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Lag coefficient of bimetal thermometer of chronometric radiosonde

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ABSTRACT. The value of lag coefficient of the bimetal thermometer used in chronometric radiosonde of the India Meteorological Department when it is exposed at various orientations to air currents with different speeds has been determined experimentally. The order of errors that may be expected due to lag at different pressure levels has also been computed.

1. Introduction

The temperature element used in the chronometric radiosonde (Mathur 1948) of India Meteorological Department consists of a steel-invar bimetal strip of thickness 0·025″, width 0·125″ and length 5·0″ ; this is fabricated in the form of a spiral coil with $2\frac{1}{2}$ turns. One end of the coil is fixed and a pen is attached to the other end ; the tip of the pen carries a fine silver contact and moves along an insulated cylinder rotated by a clock work ; the cylinder is inlaid with a helically wound contact wire.

Measurement of upper air temperature with balloon-borne radiosonde involves errors due to (*i*) lag of the temperature sensitive element and (*ii*) radiation. Both of these errors increase with decreasing air density or in other words with increase of height. This paper deals with the error due to lag of the thermometer element.

2. Theory

When a thermometer of any type at temperature θ_0 is placed in a medium of temperature θ_e the change in the indication of the thermometer with time according to Newton's Law of cooling is denoted by

$$\frac{d\theta}{dt} = -\frac{1}{\lambda}(\theta - \theta_e) \quad (1)$$

where θ is the instantaneous indication of the thermometer at time t and λ is a quantity known as the lag coefficient of the thermometer. Integrating (1) we get

$$\theta - \theta_e = (\theta_0 - \theta)e^{-t/\lambda} \quad (2)$$

assuming that the temperature θ_e of the environment does not change. From above it may be seen that after a time λ the difference in the reading of the thermometer and the temperature of environment is reduced to $1/e$ times of its initial value. This is another way of defining λ. Relation (2) can also be expressed in the form

$$\log_e (\theta - \theta_e) = \log_e (\theta_0 - \theta_e) - \frac{1}{\lambda}t \quad (3)$$

which is the equation of a straight line of slope $-1/\lambda$.

In radiosonde ascent the temperature of environment changes with time and assuming θ_e to be a linear function of time, it has been shown (Middleton 1947) that

$$\theta - \theta_e = -\beta\lambda(1 - e^{-t/\lambda}) \quad (4)$$

where β = change of temperature of environment per unit time. After a time $t >> \lambda$

$$\theta - \theta_e = -\beta\lambda \quad (5)$$

It follows, therefore, that in a medium where the temperature changes at a rate β the thermometer at any instant, would record a temperature which differs from the temperature of environment by an amount equal to the product of lag coefficient and rate of change of temperature with time.

It would appear from relation (3) that the lag coefficient of a thermometer can be determined by plotting its cooling curve when it cools from a higher temperature to take up the temperature of the medium in

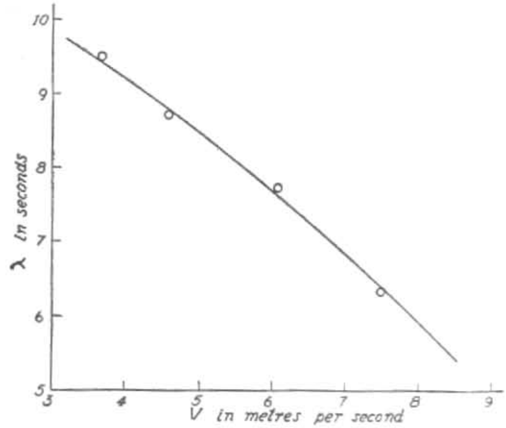

Fig. 1

which it is placed. The logarithm of the temperature difference between instantaneous and final values when plotted against time on a graph, produces a straight line, the reciprocal of the slope of which gives the value of λ. This value, however, depends on the rate of ventilation, *i.e.*, upon the ascensional rate of balloon and also on the manner the thermometer is exposed to ventilation.

The value of λ may be combined with the lapse rate of the free atmosphere to compute the errors in the reading of the thermometer telemetered to ground with the help of relation (5).

