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Atmospheric Turbulence and Diffusion—Part I

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CONTENTS

1. Definition and the field of turbulence

Inspite of recent striking advance in the study of turbulent motion in the laboratory as well as in the atmosphere, considerable vagueness still appears to exist in the definition of turbulence and our conception of the real nature of turbulent motion. The mental picture appears to be far from being complete. Prandtl (*The physics of solids and fluids*, Blackie, p. 277) conceives of turbulent motion as consisting of some kind of actual physical movement of small vortices of a fluid called eddies from one level to another carrying in the process the original values of their properties such as heat, mass and momentum and delivering them to the new level before finally mixing with the new environment. This conception of turbulence has led to the idea of a mixing length analogous to that of a mean free path in Brownian motion where discreet molecules move in a somewhat similar manner colliding against one another over varying distances in a process of diffusion. Brunt (1939) defines turbulence as an irregular motion which in general makes its appearance in fluids, gases or liquids when they flow past solid surfaces or even when the neighbouring streams of the same fluid flow past or over one another. Similarly there have been other attempts at an acceptable definition of turbulence but it may be said without, perhaps, undue pessimism that none of the definitions so far advanced can claim to have represented the nature of turbulence in a clear and satisfying manner.

But inspite of insufficient knowledge of the real nature of turbulence we all seem to know what turbulence means and what role it plays in the transfer or diffusion of physical properties such as heat, mass and momentum in the atmosphere. The almost incessant fluctuations of the natural wind in the form of gusts and lulls (Fig. 1), the rapid dispersion of matter such as smoke from a factory chimney or a locomotive engine, the continuous evaporation and diffusion in space, of water vapour from liquid surfaces exposed to the atmosphere are amply evident of a kind of turbulent motion in the natural atmosphere.

† Sections 5-9 which form part II of the article will be published in a later issue of this Journal

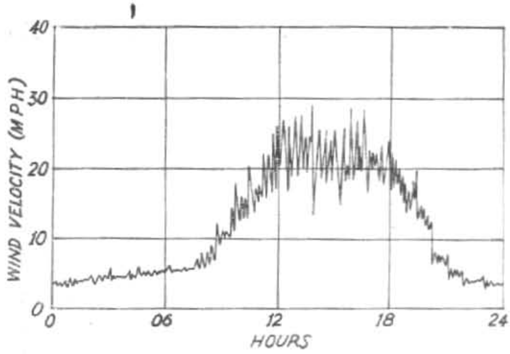

Fig. 1. Typical velocity fluctuations on a summer day in India

Acceptedly, motion in a turbulent fluid is highly complex as would be evident from its effects on physical phenomena and not amenable to an exact quantitative or mathematical treatment unless some kind of simplifying assumption is introduced to take account of the fluctuations or random motion due to turbulence. The approach must, of necessity, be through statistics. It was Reynolds (1895) who first showed that an instantaneous velocity in a turbulent fluid may be looked upon as the result of superimposition of a turbulent velocity on the mean motion of the fluid. In other words, if u, v, w be the components of an instantaneous velocity along the three rectangular axes x, y, z respectively, Reynolds assumed that

$$u=\bar{u}+u' \; ; \; v=\bar{v}+v'; \; w=\bar{w}+w' \tag{1}$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are the components of the mean motion and u', v', w' the components of the turbulent velocity along the same axes.

The field of turbulence in a medium, therefore, consists of a mean motion with components defined by (mean taken over time T).

$$\bar{u} = \frac{1}{T}\int_0^T u\,(t)\,dt\,; \quad \bar{v}= \frac{1}{T}\int_0^T v\,(t)\,dt\,;$$

$$\bar{w} = \frac{1}{T}\int_0^T w\,(t)\,dt \tag{2}$$

and a turbulent velocity with components

$$u'=u\,(t)-\bar{u}\,;\; v'=v\,(t)-\bar{v}\,;\; w'=w(t)-\bar{w}$$

When the turbulent fluid is at rest, a particle which is initially at the origin of a set of orthogonal axes ($Oxyz$) will diffuse in time t to a point (x, y, z) where the values of the space coordinates are given by the expressions

$$x = \int_0^t u'(\alpha)\,d\alpha \;;\; y = \int_0^t v'\,(\alpha)\,d\alpha \;;$$

$$z = \int_0^t w'\,(\alpha)\,d\alpha \tag{3}$$

Reynolds (1883) showed for the first time that the components of a turbulent velocity are not quite independent of one another but a certain measure of correlation obtains between them. Three types of correlation coefficients are used in experimental measurements on turbulence (Frenkiel 1952) ;

(*i*) Eulerian time-correlation coefficients for successive values of the turbulent velocity at a fixed point, considered as a function of an interval of time ;

(*ii*) Eulerian space correlation coefficients for simultaneous values of the turbulent velocities at two points in the field, considered as the function of the distance between the two points ; and

(*iii*) Lagrangian time-correlation coefficients found when the turbulent velocity of the same fluid particle is considered as a function of the time.

