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Radar echoes from monsoon rain

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ABSTRACT. The paper describes air-borne radar set AN/APQ-13, modified for storm-detection work from a static station. A few typical pictures of radar echoes from monsoon rain as observed from the Meteorological Station at New Delhi are described. The usefulness of the equipment for precipitation studies has also been discussed.

1. Introduction

The use of radar has placed in the hands of the meteorologist an entirely novel and new technique for physical meteorological research and local short-term weather forecasting. A centimetre radar can permit the measurement of rainfall almost as accurately as isohyetal patterns derived from micro-network of raingauges (1948*). In addition, it can give a pictorial representation of the growth and movement of precipitation cells, progressively with time. During and after the World War II a number of observations have been made in different countries for locating rainstorms, thunderstorms, cold fronts etc with the help of micro-wave radars having their operating wave-length in the region of S (10 cm), X (3 cm) and K (1 cm) bands.

As radars were not available commercially it was not possible to utilise them for meteorological purposes more extensively during the war. Even today the cost of the equipment is prohibitively high and observations are being taken, wherever possible, with the help of radars released from war surpluses. The present paper describes the results of observations made with a 3-cm radar (AN/APQ-13) obtained from war surpluses. This radar has peak power of 40 kilowatts, beam width of 3° and frequency of 9375 megacycles (3·2 cm). The equipment was received in a very poor condition and considerable time had to be spent to put it in commission. As the radar was essentially meant for installing in aircraft it was suitably modified to be used as a static ground installation.

Fig. 1 shows a photograph of the rack-assembly designed in the Meteorological Office workshop, New Delhi. The assembly provides easy access to all the controls on individual units. The P. P. I. indicator is mounted on a tilting table in front of the rack-assembly to enable visual observation according to individual convenience. A 35-mm camera mount is placed over the oscilloscope to photograph the P. P. I. presentations. The radar echoes appear at radial distances from the centre and suitable range-mark circles give the distances in mile from the observing station. Normally, five ranges of operation for 4, 20, 50 and 100 nautical miles are used but the radar has a range up to 200 miles for beacon-ranging which can be introduced by a sweep-delay control which inserts a delay up to 200 miles in 10 mile steps. When the rotating antenna passes true North a line flashes on the screen radially and gives the North-South line. The screen is so oriented that the top of the circular pattern indicates the north. The antenna system mounted on a specially designed platform and kept on the roof of a double-storey building is shown in Fig. 2 with the radome removed. During operation, the antenna can be switched on for either continuous rotation at 20 r. p. m. or for 45° sector scanning. The antenna is also capable of being tilted from 10° below to 30° above the horizon. Manual controls are provided for clockwise or counter-clockwise rotation of the antenna. The power supply of 27·5 volts D. C. is obtained from a petrol engine driven unit which is housed in a suitable kiosk on the ground. The A. C. power supply at 115 V., 400 c. p. s. is obtained

*Radar and Weather, *Flight*, Vol. 55, pp. 142-143 through a suitable inverter unit mounted on the roof just below the wooden platform of the antenna system.

It was just a few days before the outbreak of 1952 monsoon at New Delhi that the radar was put in commission and arrangements made to photograph the echoes. Systematic investigations for taking observations at more frequent intervals of time, employing both P. P. I. and R. H. I. presentations, are also being planned. More than one hundred pictures of precipitation echoes from the monsoon rain at New Delhi were taken ; a selection from them may be seen reproduced in Figs. 3 to 8.

2. Radar Equation

It was Ryde (1948) of the General Electric Laboratories who showed theoretically that precipitation echoes can be received by micro-wave radars. In the following paragraph the principal features of the radar equation are given. This takes into account the specifications of the radar to be used, the type of precipitation region under observation and the details of the space in which the storm centre is located.

If P_t is the transmitted power and P_r the received power, the radar equation is given by the following formula :

$$P_r = \frac{\pi^2}{18(360)^2}\left(\frac{P_t A^2 \phi \theta h}{\lambda^6}\right) \times (\eta\lambda^4) \times \left(\frac{k}{R^2}\right)$$

(Radar Factor) (Precipitation Factor) (Space Factor)

Where,

P_t=Power transmitted

A=Aperture of the parabola

ϕ=Beam width vertical

θ=Beam width horizontal

h=Pulse length

λ=Wave-length

η=Reflectivity of the storm region per unit volume

k=Attenuation factor

R=Distance of the storm

In the radar factor in the above equation it will be noticed that the back-scattered received power is inversely proportional to λ^6, provided the Rayleigh scattering holds, *i.e.*, when $2r < \lambda/10$. For observations of heavy rainfall where drop sizes are likely to be large, a 10-cm radar will be more useful than a 3-cm one.

The reflectivity η appearing in the precipitation factor is a function of number of particles per unit volume, their size distribution and the radar frequency in the back-scattered energy. It has been shown that the reflectivity η is a function of nr^6, where n is the number of drops per unit volume of average radius r, such that $2r > \lambda/10$.

In the space factor appearing at the end of the above equation, the attenuation factor k is dependent upon the absorption of electromagnetic energy by water vapour and oxygen and molecular scattering by hydrometeors. The precipitation attenuation does not become large until the drop size becomes appreciable compared to the wave-length of the radar employed.

It may also be mentioned that under the most favourable conditions the range of storm-detection is not likely to be greater than 200 miles. If h is height in feet of a horizontally directed radar beam above the surface of the earth, the range R (in miles) is given by the following relation, to a first approximation—

$$h = \tfrac{1}{2} R^2$$

If R=200 miles ; h=20,000 ft.