Experiments were conducted to determine the value of λ, (*i*) for wind speeds of 7·5, 6·1, 4·6 and 3·7 m. sec^{-1} and (*ii*) when the plane of the coil of bimetal thermometer was exposed to the draft of air at angles of 0°, 20°, 40°, 60° and 90°. The results are given in the following Section. It may be mentioned that ascensional rate above 7·5 m. sec^{-1} and below 3·7 m. sec^{-1} are not normally employed for radiosonde observations.

3. Results

The values of λ for different rates of ventilation are given below. These have been obtained with the plane of the coil normal to the direction of ventilation. The result is shown graphically in Fig. 1.

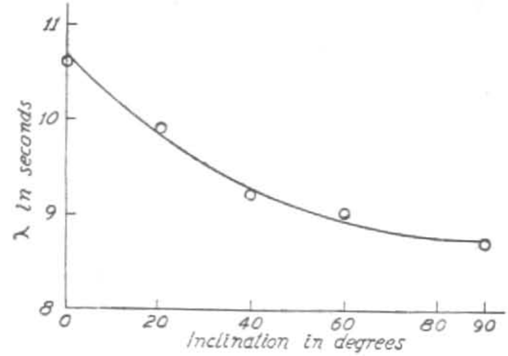

Fig. 2

Rate of ventilation (m. sec^{-1})	λ (sec)
7·5	6·3
6·1	7·7
4·6	8·7
3·7	9·5

When the plane of the coil of the thermometer was inclined at different angles to the direction of ventilation the following values of λ were obtained.

Inclination (degrees)	λ (sec)
0	10·6
20	9·9
40	9·2
60	9·0
90	8·7

These values correspond to a ventilation rate of 4·6 m. sec^{-1}. The result is shown graphically in Fig. 2.

4. Discussion

In addition to the rate of ventilation and the manner in which it is exposed to ventilation, lag coefficient of a thermometer depends upon many other factors ; of these the following are important

(*i*) The material of which the thermometer is made and its dimensions or strictly speaking on the total heat capacity of the element (*ii*) Area of surface effectively exposed to ventilation and its nature of roughness

(*iii*) Properties of the medium in which the thermometer is placed

(*iv*) Heat capacity and conductivity of auxiliary parts in thermal contact with the element.

Of these, (*i*), (*ii*) and (*iv*) depend on the material, shape and method of mounting the element on the sonde and are at the control of the designer while (*iii*) depends on the thermodynamic properties of air.

The dependence of lag coefficient on ventilation has been investigated (Middleton, Edwards and Johnson 1938) and it has been found that the relation can be represented by an expression of the type

$$\lambda = K\, V^{n} \qquad (6)$$

where K = a constant for the particular thermometer

V = wind speed

In this expression n has been found to have a value which lies in the neighbourhood of $-0 \cdot 5$. Suomi and Barrett (1952) have expressed the view that the physical significance of the factors relating to constant K are not very clear. They have quoted an empirical expression for K derived by deQuervain in the following form

$$K = 22 \cdot 4 \, \frac{c_p M}{S} \left[0 \cdot 319 \text{ exp. } \left\{ -1 \cdot 162 \, (p/p_0)^{\frac{1}{3}} \right\} \right] \qquad \ldots (7)$$

where c_p = specific heat of the material

M = mass of the material

$c_p M$ = total heat capacity of the material

S = effective surface exposed to ventilation

p/p_0 = ratio of actual pressure to a standard pressure, ($1013 \cdot 2$ mb)

From the above expression it is evident that ratio of mass to surface exposed to ventilation is the figure of merit for low lag thermometers. It follows, therefore, that by decreasing the thickness of the bimetal thermometer and increasing its exposed surface, a lower value of λ can be obtained. But the thickness cannot be reduced indefinitely without impairing the torque deflection, temperature-torque characteristics and the mechanical stability of the thermometer. For the same length of thermometal reduction in thickness would result in an increase in deflection ; for a required amount of deflection, therefore, reduction in thickness would require a proportionate reduction in length. Both these factors would reduce the mass and consequently the value of λ.

Factor (*iv*) can be avoided by thermally insulating the thermometer from the instrument frame. It is to be noted that in many radiosondes the metal frame of the instrument has a considerably greater mass compared to thermal element employed for measurement of temperature and this frame is not properly exposed to ventilation. As such, during flight the frame may possibly be at a higher temperature than the thermometer ; this will cause flow of heat from the frame to the thermometer which would consequently read higher for falling temperature.

