956), and will not be repeated here. The method is essentially to prepare charts of $z_L - \bar{z}_L - G$ and $\bar{z}_L + G$. Next, by displacing the isopleths of the former along the contours of the latter according to the geostrophic wind, a forecast is obtained of the $z_L - \bar{z}_L - G$ field. The initial field of $z_L - \bar{z}_L - G$ was then subtracted from the predicted field to obtain the 500-mb forecast change, and the same procedure was repeated with 1000-500 mb thickness charts.

The main advantage of the method is that only one time step of 24 hours is needed, because we substitute a slow moving dummy field ($z_L - \bar{z}_L - G$) for prediction instead of the original field of z_L, and subsequently compensate for the change in $\bar{z}_L + G$. Accordingly, the length of the grid chosen for our computation was 10° latitude (approximately 1000 km) and the whole operation was performed for a single time step of 24 hours.

4. Results

The methods described above were tested on two monsoon depressions in the Bay of Bengal. It became apparent quite early in our computations that the application of these methods in India was beset with difficulties. By and large the difficulties were due to the inadequate nature of data available, and also the accumulation of errors caused by using finite differences for derivatives.

In both the methods wherever data were missing, recourse had to be taken to interpolation. Secondly, it was noticed with the barotropic model that unless the data were processed before each time interval, spurious and abnormally large values of dz/dt were observed at several grid points. These were smoothed out by replacing them with the average of neighbouring values to restore consistency in the computed patterns. A curious observed feature was the fact that these spurious values tended to multiply in number after the fifth time interval, *i.e.*, after about 15 hours of the forecast period. Undoubtedly, interpolation and smoothening processes mean a certain degree of subjective selection in the final result; this was undesirable, but could hardly be avoided with the present network of stations.

The depressions studied were a fast moving one in August 1957, and a comparatively slow but recurving depression in May 1956. We shall deal with each in turn.

Case I—Bay depression (20-26 August 1957)

A shallow depression formed in the north Bay of Bengal on 20 August 1957, deepened into a cyclonic storm by the next morning and crossed the Orissa coast near Gopalpur on the morning of the 22nd. Subsequently, it moved rapidly inland, began recurving towards the north on the morning of the 23rd and finally filled up near the Punjab hills on the 26th. The track of the depression, with the approximate position of its centre on each day, is shown in Fig. 2.

The 24-hour period between 0000 GMT on 22 and 23 August 1957 was chosen for test because the depression showed rapid movement during this period. The surface pressure pattern and the 500-mb contours are shown in Figs. 2 and 3. In Fig. 3 we have also shown the rectangular area and the grid-points selected for computation with the barotropic model. Figs. 4, 5 and 6 show the contours—predicted and observed—of 1000 and 500 mb 24 hours later. The predicted contours of 500 mb obtained by the barotropic and Estoque's models are shown separately.

It would be seen from the above figures that there was some agreement between the observed and predicted fields. With the barotropic model, the computed displacement of the trough at 500 mb was less than what was observed. The computed fall in height values was also considerably more than the observed fall. There was thus a tendency to over-intensify a trough with this model. No closed low could be discerned with Estoque's model, but a well marked trough was noted in the vicinity of the observed low. The tendency for over-intensification was also less marked.

Case II—Bay depression (29 May-6 June 1956)

Unsettled conditions in the north Bay of Bengal concentrated into a shallow depression by the morning of 29 May 1956. During the next two days, the depression remained practically stationary and gained in intensity until it became a severe cyclonic storm with a core of hurricane winds by the morning of 31 May 1956. It was then centred about 60 miles south of Calcutta.

The cyclone moved slowly northwestwards, struck the coast near Contai on the night of 31 May 1956 and then recurved towards the northeast. It continued to move northeastwards, and gradually weakened into a diffused low over East Pakistan and neighbouring areas of Assam by 6 June 1956. The track of the depression is shown in Fig. 7, and Fig. 8 shows the 500-mb contours at 1500 GMT on 29 May 1956.

In the previous depression a forecast period was chosen in which there was rapid movement. In the present case, there was practically no movement but the depression was beginning to intensify. Figs. 9, 10 and 11 show the predicted and observed contours of 1000 and 500 mb on 30 May 1956.