2. The turbulent boundary layer and the hydrodynamical treatment

The early theories of turbulent motion in fluids were inspired to a large extent by a series of beautiful experiments by Reynolds (1883) on the flow characteristics by putting colouring matter in a fluid in motion in a tube. The observation was that the flow remains laminar upto a certain critical value of the free-stream velocity, beyond which it turns into the irregular motion of turbulence. The explanation of this transformation was sought in terms of the effect of the boundary on the fluid motion. The flow in contact with the tube surface is assumed to be brought to rest by frictional drag due to viscosity and attain the free-stream velocity at some distance from the boundary. The layer of the velocity gradient begins at the entrance of the tube and gradually thickens downstream the free-stream velocity being reached at the axis of the tube. Its thickness at any distance x downstream depends upon the type of the velocity gradient assumed and may be found by making use of the famous Karman's Integral Equation (Karman 1921)

$$\frac{d}{dx}\int_0^{\eta} u^2 dy - U\frac{d}{dx}\int_0^{\eta} u dy = -\frac{\eta}{\rho}\frac{\partial p}{\partial x} - \nu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (4)$$

where u is the fluid velocity at a distance y from the boundary, U the free stream velocity, η the thickness of the boundary layer, and the other terms have their usual meanings, *i.e.*, $\partial p/\partial x$ is the pressure gradient along the x-axis, ρ the fluid density, and ν the coefficient of kinematic viscosity. If we assume a velocity distribution (Lamb 1932 a)

$$u = U\sin(\pi y/2\eta) \quad (5)$$

with the boundary conditions $u=0$, $\partial^2 u/\partial y^2=0$ for $y=0$, $u=U$ and $\partial u/\partial y=0$ for $y=\eta$, substitution in Eq. (4) with the omission of $\partial p/\partial x$, gives

$$\eta = 4\cdot 80\sqrt{\nu x/U} \quad (6)$$

Hence the tangential drag on the lamina is

$$(p_{xy})_{y=0} = 0\cdot 328\,\rho\, U^2\sqrt{(\nu/Ux)}$$

an expression which is almost the same as the Blasius expression

$$(p_{xy})_{y=0} = 0\cdot 332\rho U^2\sqrt{(\nu/Ux)}$$

found from theoretical considerations (Blasius 1907).

Reynolds showed that as long as the dimensionless number Ud/ν known as the Reynolds number, where d is the diameter of the tube, remained below a certain critical value, the flow in the boundary layer was laminar. The moment the value exceeded the limit, the laminar boundary layer became turbulent. But close to the surface of the boundary, a thin laminar sublayer was still perceptible. Owing to the transfer of momentum down the velocity gradient and resultant mixing, the gradient of the mean velocity in the turbulent boundary layer is smaller than that in a laminar boundary layer

Mathematically, the flow in a laminar boundary layer in the case of an incompressible viscous fluid is given by the famous Navier-Stokes equation (Lamb 1932 b)

$$\frac{Du}{Dt} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u \quad (7)$$

where Du/Dt is acceleration following the motion of the fluid, X the x-component per unit mass of the external force, $\partial p/\partial x$ the pressure gradient along the x-axis, and ∇^2 the Laplace Operator for

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right),$$

and two other equations for flow along the y-and the z-axes. In terms of the viscous stresses in the fluid, Eq. (7) may be written

$$\frac{Du}{Dt} = X + \frac{1}{\rho}\left(\frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z}\right) \quad (8)$$

where

$$p_{xx} = -p + 2\mu\frac{\partial u}{\partial x}\,;\; p_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right);$$

$$p_{xz} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

μ being the coefficient of molecular viscosity.

Slight rearrangement of Eq. (8) gives

$$\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}(p_{xx} - \rho u^2) + \frac{\partial}{\partial y}(p_{xy} - \rho uv) + \frac{\partial}{\partial z}(p_{xz} - \rho uw) + \rho X \quad (9)$$

Now, when the boundary layer becomes turbulent, the fluid velocity and the stresses fluctuate but mean values may still be used for the turbulent boundary layer.

Substituting Eq. (1) in Eq. (9) we get

$$\rho\frac{\partial\bar{u}}{\partial t} = \frac{\partial}{\partial x}\left(p_{xx} - \rho\bar{u}^2 - \overline{\rho u'^2}\right) + \frac{\partial}{\partial y}\left(p_{xy} - \rho\bar{u}\bar{v} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial z}\left(p_{xz} - \rho\bar{u}\bar{w} - \rho\overline{u'w'}\right) + \rho X \quad (10)$$

and the equation of continuity gives

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

Comparing Eq. (10) with Eq. (9) we find that motion in the turbulent boundary layer may be represented by taking into consideration certain additional stresses, known as Reynolds stresses. These stresses are

$$\tau_{xx} = -\rho \overline{u'^2}\,; \qquad \tau_{xy} = -\rho \overline{u'v'}\,;$$
$$\tau_{yy} = -\rho \overline{v'^2}\,; \qquad \tau_{yz} = -\rho \overline{v'w'}\,;$$
$$\tau_{zz} = -\rho \overline{w'^2}\,; \qquad \tau_{xz} = -\rho \overline{u'w'}\,. \quad (11)$$

In streamline flow the viscous stresses due to molecular agitation in a boundary layer in the xz – plane, the flow being along the x–axis, is

$$p_{xz} = \mu \frac{\partial u}{\partial z} = \rho \nu \frac{\partial u}{\partial z} \qquad (12)$$

where μ is the coefficient of molecular viscosity and ν the coefficient of kinematic viscosity.

When the flow becomes turbulent, the corresponding eddy stress is given by

$$\tau_{xz} = -\rho\, \overline{u'w'} = -\rho \overline{w'l}\, \frac{\partial \bar{u}}{\partial z}$$
$$= -\rho K_M \frac{\partial \bar{u}}{\partial z} = -A \frac{\partial \bar{u}}{\partial z} \qquad (13)$$

where l is the distance over which the mean flow velocity fluctuates by u' and is called the mixing length, K_M the coefficient of eddy diffusion of momentum ($= \overline{lw'}$) and A the coefficient of Schmidt's Austausch or interchange of momentum (Schmidt 1925). Thus a change over from a laminar motion to a turbulent motion involves replacement of μ by A, and ν by K_M. In the atmosphere the values of μ and ν are practically constant or vary slightly with temperature and pressure but those of A and K_M are far from being so. It is here that further progress along these lines received a set-back because then it was not possible to solve the Navier-Stokes equation which is now to be written in the form

$$\frac{D\bar{u}}{Dt} = X - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + K_M \nabla^2 \bar{u} \qquad (14)$$

because of the variability of the term K_M.