The effective exposed surface of the element depends on how the element is oriented with respect to the direction of ventilation. If the plane of the coil is parallel to the draft, the effective exposed surface is minimum and the value of λ will be maximum ; with the plane of the coil normal to the direction of wind the effective exposed surface is maximum and the value of λ will be minimum. At any other orientation λ will have a value between these two extremes. The effect may be seen from Fig. 2.

It has been found that the experimentally obtained curve of λ against inclination is approximately exponential in nature and can be represented by a relation of the form

$$\lambda = \lambda_0 \, e^{-b \sin\alpha} \qquad (8)$$

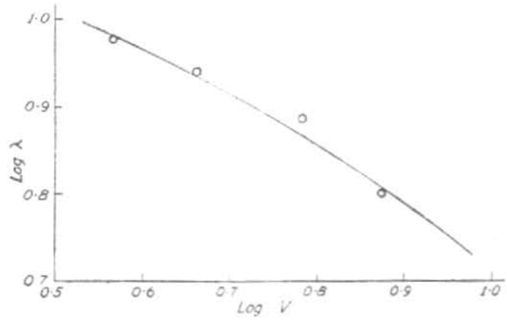

Fig. 3

TABLE 1

Errors (°C) due to lag of the bimetal thermometer for the lapse rate of 10° C km^{-1}

Rate of ascent (m. sec^{-1})	Level (mb)								
	Surface	900	800	700	600	500	400	300	200
7·5	·47	·49	·51	·54	·58	·62	·68	·76	·90
6·1	·47	·49	·51	·54	·58	·62	·68	·76	·90
4·6	·40	·42	·44	·47	·50	·53	·60	·65	·76
3·7	·35	·37	·38	·40	·43	·46	·51	·57	·67

TABLE 2

Errors (°C) due to lag of the bimetal thermometer for the lapse rate of 6·5°C km^{-1}

Rate of ascent (m. sec^{-1})	Level (mb)								
	Surface	900	800	700	600	500	400	300	200
7·5	·31	·32	·34	·36	·38	·41	·45	·50	·59
6·1	·31	·32	·34	·36	·38	·41	·45	·50	·59
4·6	·26	·27	·29	·30	·32	·35	·39	·43	·51
3·7	·23	·24	·25	·27	·28	·30	·33	·37	·44

where b is a constant, α is inclination and λ_0 the value of λ at 0° inclination. The value of b is very nearly equal to 0·2 and probably depends on the shape of the thermometer.

As already mentioned, the quantity n in the expression (6) has a value which is very nearly equal to -0·5. This value is not constant and tends to increase with rate of ventilation; according to Petterson and Womack (1937), it approaches -1 when the ventilation rate is very high. Results obtained in a different series of experiments are given below.

V (m/sec)	n
3·7	—0·50
4·4	—0·52
4·8	—0·54
5·2	—0·57
6·0	—0·60
6·9	—0·66
7·2	—0·70

These results are also in agreement with those obtained by Middleton and others. The curve log V against log λ according to relation (6) should be a straight line. From experiment it is found that this is not strictly true and the value of n has been calculated assuming that the above relation holds only over small range of velocity. Fig. 3 may be seen in this connection.

In radiosonde observations the error due to lag of the thermometer as pointed out in (5) will be equal to the product of β and λ. But the lapse rate β (degrees sec^{-1}) of the free atmosphere is not constant at all heights and during all seasons. Order of errors computed for lapse rate of 10°C km^{-1} and 6°·5 C km^{-1}, are given in Tables 1 and 2.

In the above calculation it has been assumed that the chief process of heat transfer from the element to the air flowing past it is through thermal conduction. This is true only at lower levels but at higher levels where the density of air is very low, this is no longer the major process of heat exchange; radiation at that level plays an important part and the value of λ will be different. The errors at different levels given in the above tables have been calculated on the basis that the error due to lag varies inversely as the square root of air density (Dymond 1947).

It may also be pointed out that these errors are positive for falling temperatures and negative for inversions. For isothermal layers, there is no correction. In the stratosphere where temperature variation is low, correction due to lag is negligible.

5. Conclusion

The lag coefficient of thermometal element used in chronometric radiosonde is of the order of 8 seconds for an ascensional rate of 18-20 km hr^{-1}. The computed errors caused by this amount of lag is of the order of 0·4°C at 700 mb and 0·5°C at 200-mb level.

6. Acknowledgement

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