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Atmospheric Turbulence and Diffusion—Part II

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(Continued from page 217 of Vol. 5 No. 3)

5. Recent developments in theories of turbulent motion : study of isotropic turbulence

The classical treatment of turbulent motion in a boundary layer was dealt with in Section 2. The current view of turbulent motion is that it consists of the superposition of a large number of component motions which differ from each other in their length scale and the observed properties of the turbulence are to be regarded as the statistical result of such interaction. This conception has led to the resolution of the velocity field into a series of Fourier components of the general form (Batchelor 1952).

$$V(x) = \int A(x) e^{i\kappa \cdot x} dx \quad (55)$$

where $V(x)$ is the (vector) velocity at (vector) position x , $A(x)$ the vector Fourier coefficient, κ the (vector) wave-number ($\kappa = 2\pi/\text{wave-length}$), x the wave-number space defined by $\kappa = |x|$, and the integration taken over all x -space.

$A(x) e^{i\kappa \cdot x} dx$ is the contribution to the velocity from the volume element dx of the wave-number space and

represents a sinusoidal velocity distribution with wave-length $2\pi/\kappa$, which is thus a measure of the scale of the component motion.

The equation of continuity, $\nabla \cdot V = 0$, requires

$$x \cdot A(x) = 0 \quad (56)$$

so that the component motions are sinusoidal waves with velocity parallel to $A(x)$ and spatial variations of velocity in the direction of x .

The square of the Fourier coefficient $A(x)$ being proportional to the kinetic energy of the Fourier component with wave-number x is a measure of the contribution of the component to the total kinetic energy of the turbulence. The Fourier resolution of the instantaneous velocity field thus provides a spectral distribution of kinetic energy of Fourier components of different length scale.

The velocity distribution represented by the Navier-Stokes equation, Eq. (7), may now be examined in terms of Fourier resolution of the velocity field. In vector notation, the three dimensional Navier-Stokes equation,

neglecting the external force, takes the form

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = - \frac{1}{\rho} \nabla p - \nu \nabla^2 \mathbf{V} \quad (57)$$

and the equation of continuity

$$\nabla \cdot \mathbf{V} = 0 \quad (58)$$

By substitution of the Fourier expression for the velocity in Eq. (57) and, eliminating the pressure term with the aid of the Eq. (56), we obtain, for the rate of change of the Fourier coefficient, the expression

$$\frac{\partial A(\alpha)}{\partial t} = i \int \alpha' \cdot A^*(\alpha' - \alpha) \left[\alpha \cdot A(\alpha') \frac{\alpha}{\alpha^2} - A(\alpha') \right] d\alpha' - \nu \kappa^2 A(\alpha) \quad (59)$$

where * denotes a complex conjugate.

The term with viscosity in Eq. (59) represents a linear damping action and shows that the viscous dissipation is more pronounced at high wave-numbers than at low wave-numbers. The term arising from pressure and inertia forces, however, is nonlinear and shows that there is a continual interaction, or modulation between the different Fourier components. The variation of the energy of the typical Fourier component of wave-number α is then given by

$$\frac{\partial A(\alpha) \cdot A^*(\alpha)}{\partial t} = i \int [-\alpha' \cdot A^*(\alpha' - \alpha) A^*(\alpha) \cdot A(\alpha') + \alpha' \cdot A(\alpha' - \alpha) A(\alpha) \cdot A^*(\alpha')] d\alpha' - 2\nu \kappa^2 A(\alpha) \cdot A^*(\alpha) \quad (60)$$

Eq. (60) shows that the interaction leads, in general, to a statistical flow of energy between any two components of the motion. It is this inertial transfer of energy between the different components of the motion that has been the main obstacle to the solution of the turbulence problem. A solution of Eq. (59) has not yet been found any more than is a solution of Eq. (7) and theories of turbulent motion consist essentially of physical hypotheses which permit a quantitative consideration of the energy transfer processes in the larger wave-number region of the energy spectrum. The basic theories on which these hypotheses have been built up are the theory

of universal similarity and isotropy of motion which will be considered in the following paragraphs.