The experimental value of ν is of the order of 10^{-1} cm^2 sec^{-1}, whereas studies in diffusion give K varying over a wide range of 10^3 to 10^{11} cm^2 sec^{-1} (Richardson 1926). Thus although the hydrodynamical approach failed to solve the problem of turbulent diffusion it served to reveal for the first time that the transfer or diffusion of properties in a turbulent medium is on an enhanced scale. This result marked a great advance in the early study of turbulent diffusion.

3. Eddies as diffusing agents—transfer of heat, momentum and matter in the atmosphere

(a) Transfer of heat in the vertical by eddies

Taylor (1915) showed that the net upward flux of heat by eddies across an isobaric surface of unit area per unit time is given by

$$\text{Heat flux, } F_H = -\rho\, c_p\, K_H \left(\frac{\partial T}{\partial z} + \Gamma\right) \qquad (15)$$

where c_p is the specific heat at the isobaric surface, Γ the adiabatic lapse rate of temperature, $-\partial T/\partial z$ the prevailing lapse rate of temperature, ρ the density, and K_H the coefficient of eddy conductivity.

The heat transfer equation deduced by him is

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left\{ K_H \left(\frac{\partial T}{\partial z} + \Gamma\right)\right\} \qquad (16)$$

If the variation of K_H with height be neglected,

$$\frac{\partial T}{\partial t} = K_H \frac{\partial^2 T}{\partial z^2} \qquad (17)$$

Relation (15) is important for the atmosphere because it states that in a stable atmosphere in which $-\partial T/\partial z < \Gamma$, the net heat flux is negative, *i.e.*, downward, while in the case of an unstable atmosphere characterised by a lapse rate greater than Γ, the net flux is positive, *i.e.*, upward. Sutton (1949) in a theory of convection near the ground gives for the upward flux of heat the expression

$$\frac{F_H}{(F_H)_0} = 1 - \frac{\rho c_p}{(F_H)_0} \int_0^z \frac{\partial T}{\partial t}\, dz \qquad (18)$$

where $(F_H)_0$ is the value of the flux of heat at the ground.

From observations made by Johnson and Heywood(1938) on clear June days at Leafield, England, Sutton (loc. cit.) computed values of $\partial T/\partial t$ at four heights, *viz*, 1·2m, 12·4m, 30·5m, and 87·7m and showed that in the hours around noon, 1030-1330, $\partial T/\partial t$ is very nearly constant with height and time. These enabled him to come to the conclusion that in the mid-hours of a clear summer day in the height range 10m to 100m the upward flux of heat is invariable with height. He also gave the following expressions for K_H, the coefficient of eddy conductivity, and l_H the temperature mixing-length :

$$K_H = \text{const. } l_H^{\,4/3}$$
$$l_H = \text{const. } z^{1\cdot 35} \qquad (19)$$

Priestley and Swinbank (1947) contends that Taylor's expression (15) should be modified to take account of the buoyancy forces of the eddy at the level of its origin. They give for F_H the expression

$$F_H = -\rho c_p \left\{ -\overline{w' l_H} \left(\frac{\partial T}{\partial z} + \Gamma \right) + \overline{w' T''} \right\} (20)$$

where T'' is the temperature anomaly at the level of origin of the eddy.

Eq. (17) for the transfer of heat was applied by Taylor (loc. cit.) to study some interesting cases of diffusion of heat through the lower atmosphere. An interesting case studied was the modification in temperature distribution in a current of warm land air which is advected over a cool sea. If T_0 is the surface temperature and β the vertical temperature lapse rate on reaching the coast, and T_1 the sea surface temperature, the temperature at a height z at time t after leaving the coast is given by the error-function relation :

$$T = T_0 - \beta z + (T_1 - T_0)\left\{ 1 - \frac{2}{\sqrt{\pi}} \int_0^{z/\sqrt{4K_H t}} e^{-\mu^2} d\mu \right\} \qquad (21)$$

In arriving at Eq. (21) Taylor assumed that K_H was invariable with height and that the maximum height to which the surface temperature change was appreciable could be traced to $z = \sqrt{4 K_H t}$. Analysing the data of an actual case of advection over the Great Banks of New Foundland, Taylor obtained the value of K_H of the order of 10^3 cm^2 sec^{-1}. Taylor (loc. cit.) also studied the vertical diffusion of the diurnal temperature wave from the earth's surface If the variation at the earth's surface be represented by the relation

$$T = T_0 + A \sin pt \qquad (22)$$

the solution of Eq. (17) with this boundary condition gives for the temperature at height z the relation

$$T = T_0 - \beta z + A e^{-bz} \sin (pt - bz) \quad (23)$$

where $b = \sqrt{p/2K_H}$, and

$$p = \frac{2\pi}{24 \times 60 \times 60} = 7.3 \times 10^{-5}$$

By studying the changes in the amplitudes of the diurnal wave of temperature and the times of occurrence of the maximum temperature at different heights along the Eiffel Tower, Taylor deduced for K_H a value $\sim 10^5$ cm^2 sec^{-1}. He also found in the course of the same investigation that K_H was variable with height. Its value fluctuated with the state of stability of the atmosphere, high values being found in unstable conditions. and low values in temperature inversions.