If radar observations are being taken of an approaching storm we first get the echoes from the hydrometeors in the uppermost portions of the tallest cumulonimbus clouds along the storm. With the advance of the storm we get echoes from successively lower levels and this gives the impression that we are getting echoes from cells bigger in size and intensity than the original cells observed first. When the storm actually comes over the station the distant echoes may not appear on account of appreciable rain attenuation. This will give the appearance of solid echoes

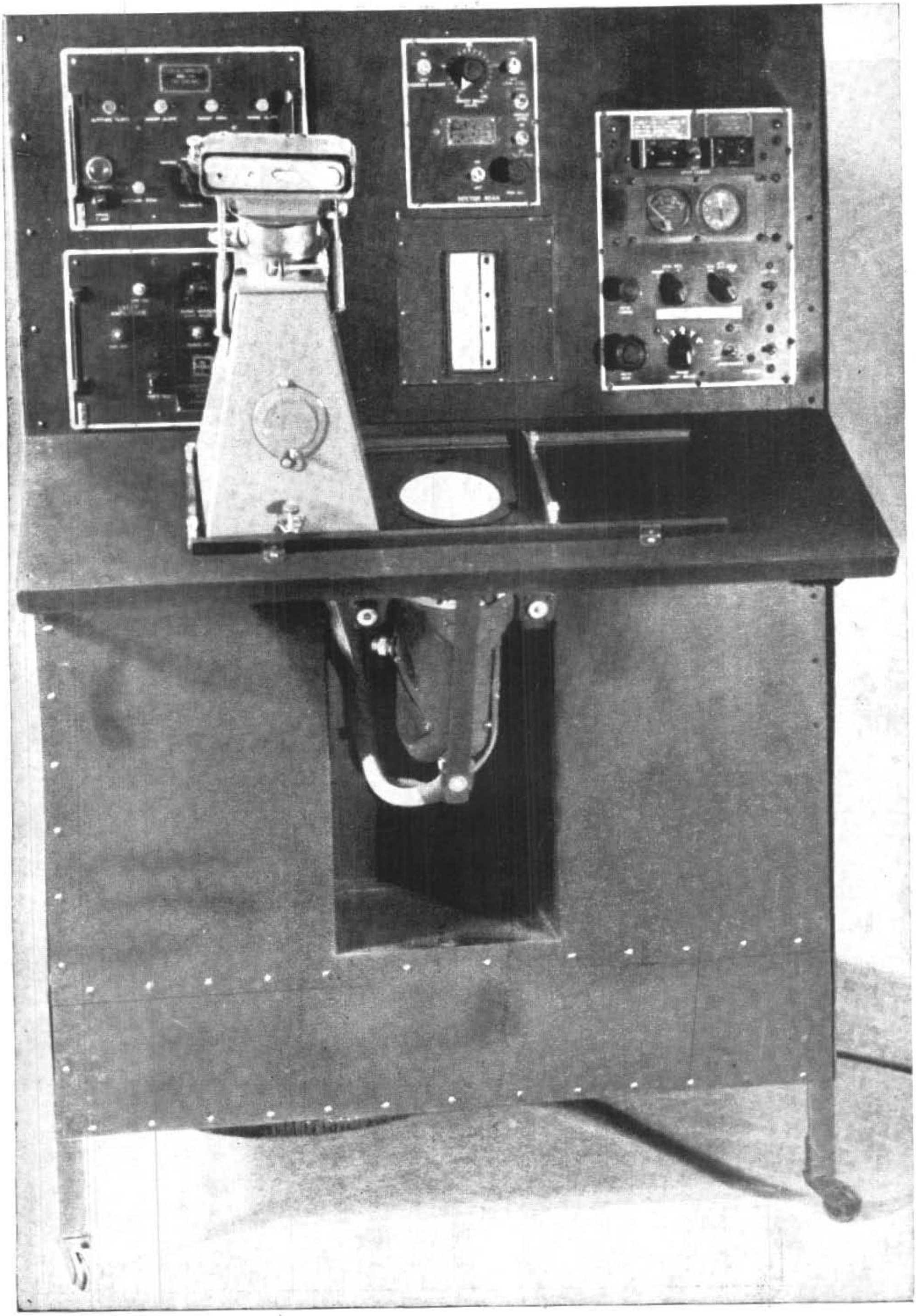


Fig. 1

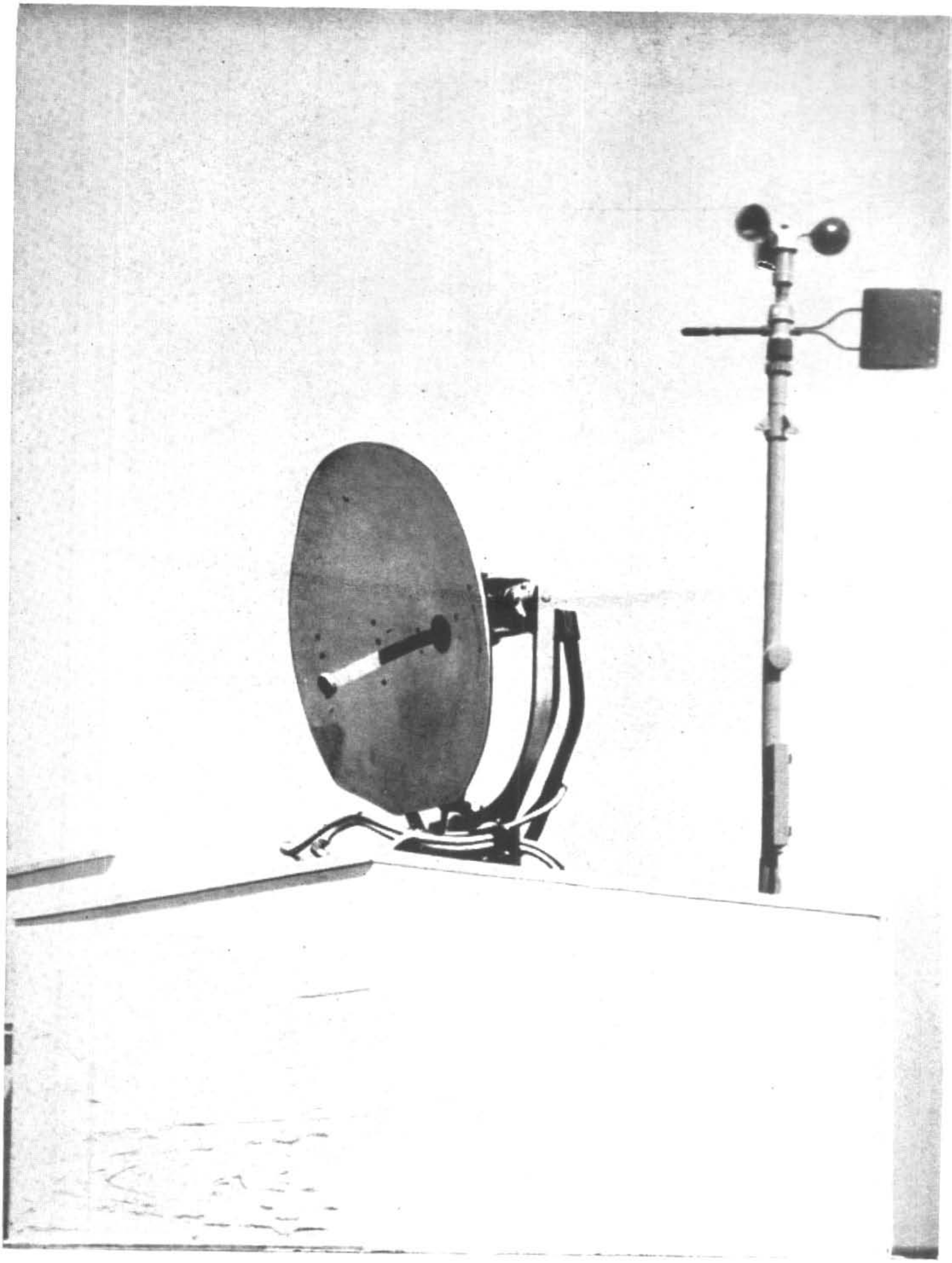
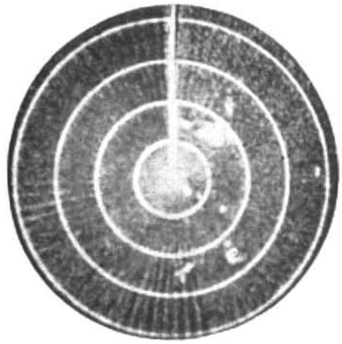