When a laminar flow breaks into turbulence at high Reynolds numbers as a result of the action of a boundary of linear dimension L , the Fourier components that are directly excited by the boundary will have wave-numbers of the order of magnitude of $1/L$. These components, in their turn, will transfer the energy by inertia to other components of larger wave-numbers until at wave-number sufficiently large the damping action of viscosity counteracts the action of inertia and further transfer of energy to larger wave-numbers is made impossible. In the statistical process of inertial transfer of energy to larger wave-numbers the direct influence of the boundary is, however, gradually lost and a stage in the wave-number scale is reached beyond which the boundary influence practically disappears. The anisotropic properties of the components at smaller wave-numbers imparted by the boundary disappear at this stage. There is also complete equipartition of energy among the different components of the motion, and the correlation between them disappears from this stage onward. Thus we conclude that at large wave-number side of the turbulent spectrum a region exists in which the disappearance of the influence of the boundary which produces the turbulence brings about a state of isotropy and similarity among the components of the turbulent motion and it is to this region that the physical hypotheses of turbulent motion will apply.

Taylor (1935) defined a microscale of turbulence, λ ,

$$\frac{1}{\lambda^2} = L \int_0^{\infty} \frac{1 - \gamma_x}{x^2} dx \quad (61)$$

where γ_x is the Eulerian correlation coefficient of two particles separated by a distance x , to give a measure of the small-scale eddies which are responsible for the viscous dissipation of energy in a field of isotropic turbulence (isotropy defined by $\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$; and $\overline{u'v'} = \overline{v'w'} = \text{etc} = 0$).

The expression for λ deduced by him from dimensional considerations was

$$\lambda = A \left(L\nu/\sqrt{E'} \right)^{\frac{1}{2}} \quad (62)$$

where L is a linear dimension of the turbulence-producing device, *i.e.*, mesh-size of grid, or, in uniform flow of the largest eddy, E' is the turbulent energy per unit mass, and A is a numerical constant of order of unity. This formula was also found to be consistent with the decay of turbulence in a wind tunnel. It will be observed that as E' increases λ decreases, *i.e.*, smaller and smaller eddies can be maintained against viscous dissipation when larger turbulent energy is available to feed them. Typical values of λ in a wind tunnel are a few mm; in the atmosphere near the ground, Taylor (1952) finds values of λ from 2 cm at a height 2 m to 13 cm at 30 m and we may expect to find much larger values in the free atmosphere where L probably increases without corresponding increase in E' .

6. Kolmogoroff's similarity hypotheses

The most striking advance in the statistical theory of turbulence since 1941 has been due to Kolmogoroff (1941) and Obukhoff (1941). Later versions of the same ideas with further developments have been put forward independently by Onsagar (1945), Weizsäcker (1948) and Heisenberg (1948).

Kolmogoroff's basic idea relates to the similarity or isotropic range of wave-numbers in a field of turbulent motion. The Fourier components in this range owe their excitation to the inertial transfer of energy from the small wave-number components of the turbulence, and the intensity of the excitation clearly depends on the rate at which this transfer of energy proceeds. In view of the

fact that all the viscous dissipation occurs in the similarity range, the rate of transfer of energy from the small wave-number components of the motion to the components in the similarity range must equal the rate of dissipation of energy (ϵ , say). The details of dissipation process will also depend on the value of ν .

When the Reynolds number of the motion is increased a step higher, the similarity range to which the above ideas apply widens and larger and larger wave-numbers come within the range of the similarity concept. When the Reynolds number is so large that the Fourier coefficients at one end of the similarity range are statistically independent of those at the other end, the Fourier coefficients at the smaller wave-number within the similarity range will no longer be influenced by viscous dissipation and will not depend on ν . The statistical flow of energy into any part of this inertial subrange from smaller wave-numbers is then balanced by an equal outflow to higher wave-numbers and the motion associated with this subrange is ununiquely determined statistically by the single parameter, ϵ .

The above considerations lead us directly to Kolmogoroff's two similarity hypotheses:

(1) At high Reynolds numbers, the average properties of the components of wave-numbers in the similarity range of turbulence are determined by ϵ and ν only;

(2) Within the similarity range described by (1) there is an inertial subrange on the smaller wave-number side whose average properties are determined by ϵ only.