Beers (1944) has solved the equation of heat transfer for the general case where the surface temperature is known as function of time on the assumption that the eddy conductivity K_H is a discontinuous step-function of elevation. Computations by this method are compared with the Lindenberg observations of diurnal variations of temperature aloft and the agreement is found to be fairly satisfactory.

Recently, direct experimental measurements have been made of the vertical eddy flux of heat by Cramer and Record (1953), and also by Swinbank (1951). In the layer from 2 to 12 metres, Cramer and Record obtained eddy velocities from hot-wire anemometers and light bivanes mounted at four intervals. Temperature fluctuations were measured with fast-response thermocouples, mounted at three levels. The observations were obtained principally over a rough land surface, under varying conditions of thermal stratification, with one set of observations for flow coming directly over a water surface. The flux-data show a maximum variation from two to four-fold within the layer. In Table 1 are presented the values of F_H and K_H computed by Cramer and Record from their measurements.

Swinbank (loc. cit) has measured the vertical flux of heat by eddies in the lower atmosphere by taking a continuous record, by photographic means, over a five-minute interval, of the detailed structure of temperature, total wind speed and its vertical component, of the air passing a fixed point. Observations made over an open grassland at two levels (2 and $\frac{1}{4}$ m above the ground) give values for the heat flux about 13×10^{-4} cal cm^{-2} sec^{-1} and for the eddy conductivity 5×10^2 cm^2 sec^{-1} in a case of temperature lapse with height. The corresponding quantities for a case of temperature inversion were found to be $-3 \cdot 4 \times 10^{-4}$ cal cm^{-2} sec^{-1} and $2 \cdot 65 \times 10^2$ cm^2 sec^{-1} respectively.

(*b*) *Eddy transfer of momentum or vorticity—the mixing-length hypothesis. The variation of wind with height*

To represent the vertical transfer of momentum by eddies in a turbulent layer, Schmidt (1925) deduced the equation

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial z} \left(\rho K_M \frac{\partial u}{\partial z} \right) \tag{24}$$

TABLE 1

Measurements of heat flux, F_H and eddy conductivity, K_H

(Time is Eastern Standard and $\rho = 1 \cdot 2 \times 10^{-3}$ g/cm^3)

z(m)	10 June 1952	11 June 1952				20 August 1952
	1500-1512	0955-1006	1310-1322	1524-1534	2119-2130	1557-1609
			F_H (*cal cm^{-2} sec^{-1}* $\times 10^{-3}$)			
11·9	0·1	2·3	1·8	0·4	—0·4	0·2
6·4	0·3	1·0	2·4	0·6	—0·7	0·1
2·3	2·9	1·6	2·7	1·4	—0·2	0·4
			K_H (cm^2 sec^{-1} $\times 10^3$)			
11·9	1·1	42·5	39·6	81·1	5·5	—1·4*
6·4	1·5	16·9	19·8	13·7	8·0	1·2
2·3	3·8	4·6	6·0	6·0	2·4	0·8

* The minus sign results from an upward heat flux and a temperature inversion

In arriving at this equation Schmidt and Prandtl introduced the concept of the mischungsweg or the mixing-length. According to this concept, an eddy which originates at a level z_0 and is a part of the mean motion at that level conserves its momentum in moving to a new level z and delivers it to the level z before mixing with new environment. The distance $(z-z_0)$ is called the mixing-length and may be said to correspond to the mean free path of molecular motion.

Taylor using the same concept of the mischungsweg derived on the assumption of the conservation of vorticity instead of momentum the relation

$$\frac{\partial u}{\partial t} = K_M \frac{\partial^2 u}{\partial z^2} \tag{25}$$

On account of inadequate knowledge of the variation of K_M with height, neither Eq. (24) nor Eq. (25) when applied to the actual atmosphere was able to explain the actual variation of wind with height.

Numerous investigations of fluid motion in pipes and tunnels have proved that under conditions of neutral stability, the velocity profile in the turbulent boundary layer over a plane surface is given by a logarithmic law of the type (Brunt 1939)

$$\frac{u}{u_*} = \frac{1}{k_0} \log \frac{z}{z_0''} \tag{26}$$

where u is the velocity at distance z from the boundary, $u_*=\sqrt{\tau_0/\rho}$ called the frictional velocity, τ_0 the surface value of the horizontal shearing stress, ρ the fluid density, z_0'' the roughness parameter, and k_0 the Kármán constant.

Prandtl (1932) suggested that Eq. (26) should be appropriate to the wind velocity profile in the lowest layers of the atmosphere under conditions of neutral stability and a logarithmic relationship between u and z has in fact been shown to be satisfactory under these conditions by Best (1935), Sverdrup (1936), Paeschke (1937) and others. Rossby and Montgomery (1935) from measurements of wind gradients over open grasslands have shown that the logarithmic law of variation of wind with height is in good agreement with observations when the atmosphere is adiabatic or unstable. It is to be noted that Eq. (26) is valid for the lowest layers of the atmosphere where τ the shearing stress is effectively constant. Experiments by Nikuradse (1932) and Dryden (1936) in smooth tubes and over flat plates showed that Eq. (26) is only satisfactory for values of $\sqrt{\tau_0/\rho}$ greater than about 30.

Deacon (1949) assumed for the velocity gradient a relation

$$\frac{du}{dz} = a\, z^{-\beta'} \tag{27}$$

where a is a constant, and $\beta'<1$, $\beta'=1$ or $\beta'>1$ according as the atmospheric condition is stable, neutral or unstable. Using the logarithmic relation for u, Eq. (26), in Eq. (27) Deacon derives a generalised velocity profile

$$u = \frac{u_*}{k_0\,(1-\beta')}\left[\left(\frac{z}{z_0''}\right)^{1-\beta'} - 1\right] \tag{28}$$

for the lowest layers of the atmosphere.