Fig. 2

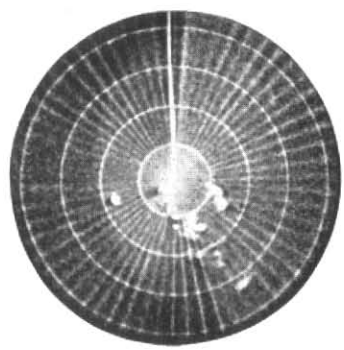
1050 IST (20; 5; –5°)
Fig. 3(a)—*Raining*



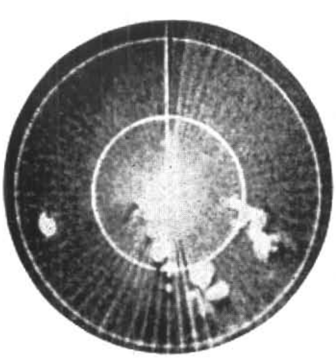
1052 IST (20; 5; –5°)
Fig. 3(b)—*Overcast*

1055 IST (20; 5; –5°)
Fig. 3(c)—*Overcast*

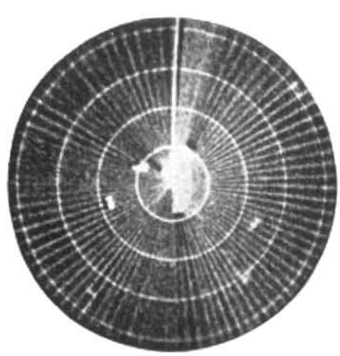
1115 IST (20; 5; –5°)
Fig. 3(d)—*Overcast*



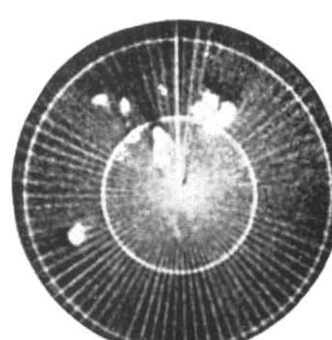
1155 IST (20; 5; –4°)
Fig. 3(e)—*Cloudy*



1157 IST (10; 5; –4°)
Fig. 3(f)—*Cloudy*

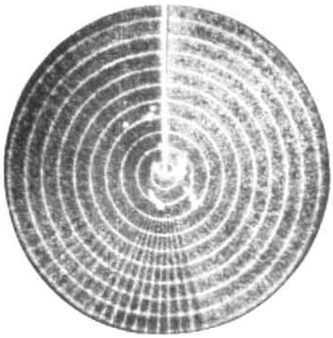


1243 IST (20; 5; –3°)
Fig. 3(g)—*Rain just started*



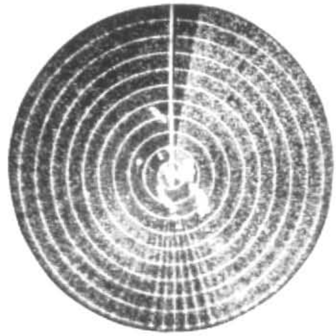
1312 IST (10; 5; –4°)
Fig. 3(h)—*Rain just stopped*

Fig. 3. 7 August 1952



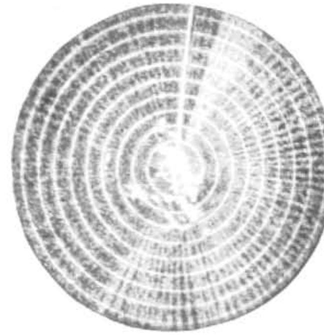
0935 IST (50; 5; −4°)

Fig. 4(a)—*Overcast*



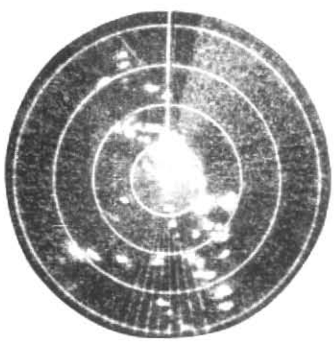
0955 IST (50; 5; −4°)