The similarity hypotheses enable predictions to be made in respect of certain statistical quantities. For instance, the spectral

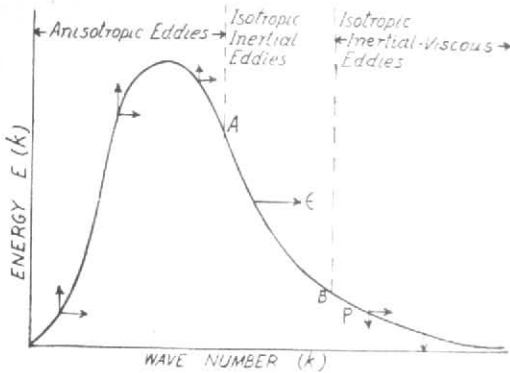


Fig. 4. Diagrammatic representation of energy transfer processes in a steady turbulence spectrum in which similarity in eddy structure is assumed to exist among eddies with wave-numbers larger than those represented by the point A and viscous dissipation is appreciable only in eddies with wave-numbers larger than B (After Priestley and Sheppard 1952)

- ↑ energy input from mean motion (total = ϵ)
- energy transfer from eddies of smaller to those of larger wave-numbers
- ↓ energy dissipated by viscosity (total = ϵ)

density of kinetic energy, $E(k)$ say, at wave-numbers within the similarity range may be expressed by dimensional arguments in terms of the relevant parameters ϵ and ν in the form

$$E(k) = \epsilon^{\frac{1}{3}} \nu^{\frac{5}{3}} F(k\nu^{\frac{2}{3}}/\epsilon^{\frac{1}{3}}) \tag{63}$$

where F is a function of universal (and undetermined) form. The form (63) applies for values of $k \gg 1/L$. Now if η_0 be the length-scale of the smallest existing eddy in the similarity range, dimensional considerations enable us to write

$$\eta_0 = (\nu^3/\epsilon)^{\frac{1}{4}} \tag{64}$$

so that relation (63) becomes

$$E(k) = \epsilon^{\frac{1}{3}} \nu^{\frac{5}{3}} F(k\eta_0) \tag{65}$$

provided $k \gg 1/L$ and provided the Reynolds

number is such that the condition

$$L/\eta_0 = L \epsilon^{\frac{1}{3}} \nu^{-\frac{3}{4}} \gg 1 \tag{66}$$

is satisfied.

The prediction which is possible to make about $E(k)$ when the Reynolds number is still higher is even more specific. Provided the Reynolds number is such that the wave numbers satisfying the relation, $1/L \ll k \ll 1/\eta_0$, can be found, then for these wave-numbers the expression (63) for $E(k)$ must be independent of ν . The consideration gives for the inertial subrange the relation,

$$E(k) = \alpha \epsilon^{\frac{2}{3}} k^{-\frac{5}{3}} \tag{67}$$

where α is an absolute constant.

The spectrum of turbulence with the process of energy transfer at different parts of the spectrum as visualised in the theory is shown in Fig. 4.

7. Experimental tests of the similarity hypotheses—application to the atmosphere

In spite of difficulties of attaining large wave numbers ($k \gg 1/L$) in a wind tunnel without unduly increasing the size of the grid dimensions and the stream velocity, sufficient experimental evidence has been accumulated which show the essential correctness of the predictions made from the similarity theory. The prediction that the motion associated with the similarity range of wave-numbers is always *statistically isotropic* has been tested by many workers (Townsend 1948 ; Corrsin 1949 ; Laufer 1950). Laufer studied the turbulent flow produced by a pressure gradient in the space between two parallel planes, and by making experimental Fourier analyses of the variations of two components of the velocity recorded at a fixed point in the channel, was able to compare the values of $\overline{B_1(k) B_1^*(k)}$ and $\overline{B_1(k) B_2^*(k)}$ for various values of k . Here $B(k)$ is the Fourier coefficient concerned and the suffixes 1 and 2

denote components parallel to the mean flow and perpendicular to the planes respectively. When the Fourier coefficient $B(\kappa)$ becomes statistically isotropic there is no correlation between $B_1(\kappa)$ and $B_2(\kappa)$ and experimental values show that there is in fact a trend towards this state as κ increased.