It will be seen that the barotropic model in this case predicted more displacement towards the northeast than was observed. The formation of a separate low was also predicted over Bihar and U.P. but this was not observed. The results obtained with Estoque's model appeared to be better in this case; in fact the computed fall in height values also showed fair agreement with what was observed.

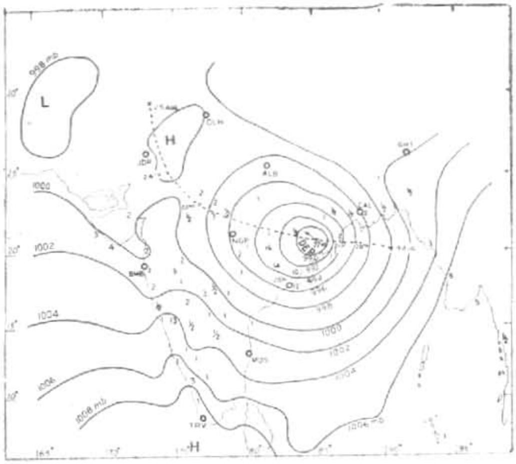

Fig. 2. Weather map for 0830 IST on 22 August 1957

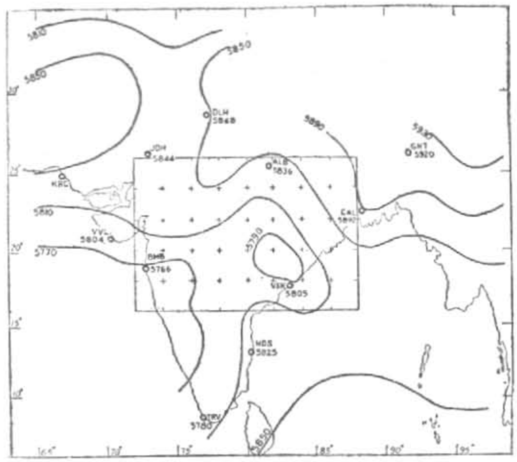

Fig. 3. Contours of 500 mb for 0000 GMT on 22 August 1957

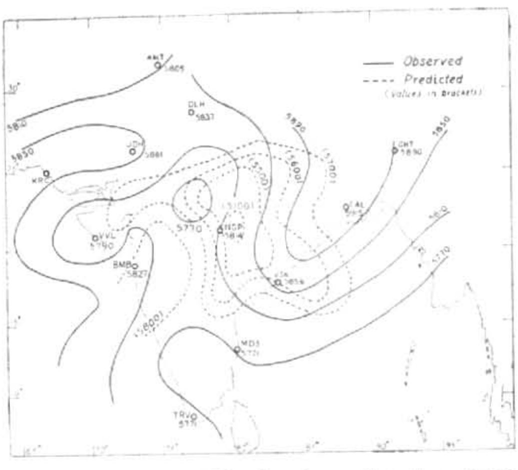

Fig. 4. Contours of 500 mb—observed and predicted (Barotropic model) for 0000 GMT on 23 August 1957

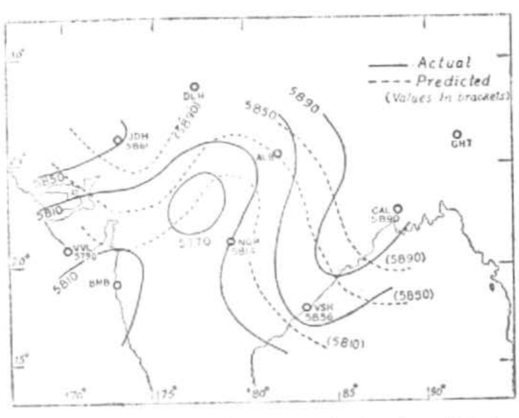

Fig. 5. Contours of 500 mb—actual and predicted (Estoque's model) for 0000 GMT on 23 August 1957

5. Summary and conclusions

While it is clearly impossible to generalize on the basis of two cases, it is felt that the main purpose for which the investigation was undertaken, namely, to gain experience and insight into the nature of difficulties, was largely achieved. The following tentative conclusions could be drawn from the present study—