Sutton (1934, 1947 a) has shown that the mean velocity distribution in the turbulent boundary layer can be represented equally satisfactorily by the conjugate power-law relationship of Schmidt, *viz.*,

$$u\,(z) \propto z^{m}$$
$$K\,(z) \propto z^{1-m} \tag{29}$$

where m has a fractional value which is 1/7 for a wide range of observations but decreases to 1/10 when the stability of the flow decreases with the Reynolds number $Re \sim 10^6$ or more (Kármán 1921).

Recent experimental measurements by Cramer and Record (loc. cit) give values of the horizontal shearing stress, τ, and the eddy viscosity K_M (see Table 2, taken from Cramer and Record's paper). The values of the Kármán constant, k_0, found by the same authors are also included in the same table.

TABLE 2

Measurements of horizontal shearing stress, τ, eddy viscosity K_M, and Karman constant k_0

z(m)	10 June 1952	11 June 1952				20 August 1952
	1500-1512	0955-1006	1310-1322	1524-1534	2119-2130	1557-1609
			τ (dynes/cm²)			
11·9	0·2	1·7	2·8	4·1	1·6	0·5
6·4	0·5	1·2	2·0	2·3	0·8	0·2
3·7	3·7	2·2	3·8	3·5	0·9	0·7
2·3	2·9	2·0	3·0	3·0	0·6	0·8
			K_M (cm² sec⁻¹ × 10³)			
11·9	1·4	16·8	25·9	53·0	15·0	17·8
6·4	1·6	7·4	11·8	16·6	5·6	4·4
3·7	7·3	9·3	14·7	14·8	4·6	7·6
2·3	3·8	6·0	8·1	8·2	2·4	5·8
			k_0			
11·9	0·1	0·3	0·3	0·7	0·2	0·8
6·4	0·1	0·3	0·3	0·6	0·2	0·5
3·7	0·3	0·5	0·5	0·7	0·2	0·8
2·3	0·3	0·5	0·5	0·7	0·2	0·8

It will be seen from Table 2 that although the mean value of k_0 works out to be approximately 0·4, its actual variation with height and stability is rather complicated and covers a wide range of values.

(c) *Turbulence in a gravitating field—Richardson's criterion*

Much, if not most, of the preceding work could be fairly described as a straight-forward application of the aerodynamical theory of the turbulent boundary layer to meteorology. The factor which sharply differentiates the meteorological problem from those of aerodynamics is undoubtedly the effect of variable temperature distribution on the nature of the flow. In other words, the meteorologist must take into account the effects of the gravitational field on the motion of the eddies. The basic result in the theory of turbulence in a gravitational field is that due to Richardson (1920) who enunciated the criterion that the kinetic energy of the eddying motion will increase or decrease according as the rate at which energy is extracted by the Reynolds stresses exceeds or falls below that at which work has to be done by the turbulence against gravity. From this it is easy to show that turbulence will increase or decrease according as

$$K_M \left\{ \left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2 \right\} > \text{ or } < \frac{g K_H}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right) \qquad (30)$$

If K_M is assumed to be identical with K_H (as assumption for which there is little justification), the critical value of the Richardson criterion

$$Ri = \frac{\frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma\right)}{\left\{\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right\}} \qquad (31)$$

tends to unity.

In the derivation of his criterion Richardson assumed an atmosphere with very little turbulence (just-no-turbulence). The matter has since been the subject of intensive research, chiefly theoretical. Taylor (1931) and Goldstein (1931) showed that in the case of an inviscid incompressible fluid with a linear velocity profile and a continuous density distribution, $Ri=0 \cdot 25$ but the most detailed and realistic investigation is that of Schlichting (1935) on the decay of turbulence in the boundary layer of a smooth plate. Schlichting found that Ri varied between $0 \cdot 041$ and $0 \cdot 029$, depending upon the inertia effects of the density distribution, a result which was later confirmed by Reichardt (1938) in the Gottingen hot-cold wind tunnel. Recently, a detailed investigation of the mathematical aspects of Richardson's derivation has been made by Calder (1949) who finds that the inclusion of certain terms, neglected by Richardson because of his assumption of 'just-no-turbulence' changes the form of the criterion to $Ri=1-\delta$, where $\delta>0$, but he was unable to give a definite value of δ, except that δ must be small if the initial degree of turbulence is small.

There have been several attempts to determine the value of Ri in the atmosphere. Durst (1933) agrees with Richardson in finding a value $Ri=1$, but Paeschke (1937) concludes that the Schlichting Value, $Ri= 0 \cdot 04$, is appropriate. Some of the best evidence is that found by Deacon (1949) who proposes $Ri=0 \cdot 15$ for conditions near ground, while for the free atmosphere Petterssen and Swinbank (1947) suggest $Ri=0 \cdot 65$.

Recently, Gifford and Mikesell (1953) have buorght to light a very interesting observation on the relation between atmospheric turbulence and stellar scintillation. Quantitative observatory measurements of stellar scintillation have been correlated by these workers with the observed wind-speeds and shears at certain levels of the atmosphere and they find that the scintillation index which expresses information on both low and high frequency components, is best related to the high level wind-speed and shear. In recent years, the existence of a layer of high-level turbulence at a height of about 30,000 ft above ground has been confirmed by a number of workers (Hislop, 1951 ; Wyatt, 1952 ; Arakawa, 1953 and others).