Stewart and Townsend (1951) has tested Eq. (65) by measuring the one-dimensional spectrum $\phi(\kappa)$ (obtained by Fourier analysis of the temporal variation of velocity at a fixed point) in grid turbulence. The results for various positions downstream from the grid and for various Reynolds numbers show that the curves of $\phi(\kappa)/\epsilon^{\frac{1}{2}}\nu^{\frac{5}{2}}$ corresponding to different Reynolds numbers come towards coincidence as κ increases, as they should according to Eq. (65).

A third test is provided by measurements of dimensionless ratios of quantities which are determined by the similarity range of wave-numbers. One such ratio is

$$\left(\frac{\partial v_1}{\partial x_1}\right)^2 \left(\frac{\partial^3 v_1}{\partial x_1^3}\right)^2 / \left[\left(\frac{\partial^2 v_1}{\partial x_1^2}\right)^2\right]^2 \quad (68)$$

where v_1 and x_1 are parallel velocity and position components.

Being dimensionless and uniquely determined by ϵ and ν in the similarity range, the value of the ratio must be a universal constant. Measurements of the ratio in isotropic turbulence and in turbulence behind a cylinder by Batchelor and Townsend (1949) have shown that there is a definite trend for this ratio to attain a constant value of 4.5 at large Reynolds numbers which will satisfy the condition embodied in relation (66). Stewart and Townsend (1951) have recently shown that for the Reynolds numbers attainable by them in a wind tunnel (C.1000) similarity obtains when the wave number $\kappa > 0.6(\epsilon/\nu^3)^{\frac{1}{2}}$, i.e., well within the viscous dissipation range. They also infer that an inertial

sub-range ($\kappa < 0.1(\epsilon/\nu^3)^{\frac{1}{2}}$) will exist only when the Reynolds number for the viscosity controlled eddies, $\lambda E^{\frac{1}{2}}/\nu > 1000$. This condition is likely to be satisfied in most cases of atmospheric turbulence so that both the Kolmogoroff hypotheses may be inferred to have meteorological application.

Batchelor (1950) has discussed those consequences of similarity hypotheses which may have possible application to problems of atmospheric diffusion. In the atmosphere the range of eddy sizes that can exist is very wide and Batchelor concludes that as the size of the viscosity-controlled eddy given by $(\nu^3/\epsilon)^{\frac{1}{2}}$ is very small, the inertial subrange of eddy-sizes must be very large in the atmosphere. Using a value of $5 \text{ cm}^2 \text{ sec}^{-3}$ for ϵ calculated by Brunt (1926) for the lowest 10 km of the atmosphere and taking $\nu = 0.15$ in the lower layers of the atmosphere the value of $(\nu^3/\epsilon)^{\frac{1}{2}}$ works out to be of the order of 10^{-1} cm . Thus the range of eddy-sizes over which the second of Kolmogoroff's hypotheses is applicable is very large, indeed. Batchelor (loc. cit) has, however, shown that the similarity hypotheses are inapplicable to diffusion from fixed sources. It is only the process of diffusion of a discrete cloud which move with the fluid to which the similarity hypotheses would apply because the spread here is dependent on the velocity differences. He deduces expression for the rate of change of mean-square separation of particles, the problem which Richardson had earlier dealt with but now with the proviso that the initial separation and time of diffusion were not too large to make the similarity hypotheses inapplicable. Suppose a large number of pairs of particles with initial separation l_0 to be released, then the mean square separation of the pairs \bar{l}^2 at time t after release, always in the inertial

subrange, is given by dimensional consideration in terms of l_0 , ε , and l by the relation

$$\frac{\bar{l}^2 - l_0^2}{l_0^2} = 4/3 c_1 \left(\frac{l \varepsilon^{1/3}}{l_0^{3/2}} \right)^2 \dots l \ll \frac{l_0^3}{\varepsilon^{1/3}} \text{ or} \\ \bar{l}^2 - l_0^2 \ll l_0^2 \quad (69)$$

