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Contribution of the humidity term in the formula for reduction of pressure to sea level

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(*Received 13 August 1956*)

List of symbols

p_s = Station pressure in mb

p_0 = Pressure reduced to sea level in mb

T = Station temperature argument in °K

H_p = Station elevation above sea level in geopotential metres

a = Assumed lapse rate in the air column extending from sea level to the level of the station in °C per gpm

e_s = Surface vapour pressure in mb

C_h = A function of H_p (expressed in °C/mb)

T_m = Mean temperature of the air column in °K

T_v = Virtual temperature in °K

T_{mv} = Integral mean virtual temperature of the air column in °K

Z = Geometric altitude in metres above mean sea level

Z_p = Elevation of the station in metres above sea level

$g_{\phi,0}$ = Acceleration of gravity at sea level at latitude ϕ in m/sec^2 on the 'Meteorological Gravity System'

R = Gas constant for dry air

1. Introduction

Towards the end of 1954, World Meteorological Organisation brought out a Technical Note on the Reduction of Atmospheric Pressure (Preliminary Report on problems involved—*Tech Note*, 7, WMO, No. 36, TP 12). This consists of two parts. Part I gives a summary of the methods used by Meteorological Services for reducing pressure to mean sea level. Part II is the report prepared for the C.I.M.O. Working Committee II on 'Reduction of Pressure' by the Chairman. According to this 'A rather chaotic situation exists regarding methods used for pressure reduction'. Of the eight points mentioned in the summary review of practices of Members regarding pressure reduction, the last but one of them relates to the correction for humidity. It has been indicated by an example that there is no more justification for neglecting the humidity correction than for neglecting the lapse rate correction in those cases where the dew point is moderate or high. In taking the mean integral virtual temperature of the air column, T_{mv}, the note introduces a new form which appears to be used by the U.S.A. Weather Bureau and Canadian Meteorological Service. The expression suggested for mean virtual temperature T_{mv} is

$$T_{mv} = T_s + \frac{a\,H_p}{2} + e_s\,C_h$$

The last term $e_s\, C_h$ on the right hand side denotes the contribution due to humidity. It is also mentioned in the second part of the Technical Note that this form is an excellent approximation for T_{mv}. This approximation, however, is based on two principal assumptions. The first is the constancy of the lapse rate and the second is that the variation of vapour pressure with height is in accord with Hann's well-known formula. The purpose of the present study is to examine the order of humidity correction due to variation of vapour pressure in space and time and to see to what extent the new form suggested may be regarded as an improvement over existing practice. The suggestion that surface vapour pressure e_s may be represented by surface temperature T_s by means of e_s, T_s curves is also considered.

2. Expression for humidity correction

Starting from the well-known equations

$$dp = -g_\phi \rho\, dZ \text{ and } p = R\rho T_v$$

$$\int_{p_s}^{p_0} \frac{dp}{p} = \int_0^{Z_p} \frac{g_\phi \rho\, dZ}{RT_v} \qquad (1)$$

Using the relation between geometric and geopotential units (u), $\frac{1}{9\cdot 8}\, g_\phi dZ = du$ (*vide* Resolution 164 of the Conference of Directors, Washington, 1947) and assuming the mean virtual temperature of the air column as T_{mv}, the general hypsometric equation takes the form

$$\log_e \frac{p_0}{p_s} = \left(\frac{9\cdot 8}{R}\right) \frac{H_p}{T_{mv}}$$

$$= 0\cdot 03414\, \frac{H_p}{T_{mv}} \qquad (2a)$$

$$\text{or} \quad \log_{10} \frac{p_0}{p_s} = (0\cdot 0148275)\, \frac{H_p}{T_{mv}} \qquad (2b)$$

From the expression for virtual temperature

$$T_v = T\left(1 - 0\cdot 378\, \frac{e}{p}\right)^{-1},$$

the mean virtual temperature of the air column

$$T_{mv} = T_m\left(1 - 0\cdot 378\, \frac{e_m}{p_m}\right)^{-1} \qquad (3)$$

where T_m is the mean temperature of the air column and is equal to $T_s + \frac{aH_p}{2}$ (T_s is the surface temperature, a is the assumed uniform lapse rate of temperature). e_m and p_m are the mean vapour pressure and pressure of the air column respectively. With these, the general hypsometric equation may be written as follows—

$$\log_e \frac{p_0}{p_s} = (0\cdot 03414)\, \frac{H_p}{T_m}\left(1 - 0\cdot 378\, \frac{e_m}{p_m}\right)$$

$$= 0\cdot 03414\, \frac{H_p}{T_s + \frac{aH_p}{2}}\left(1 - 0\cdot 378\, \frac{e_m}{p_m}\right)$$

$$\text{or} \quad \log_{10} \frac{p_0}{p_s} = (0\cdot 0148275)\, \frac{H_p}{T_s + \frac{aH_p}{2}} \times \left(1 - 0\cdot 378\, \frac{e_m}{p_m}\right) \qquad (4)$$