(*d*) *Diffusion of matter in a turbulent medium*

Fick's equation :

$$\frac{D\chi_c}{Dt} = \frac{\partial}{\partial x}\left(K_{Dx}\frac{\partial \chi_c}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{Dy}\frac{\partial \chi_c}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{Dz}\frac{\partial \chi_c}{\partial z}\right) \qquad (32)$$

has been employed in an attempt to represent the rate of change of concentration of mass, χ_c, at a point (x, y, z) in space, emitted from various types of ground sources. In this equation, the x-axis is chosen downwind, the y-axis cross-wind, the z-axis vertical and K_{Dx}, K_{Dy}, K_{Dz} are the components of the coefficient of eddy diffusion along these axes respectively. The law of variation of the diffusion coefficient components along the axes not being known with any certainty, a solution of Eq. (32) was tried assuming $K_{Dx}=K_{Dy}= K_{Dz} = K_D$, an assumption for which there is no physical justification. Obviously, even with this assumption the solution showed no agreement with observation. To fit in with observation further assumption had to be made that the absolute value of K_D increased with distance from the source, or with the scale of the phenomena This further assumption was mathematically indefensible and it virtually marked the end of the approach to the problem along these lines. As we shall see in the next Section a statistical theory of turbulence which was put forward by Taylor (1921) has been applied to the problem of atmospheric diffusion with remarkable success and of late there has been great advance along these lines. The classical

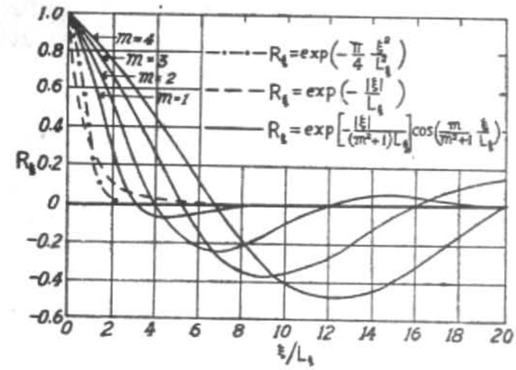

Fig. 2. Form of Lagrangian correlation curve, R_ξ, as a function of ξ. (After Frenkiel 1952)

studies by Taylor (loc. cit), Richardson (1926), Kármán (1936) and others have been lately followed by more recent studies of Kolmogoroff (1941), Onsagar (1945), Weizsäcker (1948), Heisenberg (1948), and Batchelor (1947) and others who have devoted considerable attention to the study of the structure of isotropic turbulence. To these recent studies we shall return later in the paper.

4. The statistical theory of turbulence and the treatment of diffusion

Taylor (1921) expressed the diffusion of a group of particles suspended in a medium with statistically uniform and steady turbulence ($\overline{u'^2}$ independent of locality and time) by the famous equation

$$\overline{X^2} = 2\,\overline{u'^2}\int_0^T \int_0^t R_\xi \, d\xi dt \tag{33}$$

where $\overline{X}$ is the distance travelled by a particle in time T and R_ξ the correlation coefficient between the fluctuating velocities acting on the same particle at times t and $t+\xi$. Taylor's theory in effect means that the diffusion in such a field is completely described by a knowledge of the mean eddying energy $\overline{u'^2}$ and the Lagrangian correlation coefficient R_ξ. To investigate the variation of $\overline{X^2}$ with time, attempts have been made to express R_ξ in terms of different functions of ξ (Frenkiel 1952). Some of these functions are shown in Fig. 2. In these functions of R_ξ, L_ξ is the Lagrangian scale of turbulence defined by

$$L_\xi = \int_0^\infty R_\xi d\xi$$

There are two cases in which the standard deviation does not depend on the shape of the R_ξ-curve.

1. When the dispersion time is very small compared to the Lagrangian scale of turbulence L_ξ, *i.e.*, when $\xi << L_\xi$

$$\overline{X^2} = \overline{u'^2}\, T^2 \tag{34}$$

The outline of the dispersing matter in this case is a cone.

2. When the dispersion time ξ is very large compared to L_ξ, *i.e.*, when $\xi >> L_\xi$,

$$\overline{X^2} = 2\overline{u'^2}\, L_\xi\, T \tag{35}$$

The outline of the dispersing matter is a paraboloid in this case.

Sutton (1932) suggested for R_ξ the form

$$R_\xi = \left(\frac{a}{\bar{u}\xi}\right)^n \tag{36}$$

where a is a constant length, $\bar{u}$ the mean wind velocity, and n a real quantity ($0<n<1$). In a later paper, Sutton (1934) proposed the form

$$R_\xi = \left(\frac{\nu}{\nu+\overline{w'^2}\xi}\right) \tag{37}$$

where ν is the coefficient of kinematic viscosity and w' the vertical component of the eddy velocity.