(*a*) Errors and spurious values of the height tendency shows a tendency to increase after 12-15 hours from the beginning of the forecast period. The possibility of overcoming this tendency by using shorter

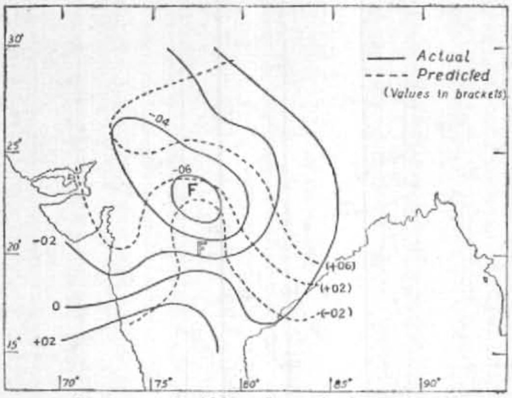

Fig. 6. Contours of 1000 mb—actual and predicted (Estoque's model) for 0000 GMT on 23 August 1957

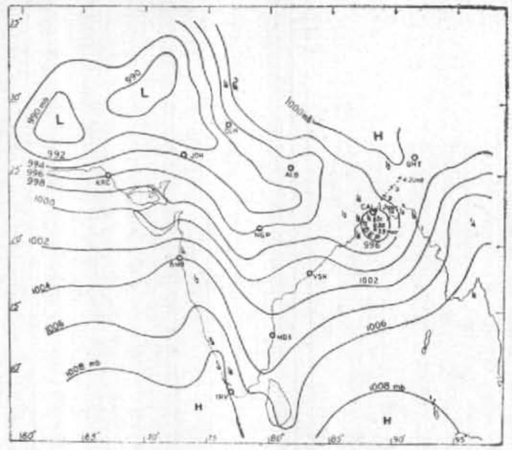

Fig. 7. Weather map for 1730 IST on 29 May 1956

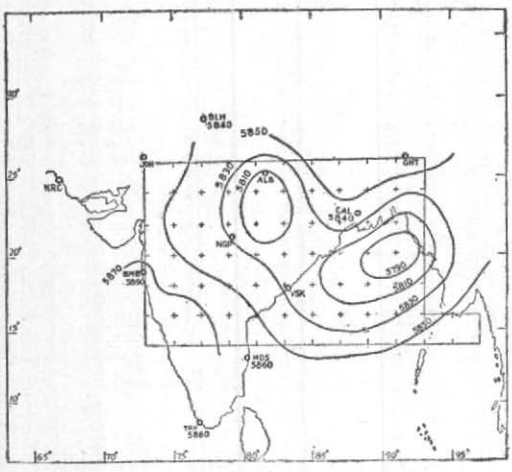

Fig. 8. Contours of 500 mb for 1500 GMT on 29 May 1956

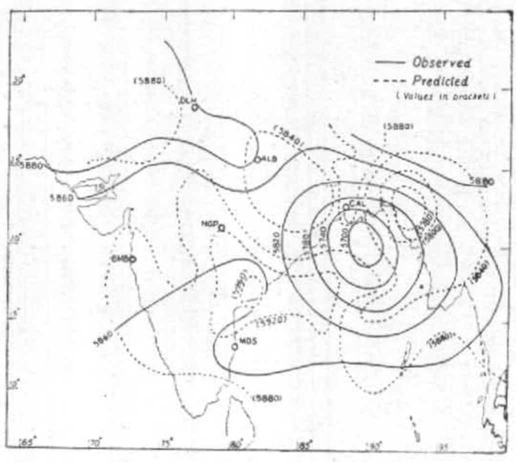

Fig. 9. Contours of 500 mb—observed and predicted (Barotropic model) for 1500 GMT on 30 May 1956

time steps and different boundary conditions needs further examination.

(*b*) Estoque's model generally yielded better results, but the graphical model needs considerable amount of extrapolation of height contours to be really effective. In the absence of high-speed computers, there is little doubt that this method would have to be adopted in preference to the barotropic model because of the amount of labour involved in the latter.