This form of the hypsometric equation is the same as the formula given in International Meteorological Tables (1890) except that in calculating H_p now, the new expression for gravity will be utilised. The form of the hypsometric equation for reduction of pressure to sea level recommended in the Technical Note is

$$\log_{10} \frac{p_0}{p_s} = \frac{(0\cdot 0148275)\, H_p}{T_s + \frac{aH_p}{2} + e_s\, C_h}$$

$$= \frac{(0\cdot 0148275)\, H_p}{T_{mv}} \qquad (5)$$

The chief difference between (4) and (5) is in the expression for T_{mv}. If (4) and (5) are the same, then

$$e_s\, C_h = T_m \left(1 - 0{\cdot}378\, \frac{e_m}{p_m}\right)^{-1} - T_m \qquad (6)$$

$$= T_m \left(0{\cdot}378\, \frac{e_m}{p_m} \right)$$

The second term in the expansion is of the order of 1/120th of the first; all terms after the first may, therefore, be neglected.

$$\therefore\ C_h = 0{\cdot}378\, \frac{e_m}{e_s} \cdot \frac{T_m}{p_m} \qquad (7)$$

Except that higher terms are neglected in deriving (7), form (5) may also be used as an approximate hypsometric formula for reduction of pressure.

Results from eq.(4) and (5) will practically be the same if C_h from equation (7) is evaluated by using values for e_m, T_m and p_m as at present. *It is mainly at this point that the Tech. Note has recommended a new procedure for the calculation of* C_h. C_h is a function of H_p and is expressed in °C per mb. The derivation is based on two assumptions. The first one is that the vertical distribution of vapour pressure in the fictitious atmosphere follows Hann's equation

$$e_Z = e_0\, 10^{-Z/6300} = e_0 \exp(-Z/\lambda) \qquad (8)$$

where $\lambda = 2736$, e_Z the vapour pressure at height Z (metres) and e_0 the vapour pressure at sea level. The second assumption is that pressure and temperature distribution are in accord with the standard atmosphere merely for the purpose of permitting a unique determination of the function C_h. Besides a statement of these assumptions, there is no indication of the exact values of T_m and p_m used and approximations made in obtaining the values of C_h given in Table I of the Note.

An attempt has been made in the following to calculate C_h using equation (7) and the above two assumptions.

e_m = Mean value of vapour pressure from 0 to Z

$$= \frac{1}{Z} \int_0^Z e_Z\, dZ$$

$$= \frac{e_0}{Z/\lambda} \left\{ 1 - \exp(-Z/\lambda) \right\}$$

Using the relation for e_Z for e_s (surface vapour pressure at height Z),

$$\frac{e_m}{e_s} = \frac{\exp(Z/\lambda) - 1}{Z/\lambda} \qquad (9)$$

$$\therefore\ C_h = \left(0{\cdot}378\, \frac{T_m}{p_m}\right) \cdot \frac{\exp(Z/\lambda) - 1}{Z/\lambda} \qquad (10)$$

$$\simeq \left(0{\cdot}378\, \frac{T_m}{p_m}\right)\left(1 + \frac{Z}{2\,\lambda} + \frac{Z^2}{6\,\lambda^2}\right) \qquad 10(a)$$

$$\simeq \left(0{\cdot}378\, \frac{T_m}{p_m}\right)\left(1 + \frac{Z}{2\,\lambda}\right) \qquad 10(b)$$

C_h has been calculated using values of p_m and T_m for the standard atmosphere given in the Manual of I.C.A.O. Standard Atmosphere. Table 1 gives C_h corresponding to equations 10, 10(a) and 10(b). Values of C_h given in the Tech. Note are also included for purposes of comparison.