Using the methods developed by him in 1932 and 1934, Sutton gives the following expressions for the distribution of concentration of matter such as smoke, gases, and other particulate clouds emitted from continuous point and line sources at ground level ($z=0$) :

Continuous point source at (0, 0, 0) *emitting Q grams per second*

$$\chi_c\,(x,y,z) = \frac{Q}{\pi C_y C_z\, \bar{u}\, x^{2-n}} \exp\left\{-\frac{1}{x^{2-n}}\left(\frac{y^2}{C_y^2}+\frac{z^2}{C_z^2}\right)\right\} \tag{38}$$

Continuous infinite line source along $x=0$, $z=0$, emitting Q grams per second per centimetre

$$\chi_c\ (x,z) = \frac{Q}{\pi^{\frac{1}{2}}\ C_z\ \bar{u}\ x^{(2-n)/2}} \exp\left\{\frac{-z^2}{C_z^2\ x^{2-n}}\right\} \quad (39)$$

where $\chi_c\ (x,y,z)$ is the concentration of smoke or gas (g/cm^3) at the point (x, y, z), $\bar{u}$ the mean wind speed assumed constant with height, and C_y, C_z are the generalised diffusion coefficients along the y– and the z–axes respectively given by the expressions

$$C_y^2 = \frac{4\nu^n}{(1-n)\ (2-n)\bar{u}^n}\left(\frac{\overline{v'^2}}{\bar{u}^2}\right)^{1-n} \quad (40)$$

$$C_z^2 = \frac{4\nu^n}{(1-n)\ (2-n)\bar{u}^n}\left(\frac{\overline{w'^2}}{\bar{u}^2}\right)^{1-n} \quad (41)$$

From the above expressions it is immediately seen that the principal quantities measured in diffusion experiments are given by the following relations.

Peak concentration in the cloud from a continuous point source at $x=y=z=0$

$$(\chi_c)_m = \chi_c\ (x,0,0) = \frac{2Q}{\pi C_y\ C_z\ u\ x^{2-n}} \quad (42)$$

(NOTE : The figure 2 is used to take account of the reflection of the cloud from the ground).

Peak concentration in the cloud from a continuous infinite crosswind line source at $x=z=0$

$$(\chi_c)_m = \chi_c\ (x,\ 0) = \frac{2Q}{\pi^{\frac{1}{2}}\ C_z\ u x^{(2-n)/2}} \quad (43)$$

Width of the cloud from a continuous point source

$$2y'_0 = 2\ (\log_e 10)^{\frac{1}{2}}\ C_y\ x^{(2-n)/2} \quad (44)$$

Height of the cloud (point or line source)

$$z'_0 = (\log_e 10)^{\frac{1}{2}}\ C_z\ x^{(2-n)/2} \quad (45)$$

Sutton (1947a) has presented the results of a large number of field experiments at Porton, England, using true gases (chlorine, phosgene, or sulphur dioxide) or particulate clouds (screening smokes, dyestuffs, or arsenical compounds, generated by sublimation or spraying) and shown that the computed values of concentration as given by the equations (42) to (45) are in satisfactory agreement with observations.

Sutton (1947b) has also studied the problem of diffusion of matter from an elevated continuous point source such as a factory chimney and gives for concentration from such a source the relation

$$\chi_c\ (x, y, z) = \frac{Q \exp\ (-y^2/C_y^2 x^{2-n})}{\pi C_y\ C_z\ u\ x^{2-n}} \left[\exp\left\{-(z-h)^2/C_z^2\ x^{2-n}\right\} + \exp\left\{-(z+h)^2/C_z^2\ x^{2-n}\right\}\right] \quad (46)$$

where h is the height of the source, *i.e.*, the height of the chimney above the ground level.

In this paper on airborne pollution from factory chimneys, the main result derived by Sutton (loc. cit.), for an atmosphere in which the wind and eddy diffusivity increase with height in accordance with the conjugate power-law, is contained in the relation

$$(\chi_c)_{\max} \propto 1/h^{4/(2-n)} \quad (47)$$

where $(\chi_c)_{\max}$ is the maximum concentration at the ground level. Eq. (47) means that raising the height of the chimney-top is equivalent to moving the source into a faster wind with the result of increased dilution of the cloud at the ground level.

In a recent paper Sutton (1950) has discussed the dispersion of a stream of hot gas emitted from a point source in the atmosphere. An interesting part of this investigation which conforms more to the actual conditions in the atmosphere is contained in his analysis of the dispersion in the presence of a horizontal wind. An approximate relation is given for the shape of the plume and it is shown that the reduction of maximum concentration at ground level caused by adding heat to the effluent from a stack is directly proportional to the strength of the heat source and inversely proportional to the height of the chimney and the cube of the horizontal wind speed.

In the field of smoke-screening, Sherwood (1949) has made an important contribution to the study of geometry of smoke-screens. He has applied Sutton's expressions for diffusion to predict the shape and size of areas screened by smoke emitted from a single smoke generator as well as from multiple generators placed in a crosswind line under various conditions of atmospheric stability.

The relative diffusion of neighbouring particles has been studied by Richardson (1926), Richardson and Stommel (1948) and others, and recently by Brier (1950). Richardson (loc. cit.) showed in his 'Distance-neighbour graph' theory that if l is the projection of the separation of a pair of marked particles on a fixed direction and $q(l)$ is the number of neighbours per unit of l,

$$q(l) = \int_{-\infty}^{+\infty} \chi_c\,(x)\;\chi_c\,(x+l)\,dx \qquad (48)$$

where $\chi_c\,(x)$ is the concentration at the distance x from a fixed origin. The equation of diffusion then becomes

$$\frac{\partial q}{\partial t} = \frac{\partial}{\partial l}\left\{F(l)\,\frac{\partial q}{\partial l}\right\} \qquad (49)$$

where $F\,(l)$ is a kind of diffusion coefficient. If l_0 is the initial value of l, and l_1, its value at t seconds later,

$$F(l_0) = \frac{\text{mean of } (l_1-l_0)^2 \text{ for all pairs}}{2t} \qquad (50)$$

Observations by Richardson and Stommel (loc. cit.) on small objects floating in water indicate that

$$F\,(l) = 0{\cdot}07\; l^{\,1.4} \qquad (51)$$

This is a result of considerable importance because the recent statistical theories of Weizsäcker and Heisenberg predict a similar law of diffusion.