and

$$\frac{\bar{l}^2 - l_0^2}{l_0^2} = c_2 \left(\frac{l \varepsilon^{1/3}}{l_0^{3/2}} \right)^3 \dots l \gg \frac{l_0^3}{\varepsilon^{1/3}} \text{ or} \\ \bar{l}^2 - l_0^2 \gg l_0^2 \quad (70)$$

where c_1 and c_2 are constants of order unity. Charnock (1951) has tested Eq. (69) by examining the relative scatter of two smoke puffs in the free air reported by Durst (1948) for l_0 between 40 m and 800 m and l between 10 sec and 50 sec. The results were not conclusive but there was some evidence in favour of the validity of Eq. (69) in winds of less than 5 m/sec for which ε was between 0.2 and 0.9 ergs $g^{-1} \text{ sec}^{-1}$. The minimum value of $l_0^3/\varepsilon^{1/3}$ in Charnock's analysis was 800 sec, *i.e.*, well above the values of l used in the analysis.

Eq. (70), for large l , may be written in the differential form

$$\frac{d\bar{l}^2}{dt} = 3(c_2 \varepsilon)^{1/3} \bar{l}^{2/3} \quad (71)$$

If this be treated as the Fickian equation for the spread of a set of particles about their mean position then the corresponding coefficient of diffusion κ ($= \frac{1}{2} \frac{d}{dt} \bar{l}^2$) is

$$\kappa \propto \varepsilon^{1/3} \bar{l}^{2/3} \quad (72)$$

where l is the scale of the diffusion process. This 4/3-power relation is the same as that derived recently by Heisenberg by an entirely independent method and found by Richardson over a very wide range of l , much wider in fact than would be expected from the

similarity hypotheses and our present ideas about the isotropy of atmospheric turbulence. Moreover, the values of the numerical coefficients with $\varepsilon \approx 1 \text{ erg } g^{-1} \text{ sec}^{-1}$, are about the same in the two cases.

8. Weizsäcker—Heisenberg theory of turbulent energy transfer

Weizsäcker (1948) has advanced a workable hypothesis for the processes of transfer of energy from one part of the energy spectrum to another. He considers the energy, ε_p , lost by the Fourier components of wave-numbers of magnitude smaller than κ_p (see Fig. 4). Of this energy, a part is dissipated by molecular viscosity whereas the remainder is transferred to the Fourier components of wave-numbers larger than κ_p . Weizsäcker's main idea is that the energy that is handed over by inertia across κ_p to larger wave-numbers may be taken to have been completely lost by viscous dissipation in the wave-number range, κ_p to ∞ . The action of these components with wave-numbers larger than κ_p may, therefore, be taken into account by introducing the concept of a coefficient of eddy transfer or eddy viscosity, N_p , to which Weizsäcker gave the form

$$N_p = c \int_{\kappa_p}^{\infty} [E(\kappa'')/\kappa''^3]^{1/2} d\kappa'' \quad (73)$$

on dimensional grounds, where c is a constant, and $E(\kappa)$ is the spectral density (with respect to wave-number magnitude) of energy.

On Weizsäcker's hypothesis, the equation for the variation of the spectral density of energy may be written

$$B(\kappa_p) - \frac{\partial}{\partial t} \int_0^{\kappa_p} E(\kappa') d\kappa' = 2(N_p + \nu) \\ \int_0^{\kappa_p} \kappa'^2 E(\kappa') d\kappa' \quad (74)$$

where $B(\kappa_p)$ is the rate of supply of energy to Fourier components with wave-numbers magnitudes smaller than κ_p from the boundaries or any external source.