(*c*) The relaxation process may be considerably shortened by using suitable formulae to obtain the initial trial values for liquidation of

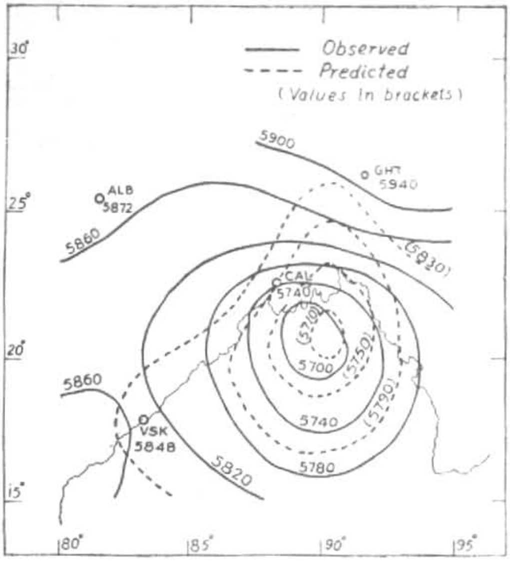

Fig. 10. Contours of 500 mb—observed and predicted (Estoque's model) for 1500 GMT on 30 May 1956

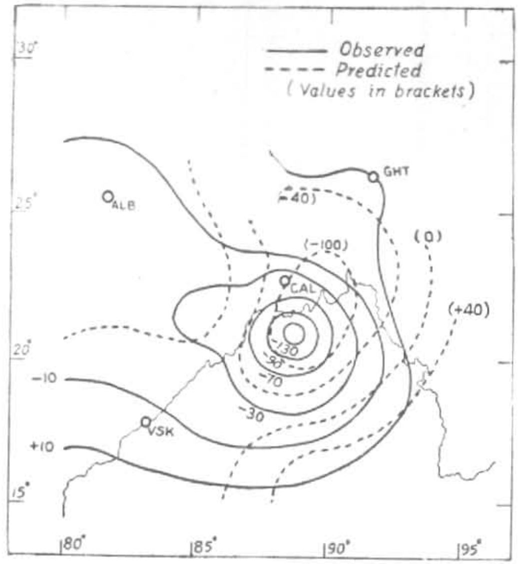

Fig. 11. Contours of 1000 mb—observed and predicted (Estoque's model) for 1500 GMT on 30 May 1956

residues. Some improvement has been also reported by using the final solution of the previous time step as the initial trial for the next iteration. These aspects were not considered in the present study and would have to be examined further.

REFERENCES

Blackburn, J. F. and Gates, W. L.	1956	*J. Met.*, **13**, 1, pp. 59-63.
Charney, J. G.	1949	*Ibid.*, **6**, 6, pp. 371-385.
Charney, J. G. and Phillips, N. A.	1953	*Ibid.*, **10**, 2, pp. 71-99.
Charney, J. G., Fjortoft, R. and Neumann, J. V.	1950	*Tellus*, **2**, pp. 237-254.
Das, P. K.	1957	Commemoration Volume of Met. Soc. of Japan.
Estoque, M. A.	1956	*J. Met.*, **13**, 2, pp. 195-202.
	1957	*Ibid.*, **14**, 1, pp. 38-42; **14**, 3, pp. 191-205.
Fjortoft, R.	1952	*Tellus*, **4**, pp. 179-194.
Jordan, C. L.	1956	*J. Met.*, **13**, 3, pp. 223-235.
Petterssen, S.	1955	*Ibid.*, **12**, 1, pp. 36-42.
Richardson, L. F.	1922	*Weather prediction by numerical process*, Camb. Univ. Press, pp. 236.
Sawyer, J. S. and Bushby, F. H.	1953	*J. Met.*, **10**, 1, p. 54.
Southwell, R. V.	1946	*Relaxation methods in Theoretical Physics*, Oxford Univ. Press.
Thompson, P. D.	1953	*Quart. J. R. met. Soc.*, **79**, pp. 51-69.
Thompson, P. D. and Gates, W. L.	1956	*J. Met.*, **13**, 2, pp. 127-141.