Fig. 4(b)—*Drizzle*



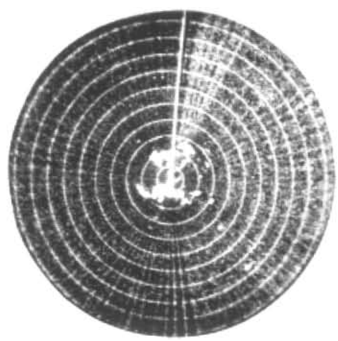
1048 IST (50; 5; −4°)

Fig. 4(c)—*Thunder heard*



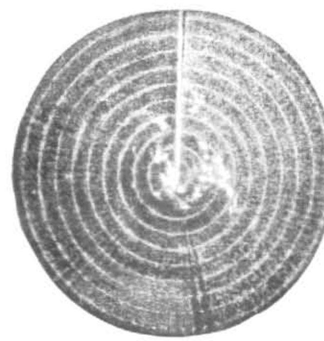
1049 IST (20; 5; −4°)

Fig. 4(d)—*Drizzle*



1143 IST (50; 5; −5°)

Fig. 4(e)—*Overcast*

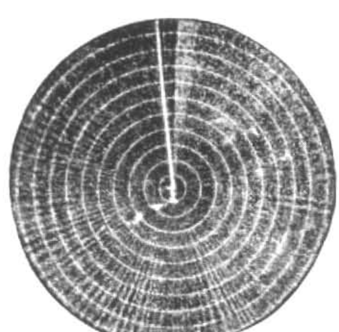


1255 IST (50; 5; −4°)

Fig. 4(f)—*Slight rain*

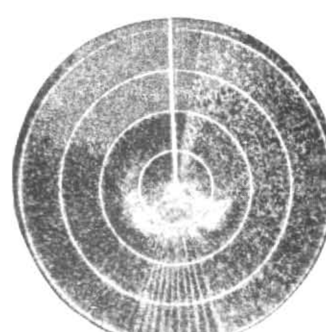
1350 IST (50; 5; −5.5°)

Fig. 4(g)—*Raining*



1430 IST (50; 5; −5.5°)

Fig. 4(h)—*Rain stopped*



1523 IST (20; 5; −5.5°)

Fig. 4(i)—*Drizzle*

Fig. 4. 8 August 1952

as observed by us on 25 August 1952 and shown at (c), (d) and (e) of Fig. 8.

In the following paragraphs are described the results of observations made at New Delhi on 7, 8, 21, 22 and 25 August 1952. Under each picture in Figs. 3 to 8, the first number within brackets gives the range in miles, the second number the distance in miles between range markers and the third the tilt of the aerial in degrees.

3. Observations of Radar Echoes from Monsoon Rain at New Delhi during August 1952

7-8-1952

The radar set was switched on at 1050 IST after seeing large cumulus and cumulonimbus clouds towards NE. The sky was then generally overcast. Several pictures were taken in succession as shown in Fig. 3. The most predominant precipitation echoes were seen towards NE. At the same time a large number of faint echoes could also be seen towards SE in the subsequent photographs taken at 1052, 1055 and 1115 IST.

By about 1155 IST the main patch of precipitation to NE completely dissipated and a large conglomeration of precipitation cells appeared in the south and southeasterly directions extending up to a distance of about 16 miles. These cells can be seen quite clearly in the next picture taken at 1157 IST with a range of 10 miles. Although the sky remained overcast, it did not start raining at the station. This is evident from the radar pictures taken at 1155 and 1157 IST in which no precipitation cell is directly over the station although they have come very near it. It started raining at the station at 1243 IST and stopped at 1312 IST. The last two pictures of the day were taken at the time of starting and stopping of the rain over the station. From the chart of the self-recording raingauge at the Meteorological Office at Safdarjung airport it was seen that 0·33″ of rain was recorded there during the above period.

The surface wind was easterly throughout the period and the upper winds were reported to be southwesterly from 2000 to 17,000 ft as revealed by the pilot-balloon observations. The cells towards SE (*e* and *f*, Fig. 3) appear to have developed a little later independent of the cells noticed at 1050 IST at a distance of about 8 miles towards NE.

The prominent precipitation cells at 1312 IST, almost to the north of the station, at a distance of about 5 miles, appear to be of the same group which was located southeast of the station at 1157 IST. It will also be observed that the isolated cell in the west-southwest direction at 1155 IST moved a few miles towards the station and could be clearly seen even at 1312 IST.

On this day, although Safdarjung reported 0·40″ of rain, the total rain at Palam was only 0·20″. Palam is at a distance of 6½ miles from the Meteorological Office, Lodi Road, in the direction 256° from the true North with respect to the station. It will be seen from the radar pictures that the concentration of precipitation cells in the direction of Palam is also less, which is probably an indication of the fact that there was less amount of rain in that region.

8-8-1952

On this day, at 0935 IST, the sky was completely overcast, distant cumulonimbus clouds could be seen in the NW direction and large cloud patches were also seen towards northnortheast, south and southwest directions and rain was expected at any moment. The first picture shown at Fig. 4(*a*) was taken at this time with a range of 50 miles. The location of precipitation echoes by the radar conforms to description of observed cloud distribution. From 0935 IST to about 0955 IST the nature of various precipitation echoes remained the same as seen from Figs. 4 (*a*) and (*b*). A slight drizzle started at the station at 0955 IST. After about an hour thunder was also heard several times over the station and there was occasional drizzle. The radar picture taken at 1048 IST showed a large number of precipitation echoes, but there was a clear area to the southwest of the station. No rain was recorded by the self-recording raingauge at Safdarjung at this time. A closer view of the precipitation echoes at this time can be seen in Fig. 4(*d*) which shows distinctly the clear areas.

In the radar picture taken at 1143 IST—Fig. 4(*e*)—the precipitation cells appear to have come closer to the station and re-aligned themselves. It is possible to see some faint echoes up to a distance of 38 miles in the southeasterly direction. The raingauge at Safdarjung recorded 0·02″ of rain from 1200 to 1212 IST. At the Meteorological Observatory, Lodi Road, New Delhi slight rain started at about 1250 IST and continued up to 1258 IST.