The values of C_h now computed from equation (10) differ mainly in the third or fourth place of decimal from those in the Tech. Note. The maximum difference in the value of $e_s\, C_h$ between using columns (6) and (7) even at 1000 gpm and for $e_s = 30$ mb is only ·11 which is small enough to be neglected. Small variations in p_m and T_m do not also affect C_h.

3. Changes in pressures reduced to sea level due to variations in T_{mv}

If p_0 and p_{01} are the values of sea level pressure for the same value of surface

TABLE 1

Values of C_h

Height (gpm)	$\left(0\cdot378\,\frac{T_0}{p_0}\right)\frac{e_m}{e_s}$	$0\cdot378\,\frac{T_m}{p_m}$	Eq. 10(a)	Eq. 10(b)	Complete expression (Eq. 10)	Technical Note
0	·1075	·1075	·1075	·1075	·1075	·1074
100	·1095	·1080	·1100	·1100	·1100	·1097
200	·1115	·1085	·1125	·1126	·1126	·1120
300	·1136	·1091	·1150	·1153	·1153	·1144
400	·1158	·1096	·1176	·1180	·1180	·1168
500	·1180	·1101	·1202	·1208	·1208	·1192
600	·1202	·1106	·1228	·1237	·1237	·1218
700	·1225	·1112	·1254	·1266	·1267	·1244
800	·1249	·1117	·1281	·1297	·1293	·1270
900	·1298	·1128	·1334	·1359	·1362	·1298
1000	·1298	·1128	·1334	·1359	·1362	·1325

NOTE—(*i*) T_0, p_0 refer to values at sea level

(*ii*) Z (in eq. 10) is in metres. As the difference between geometric metre and geopotential metre does not numerically exceed 3 per 1000, whatever the latitude, it is neglected for purposes of computations.

pressure p_s but for T_{mv} and $T_{mv}+1$ then it follows from equation (2),

$$\log \frac{p_{01}}{p_0} = -0\cdot03414 \; \frac{H_p}{T_{mv}(T_{mv}+1)}$$

$$\text{or} \quad p_{01} - p_0 = -0\cdot03414 \; \frac{H_p}{T^2_{mv}} \cdot p_0 \qquad (11)$$

(neglecting higher powers of $1/T_{mv}$ and $(p_{01}-p_0)/p_0$

For $H_p = 1000$ gpm, $T_{mv} = 300°$A and $p_0 = 1000$ mb, the change in sea level pressure is $p_{01} - p_0 = -0\cdot4$ mb. Thus for an increase of 1°C in the mean virtual temperature T_{mv} due to increase of mean temperature or the contribution of humidity, the variation in sea level pressure for a station at an elevation of 1000 gpm is about $0\cdot4$ mb. Increase in mean temperature of the air column causes a decrease in sea level pressure. As pressure readings of stations below 800 gpm only will be reduced to sea level the decrease in sea level pressure will generally be less than $0\cdot3$ mb. For other elevations and temperature variations, $p_{01} - p_0$ will nearly be linearly proportional.

4. Changes in sea level pressure due to variations from the assumptions made in calculating C_h

C_h is the product of the two ratios $0\cdot378\; T_m/p_m$ and e_m/e_s. Values of these ratios are given in Table 1.

(*i*) If it is assumed that vapour pressure does not vary with height, then $C_h = 0\cdot378\; T_m/p_m$. From Table 1, col. (3),

$$(C_h)_{1000} - (C_h)_0 = 0\cdot0053$$

If for a station at an elevation of 1000 gpm. one uses the value of $(C_h)_0$ instead of $(C_h)_{1000}$, then the mean virtual temperature T_{mv} is decreased by $0\cdot16$°C for a surface vapour pressure value of 30 mb. This introduces a small change in reduced sea level pressure of about 0·06 mb. As we are generally interested in levels below 800 gpm, this error will be less than 0·05 mb. The error introduced in neglecting variations in T_m/p_m with height is, therefore, very small and negligible. It also shows that the contribution to C_h is mainly due to (e_m/e_s). C_h could, therefore, for all practical purposes be regarded as representing the contribution due to the variation of vapour pressure with height.