Brier (1950) studied in detail the relative diffusion of a set of balloons released in the atmosphere. He took into account the initial separation of the balloons but the results of his experiments failed to supply any consistent data on diffusion process in the atmosphere.

The problem of evaporation and eddy diffusion of water vapour

Recently, there has been renewed interest in the study of evaporation from water surfaces and eventual diffusion of the vapour into space. It is noteworthy that although most of the workers in this field (Sverdrup 1936, 1937-38, 1946 ; Millar 1937 ; Montgomery 1940 ; Norris 1948 ; Craig 1949 and others) agree to give for evaporation from a smooth sea surface an expression of the type

$$E = k_0 \gamma_b\;\Gamma_b\,\rho u_b\,(q_s - q_b) \qquad (52)$$

where k_0 is Kármán constant, γ_b the resistance coefficient referred to a standard level b within the logarithmic layer, Γ_b the evaporation coefficient at level b, ρ the density of the air, u_b the wind velocity at the level b, q_s the saturated specific humidity at the temperature of the sea surface, and q_b the specific humidity at the level b, there are wide differences in the results found for evaporation from a hydrodynamically rough surface. Montgomery (loc. cit.) concludes that the rate of evaporation from a rough surface is about the same as that from a smooth surface. Sverdrup (loc. cit.) puts the rate at twice the value from a smooth surface. Norris (loc. cit), however, has shown that the rate of evaporation from a sea surface roughened by waves is about four times that to be expected from a hydrodynamically smooth surface.

The problem of diffusion of water vapour in space, although a matter of great importance has unfortunately failed to receive as much attention from meteorologists as that of smoke and other gases and vapours. This is particularly so in respect of diffusion over land surfaces where the wide variability of land surface condition with respect to type of soil, moisture content available for evaporation, vegetation etc, has been a discouraging factor. Some measurements of the distribution of water vapour in the lower layers have been made by Rossi (1933), Sverdrup (1936), Fireash (1940) Ramdas (1943), Best and others (1952). Swinbank (1951) has devised a method for continuously recording the fluctuations in the vertical eddy flux of water vapour near the Earth's surface.

The distribution of water vapour over the sea surface has been measured and studied by Taylor (1915), Wust (1920, 1937), Ficker (1936), Jaw (1937), Montgomery (1940), Sverdrup (1946), Craig (1949) and others. Of particular interest has been the study of the vertical diffusion of water vapour from a sea surface when a relatively warm and dry air current from the land flows out over a cool sea. Taylor's analysis of this problem so far as temperature and humidity modification is concerned was expressed in terms of an error-function and this has already been presented in Eq. (21). Recently, Craig (loc. cit.) has reconsidered this problem of advection and diffusion in the light of new data in the first 1000 ft of the atmosphere over Massachusetts Bay and given expressions for the proportional change of specific humidity that would occur in the advected air mass at different heights and distances off-shore.

A theoretical treatment of the same problem of diffusion in an advected air mass has been given (Booker, 1948) in connection with a study of microwave propagation in lower atmosphere over the sea. The equation governing the vertical distribution of humidity is

$$\frac{\partial}{\partial z}\left\{ K(z)\ \frac{\partial q}{\partial z} \right\} = u(z)\ \frac{\partial q}{\partial x} \qquad (53)$$

where $K(z)$ is the coefficient of eddy diffusion along the z-axis, $u(z)$ the horizontal wind speed also variable with height, q the specific humidity, x the distance off-shore, and z the height above the sea surface.

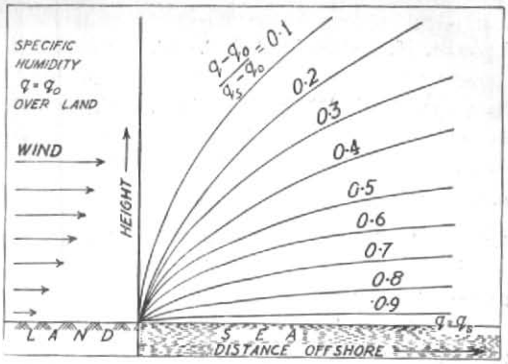

Fig. 3. Advection of air mass from land to sea : Specific humidity contours ($m=\frac{1}{2}$) (After Booker 1948)

Using the conjugate power-law relationship of Schmidt, *viz.*, Eq. (29), the solution of Eq. (53) is practicable if it is assumed that q is a function of $z^n/(n^2x)$. Substitution of this assumption in Eq. (53) shows that this is possible if $n = 2m+1$, and the function turns to be the incomplete Gamma function. With the boundary conditions $q=q_0$ at $x=0$ and $q = q_s$ at $z = 0$, the required solution is

$$\frac{q-q_s}{q_0-q_s} = I(z/z_0,\ m) \qquad (54)$$

where

$$I(\zeta, m) = \frac{1}{\Gamma\left(\frac{m}{2m+1}\right)} \int_0^{\frac{m}{2m+1}\,\zeta^{2m+1}} e^{-\eta}\cdot\eta^{-\frac{m+1}{2m+1}}\cdot d\eta$$

and $z_0 = \left\{(K_1/u_1)m(2m+1)x\right\}^{\frac{n}{2m+1}}$

K_1 and u_1, being the values of $K(z)$ and $u(z)$ at unit height.

If curves of constant specific humidity are drawn in a vertical plane containing the wind direction, a diagram of the type shown in Fig. 3 is obtained.

Recently, Gifford (1953) has provided us with an alignment chart for ready calculation of atmospheric diffusion quantities of the type given by Sutton's equations (38) to (45).

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