The next picture was taken at 1350 IST while it was raining over the station, a second time during the day. At the Observatory, Lodi Road, it started raining at 1345 IST and continued up to 1425 IST and Safdarjung airport (6 furlongs from Meteorological Office, Lodi Road) recorded 0·08″ of rain during the period. A few precipitation cells can be seen in the direction of Safdarjung airport and also at the Observatory, Lodi Road, in the radar picture at Fig. 4(*g*). The precipitation cells appear to have aligned themselves from east to west extending up to 15 or 20 miles on either side of the station. The upper winds as obtained from pilot-balloon observations at 1516 IST were almost westerly from 1000 to 12,000 ft.

The next two pictures were taken at 1430 and 1523 IST when the sky was still overcast. Only at 1523 IST there was slight drizzle. Safdarjung airport recorded 0·04″ of rain between 1510 and 1615 IST.

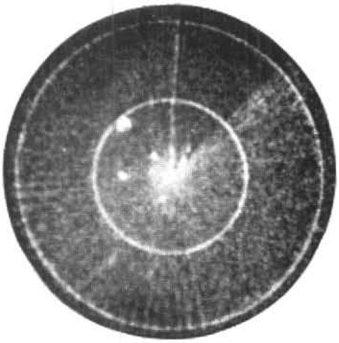
21-8-1952

It was raining practically throughout the night of 20-21 August 1952, but the radar was switched on only at 1010 IST on 21 August 1952. From midnight to about 0915 IST Safdarjung Observatory recorded 1·14″ of rain. At 1010 IST when the first picture—Fig. 5(*a*)—was taken there was rain at the station. Four more pictures were taken at 1020, 1022, 1040 and 1106 IST. Throughout this period the sky at the station was overcast and there was occasional rain. Safdarjung Observatory recorded 0·11″ of rain from 1050 to 1130 IST. It will be seen that in the first three pictures—Figs. 5(*a*), (*b*) and (*c*)—there was practically no precipitation cell in the direction of Safdarjung airport. At 1040 IST a slight appearance of precipitation cells in that direction could be noticed. In the fifth radar picture at (*e*) taken at 1106 IST, it can be seen that the raining area is over the radar site as well as Safdarjung airport. The location of precipitation cells in the pictures taken at 1040 and 1106 IST support the data of rainfall as recorded by the raingauge at Safdarjung Observatory. After the rain had stopped at 1220 IST at the station another picture was taken at 1245 IST to see the location of precipitation areas. It can be seen from the picture (*g*) of Fig. 5 that precipitation cells are about 5 miles from the station and can be seen in south, southeast and westnorthwest directions.

It started raining at the station again at 1414 IST. The radar picture (*h*) of Fig. 5 was taken at this time. Safdarjung Observatory recorded 0·03″ of rain from 1405 to 1415 IST. The picture (*h*) shows an elongated type precipitation echo extending up to 2½ miles northeast of the station. There was another echo at a distance of about 7 miles in the eastsoutheast direction. A closer view of this elongated echo can be seen in Fig. 5(*i*) with a range of 4 miles. The afternoon pilot-balloon observations showed the upper winds to be southwesterly to southerly from 1000 to 12,000 feet.

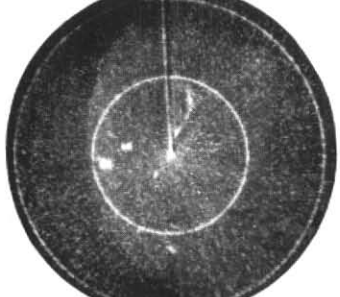
22-8-1952

The sky was partly cloudy and clouds were gathering when the radar was started at 1246 IST. The radar picture at this time—Fig. 6(*a*) — revealed the presence of well-defined precipitation echoes in the southwest direction at a distance of about 5 miles. After about 47 minutes the sky became completely overcast and rain was noticed towards the south. The photograph at (*b*) taken at this time showed the presence of precipitation echoes in the southerly direction. The picture was taken with a range of four miles to get a closer view of the echoes. A few minutes later slight rain fell at the station at 1345 IST and another photograph was taken at this time with a range of 4 miles. The



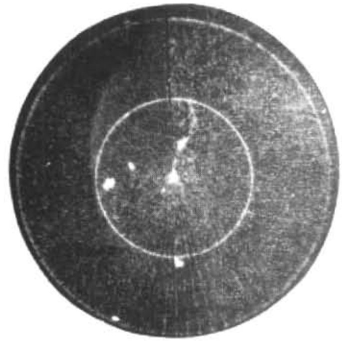
1010 IST (10; 5; −5°)

Fig. 5(a)—*Occasional rain*



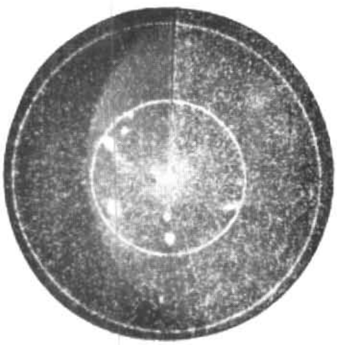
1020 IST (10; 5; −5°)

Fig. 5(b)—*Occasional rain*



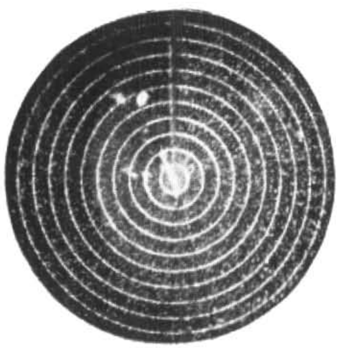
1022 IST (10; 5; −5°)

Fig. 5(c)—*Occasional rain*



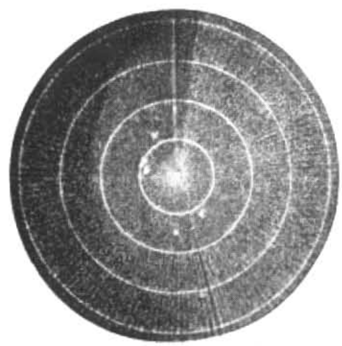
1040 IST (10; 5; −5°)