Or, substituting the value of N_p from (73), the equation (74) becomes

$$B(\kappa_p) - \frac{\partial}{\partial t} \int_0^{\kappa_p} E(\kappa') d\kappa' \\ = 2 \left\{ \nu + c \int_{\kappa_p}^{\infty} [E(\kappa'')/\kappa''^3] d\kappa'' \right\} \times \\ \int_0^{\kappa_p} \kappa'^2 E(\kappa') d\kappa' \quad (75)$$

Eq. (75) now contains E alone as a function of κ and t , and can be solved if $B(\kappa_p)$ is known. For values of κ_p so large that

$$\int_0^{\kappa_p} E(\kappa') d\kappa'$$

approximates to the total energy, and $B(\kappa_p)$ approximates to the total rate of supply of energy, we can write the left-hand side of Eq. (75) as the rate of dissipation ε and the corresponding solution is

$$E(\kappa) = \left(\frac{8\varepsilon}{9c} \right)^{\frac{2}{3}} \kappa^{-\frac{5}{3}} \left[1 + \frac{8\nu^3}{3c^2\varepsilon} \kappa^4 \right]^{-\frac{1}{3}} \quad (76)$$

A result of great interest is derived from Eq. (73) if the expression for $E(\kappa)$ for the inertial subrange, *viz.*, Eq. (67), is substituted in this equation. The result of this substitution gives the relation

$$N_p \propto \varepsilon^{\frac{1}{3}} \kappa_p^{-\frac{2}{3}} \quad (77)$$

which may now be compared with the relation (51) found by Richardson, and also with Eq. (72) assumed to hold good for the similarity range. Further, the integral in Eq. (73) may be called the mixing-length,

no longer a constant in a given position but increasing with increase in scale of the transfer processes concerned.

9. Conclusion

In concluding this brief survey of the development of the subject of turbulence from early times to the present day, we have to admit in all humility the fact that notwithstanding considerable progress in our study of the true nature of turbulent motion and its effect on physical phenomena, perhaps, along correct lines, much remains to be done. We have yet to find the detailed structure of turbulence in the atmosphere where the range of length-scale is a very wide one. From the size of a big vortex in the general circulation of the atmosphere, *viz.*, a cyclone or a Berard cell, to a size which will extend far into the similarity range of turbulence we have an immensely wide length-scale. To date we have devoted attention to the study of the structure of turbulence at the two extremes of the spectrum only; vast regions in between lie unexplored. Of the intermediate scales of turbulence we have as yet little or no knowledge. Believing as we do in a spectrum of turbulence and the presence of a vast range of intermediate sizes of eddies, we have to admit that we have yet to go a long way before we can say that all that is to be known about turbulent motion has been known. There is no doubt that the present interest in the subject gives great hope for the future and we may hope to see further progress along right lines not only behind the closed walls of the laboratory but also in the realm of the actual atmosphere where nature unfurls the whole range of the turbulence spectrum as a challenge to man's power of perception and understanding.

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Hydrometeorology of Mayurakshi Catchment

by

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(pp. 99, diagrams 3)

The above mentioned memoir has recently been issued and is priced at Rs. 3-14-0 or 6 sh. 3d. Copies are available for sale with the Manager of Publications, Civil Lines, Delhi.

In view of the large river valley projects undertaken by the Government of India, the need for detailed hydrometeorological studies in the various river catchments has assumed considerable importance. The authors have recently carried out hydrometeorological studies of the Mahanadi (1951), Koyana (1952) and Damodar (1952) catchments. In this paper a similar detailed study of the hydrometeorology of the Mayurakshi Catchment has been carried out. The catchment is just above that of the Damodar and has an area of about 720 sq. miles up to the Massenjer Dam site.

The main results of the analysis which are based on data for the period 1891 to 1950 are given below:—

The mean rainfall of the catchment for June to October is 46" and for the year 52". On an average, about 87 per cent of the annual rainfall of the catchment occurs during the period June to October. The variability of rainfall for the monsoon period as a whole is 19 per cent. The distribution of June to October rainfall is normal. There has been no significant change in the June to October rainfall of catchment since 1891 and there is no trend or periodicity in the rainfall. Frequency distributions of daily rainfall and of two to seven consecutive days have been prepared. The highest recorded rainfall is 8" in one day, 12" in 3 days and 15" in 5 days. The maximum rainfall which may occur in the catchment is 11.5" in one day, 19" in three days and 22" in five days.

Daily, monthly, seasonal and annual rainfall in the Damodar and Mayurakshi catchments are significantly and highly correlated. It is also found that the monthly value of discharge of the river at Massenjer is significantly correlated with the rainfall in the catchment.