Fig. 5(d)—*Occasional rain*



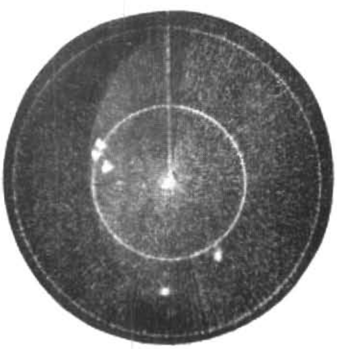
1106 IST (10; 1; −4°)

Fig. 5(e)—*Raining*



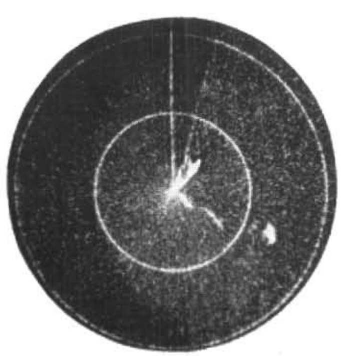
1240 IST (20; 5; −4°)

Fig. 5(f)—*Overcast*



1245 IST (10; 5; −4°)

Fig. 5(g)—*Overcast*



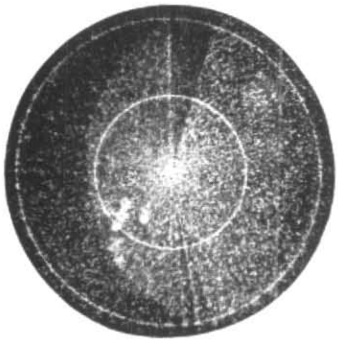
1414 IST (10; 5; −4°)

Fig. 5(h)—*Raining*

1420 IST (4; 5; −4°)

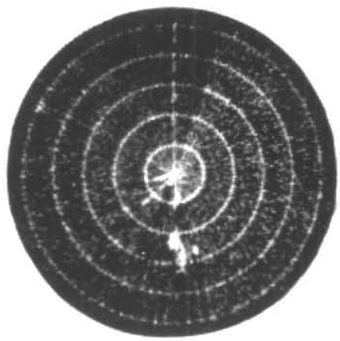
Fig. 5(i)—*Rain stopped*

Fig. 5. 21 August 1952



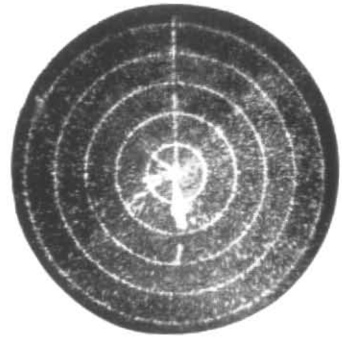
1246 IST (10; 5; −5°)

Fig. 6(a)—*Cloudy*



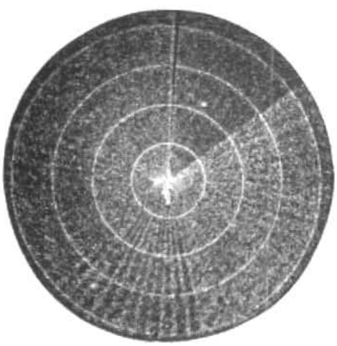
1333 IST (4; 1; −4°)

Fig. 6(b)—*Overcast*



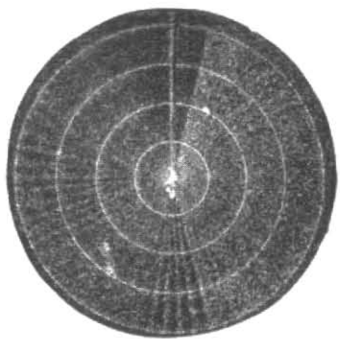
1345 IST (4; 1; −4°)

Fig. 6(c)—*Drizzle*



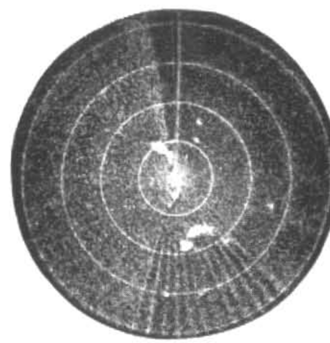
1352 IST (20; 5; −4°)

Fig. 6(d)—*Drizzle*



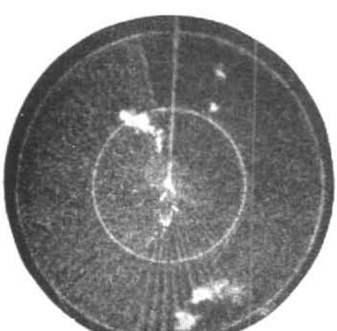
1401 IST (20; 5; −4°)

Fig. 6(e)—*Slight rain*



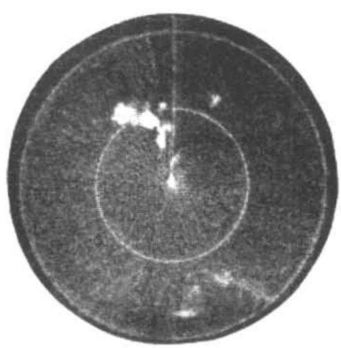
1445 IST (20; 5; −7°)

Fig. 6(f)—*Rain stopped*



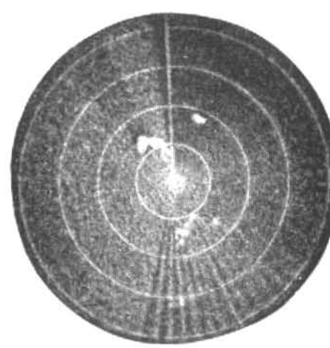
1447 IST (10; 5; −7°)

Fig. 6(g)—*Overcast*



1448 IST (10; 5; ±0°)

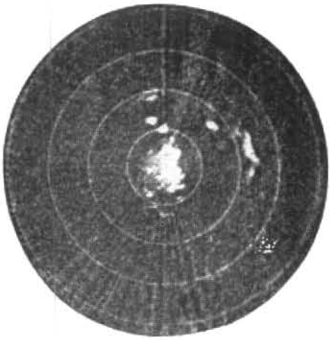
Fig. 6(h)—*Overcast*



1500 IST (20; 5; −5°)

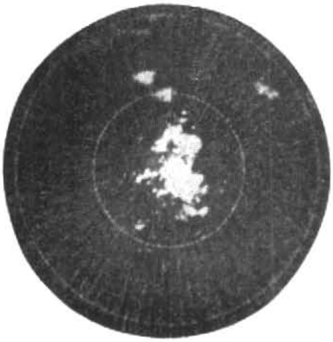
Fig. 6(i)—*Overcast*

Fig. 6. 22 August 1952



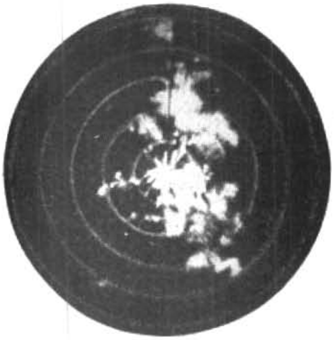
1138 IST (20; 5; −5°)

Fig. 7(a)—*Overcast*



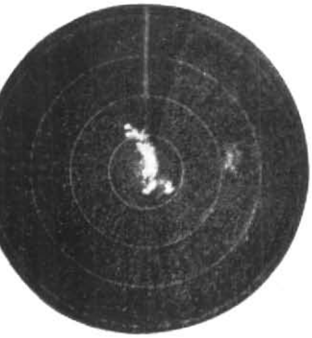
1140 IST (10; 5; −5°)

Fig. 7(b)—*Overcast*



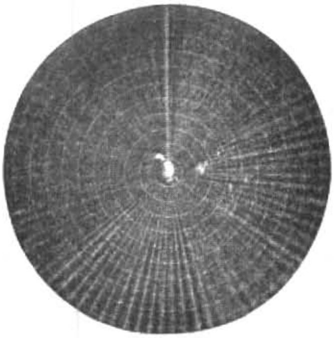
1142 IST (4; 1; −5°)

Fig. 7(c)—*Overcast*



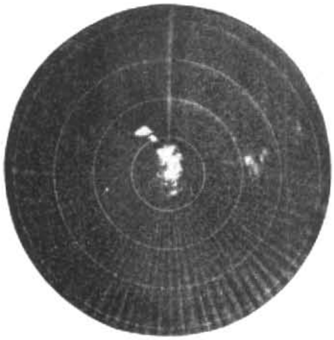
1154 IST (20; 5; −5°)

Fig. 7(d)—*Overcast*



1204 IST (50; 5; −5°)

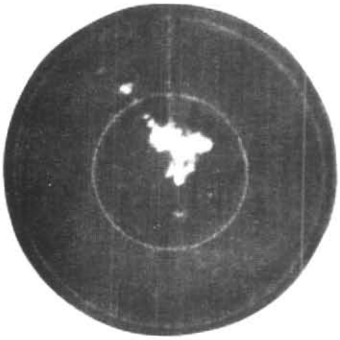
Fig. 7(e)— *Drizzle*



1206 IST (20; 5; −5°)

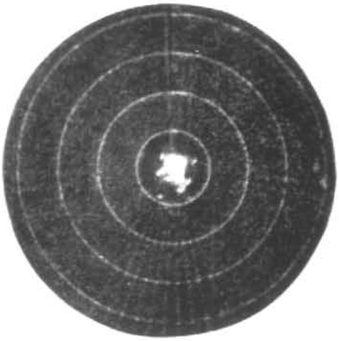
Fig. 7(f)—*Overcast*

Fig. 7. 25 August 1952



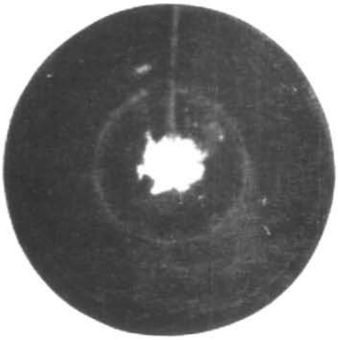
1208 IST (10: 5: +13°)

Fig. 8(a)—*Overcast*



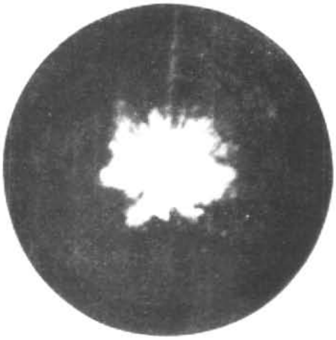
1217 IST (20: 5: −5°)

Fig. 8(b)—*Heavy rain*



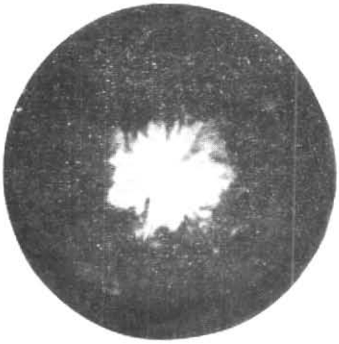
1233 IST (10: 5: −5°)

Fig. 8(c)—*Heavy rain*



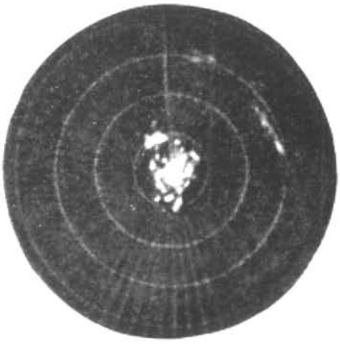
1235 IST (4: 1: −5°)

Fig. 8(d)—*Heavy rain*



1237 IST (4: 1: −5°)

Fig. 8(e)—*Heavy rain*



1250 IST (20: 5: −5°)

Fig. 8(f)—*Raining*

1328 IST (20: 5: −5°)

Fig. 8(g)—*Overcast*

Fig. 8. 25 August 1952

raingauge located at Safdarjung recorded 0·14″ of rain between 1350 to 1415 IST.

Two subsequent pictures (*d*) and (*e*) of Fig. 6 taken at 1352 and 1401 IST showed the presence of faint precipitation echoes in the northeast and southwest directions. At 1352 these cells were at distances of 11 and 16 miles respectively from the station. These moved towards the station and covered a distance of about a mile within 9 minutes. The precipitation cells over the station continued to stay on until 1445 IST. In the radar picture taken at this time it appears that both sets of cells reported above towards NE and SW have come closer to the station. The cell which produced rain probably moved northwards as shown in Fig. 6(*g*) at 1447 IST.

There is, however, an interesting picture taken a minute later, *i.e.*, at 1448 IST. In this picture the elevation angle of the radar antenna was raised by about 7°. The echo pattern from the cell to the north of the station was the same as in the earlier picture with lower elevation. The echo from the cell to the SSW of the station became fainter and its top was calculated to be at a height of 5000 ft approximately. The last picture of the day was taken at 1500 IST when it stopped raining and occasionally sunshine was seen at the station. In this picture no more precipitation cells can be seen over the station although there are a few at distances of 5 to 15 miles.

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On this day the sky was completely overcast by 1138 IST and a few drops of rain started falling at the station. Distant rain was also visible. Fig. 7(*a*) shows a large conglomeration of precipitation cells over the station within a circle of 3 miles. In spite of the large patch of the echo over the station there was no appreciable rain either at the station or Safdarjung airport. It appears that the precipitation did not reach the ground. In the picture at (*a*) a few precipitation cells could be seen in the northerly direction up to a distance of 10 miles and in the easterly direction up to 12 miles. The two subsequent pictures—Figs. 7 (*b*) and (*c*) taken at 1140 and 1142 IST with ranges of 10 and 4 miles respectively give a closer view of these echoes.

Another picture with a range of 20 miles was taken at 1154 IST when the sky was completely overcast, but no rain was falling at the station. At this time the precipitation cells seem to have aligned themselves along N–S direction. The precipitation cells reported at a distance of 12 miles to the east of the station at 1138 IST decreased in intensity by 1154 IST when the picture at (*d*) was taken. At 1204 IST the situation changed again and the sky became completely overcast and a few drops of rain fell over the station. The radar picture at (*e*) shows a large echo at the station and a few scattered echoes in the easterly direction up to a distance of 25 miles. A closer view of the echoes at this time can be seen in Fig. 7 (*f*) and Fig. 8 (*a*) with ranges of 20 and 10 miles respectively. In the picture at (*a*) the elevation angle was raised by 18° and yet a prominent echo could be photographed over the station, showing thereby that the precipitation cell was of considerable thickness extending beyond 5000 ft. Some of the small cells also appeared to possess large depth as one such cell can be seen in the southerly direction at a distance of about 2½ miles from the station.

With these thick precipitation cells over the station it started raining at 1215 IST. The elevation of the radar antenna was restored to its normal position and the range was set to 20 miles, and another photograph was taken at 1217 IST (Fig. 8*b*). Three subsequent pictures—(*c*), (*d*) and (*e*) of Fig. 8 —at 1233, 1235 and 1247 IST were taken when it was raining heavily over the station. The pictures show an area of uniform precipitation, about two miles in diameter, persisting over the station. From these pictures it can be seen that the precipitation echoes beyond this area could not be located by the radar on account of the radio-energy suffering considerable attenuation in passing through this thick layer. Safdarjung airport recorded 0·28″ of rain from 1225 to 1255 IST.

The next picture—Fig. 8(f)—was taken at 1250 IST when the rain became less intense. With the decreased intensity of rain over the station some precipitation cells could be located in the northeast direction at a distance of 15 miles. At this time the rain patch over the station developed into a well-defined cellular structure. The rain actually stopped at 1300 IST over the station. Another picture was taken at 1323 IST after the rain had stopped. In this picture—Fig. 8(g)—a large number of isolated precipitation cells can be seen in the northerly sector up to a distance of 18 miles.

4. Conclusions

From the preliminary observations made at New Delhi, it is evident that it is possible to study micro-distribution of rainfall over a particular region with radar AN/APQ-13. For a quantitative study it is necessary to have a very close net-work of raingauge stations around Delhi within a radius of 50 miles. It will then be possible to draw isohyetal lines and check their distribution by comparing them with P. P. I. pictures as obtained from the radar.

From the nature of precipitation as observed by the radar it is clear that during the monsoon, even over an area of 50 miles radius, there can be frequent occasions when it is raining in one part and not in the others. It will help the meteorologist considerably if he has at his disposal one 3-cm or 10-cm radar to enable him to issue short-range local forecasts and early warnings.

With radars employing 3-cm wavelength, there is considerable attenuation during heavy rain and it is not possible to obtain echoes beyond a radius of 5 miles or so. In order that the rain attenuation can be minimised it is desirable to have a longer wavelength, say with an operating wave-length of 6 cm.

With P. P. I. indication alone it is not possible to get a complete picture of the storm structure. It is desirable to have another radar of the same operating wavelength with a R. H. I. presentation. The range-height indicator will enable us to observe the vertical distribution of rain inside the cloud. If two sets of radars having P. P. I. and R. H. I. presentations and with their operating wave-lengths at 3 cm and 10 cm can be located at one and the same place it will be a good arrangement for the study of storm detection with the help of radar.

At New Delhi systematic study of monsoon precipitation, thunderstorm location and western disturbances is in progress. Further reports of these investigations will appear in due course.

5. Acknowledgements

The senior author and his colleagues take this opportunity of offering their heartfelt thanks to the Director General of Observatories and the Deputy Director General of Observatories (Instruments), New Delhi for encouragement given in the prosecution of these investigations. Our thanks are also due to Sri A. K. Roy, President, Meteorological Research Group, New Delhi in 1952 and other members of the group with whom the subject matter of this paper was discussed and valuable suggestions received in one of the group meetings. In the end, we would like to place on record the valuable help which Sri J. Das Gupta rendered in the earlier days of the setting up of the radar, but who had to leave this work due to his transfer to another establishment.

REFERENCE

Ryde, J. W. (1948). *Meteorological Factors in Radio-wave Propagation*, Lond. Phys. Soc., 169-188.