(*ii*) Combining the variation of both vapour pressure and (T_m/p_m) with height and using values of C_h in col. (6),

$$e_s \left\{ (C_h)_{1000} - (C_h)_0 \right\} = e_s \; (0\cdot0287)$$

$$= 0\cdot861, \quad \text{if } e_s = 30 \text{ mb}$$

If we, therefore, consider reduction of pressure readings of a station at 1000 gpm and use in equation (5) for C_h the value for $H_p = 0$ instead of for H_p=1000 gpm, the contribution to T_{mv} will be diminished by 0·861°C (for e_s = 30 mb). The reduced sea level pressure will, therefore, be increased by about 0·3 mb. The following table shows the changes in sea level pressure if the value of C_h for H_p = 0 is used instead of C_h for the actual height of the station.

e_s (mb) / H_p (gpm)	1000	800	600	500	300
30	·3	·25	·19	·16	·09

As pressure readings of only those stations whose heights are less than 800 gpm are reduced to sea level, the maximum error in neglecting the variation of vapour pressure due to height, temperature and pressure even on occasions when surface vapour pressure is 30 mb is about 0·2 mb. As most of our observatories are at elevations of less than 500 metres and high values of vapour pressure occur mostly during the monsoon and at or near coastal stations, the error in neglecting variations in C_h due to height will, therefore, be generally less than 0·1 mb.

The above analysis shows clearly that variations in sea level pressure due to the assumption of variation of vapour pressure with height in accord with Hann's formula and of temperature and pressure according to I.C.A.O. are less than 0·1 mb generally and, therefore, negligible. It may, therefore, be enough for all practical purposes to use for the mean value of vapour pressure e_m, the surface vapour pressure value itself.

Another point worth mentioning in this connection is that the error due to assuming a uniform lapse rate of temperature irrespective of day and season may frequently introduce differences of 1°C or more in mean temperature (T_m). This assumption alone would affect reduced pressure values very much more than neglecting variations in vapour pressure due to height.

5. Changes in reduced pressure for variations from an assumed value of e_m

It is usual in preparing barometer reduction tables to assume one fixed value for the mean vapour pressure of the air column (e_m). We will now indicate the order of difference introduced in reduced pressures due to variations from e_m. If the reduced pressures corresponding to e_s and e_{m1} of e_m are p_0 and p_{01} respectively, it can be shown from equation (4) that

$$p_{01} - p_0 = -\left(0\cdot03414 \; \frac{H_p}{T_m}\right) \times \left(0\cdot378 \; \frac{e_{m1} - e_s}{p_m}\right) p_0$$

Assuming p_m = 950 mb, T_m = 300° A, p_0 = 1000 mb and $e_{m1} - e_s$ = 15 mb, $p_{01} - p_0$ for different H_p are given in Table 2.

TABLE 2

H_p (gpm)	$e_{m_1}-e_s$ (mb)	$p_{01}-p_0$ (mb)
1000	15	—3·7
800	15	—0·5
500	15	—0·3
300	15	—0·2

Increase in vapour pressure from the assumed value of e_m means increase in T_{mv} and reduction of sea level pressure. This is further referred to in the subsequent Sections.

6. Variations in vapour pressure

In preparing tables for the reduction of pressure to sea level for a station, the India Meteorological Department employs the following method for finding out the mean values e_m and p_m of the air column. For p_m, mean pressure according to height ranges is used. For e_m, however, the mean annual surface vapour pressure value of a neighbouring station at nearly the same height is utilised. When a station is newly started we have no alternative but to employ this method for finding out e_m. The tables do not make provision for variations from the assumed values of e_m. This method of fixing the value of e_m has the advantage that the observer would not have to worry about the value of vapour pressure at the time of observation.

We will now study the likely extreme variations of vapour pressure from the values used in the preparation of barometer tables. Charts have been prepared showing the difference between vapour pressure values used in the preparation of tables for reduction of pressure and the highest and lowest monthly averages of surface vapour pressure for 0800 and 1700 IST. It may be mentioned that in India 50 per cent of our observatory stations are less than 600 ft (200 metres) in height, 75 per cent less than 1800 ft (500 metres) and 88 per cent less than 2600 ft (800 metres).

Figs. 1(a) and 1(b) give the highest monthly average of vapour pressure at 0800 and 1700 IST respectively *minus* value of vapour pressure used in the preparation of reduction tables.

Below latitude 20°N, the difference is less than 7 mb. Below 15°N the difference is less than 5 mb. Over most of the plateau area it is less than 10 mb excepting Central India and adjoining areas where the difference is 10–12 mb. Where the differences in vapour pressure are more than 12 mb, the elevation is generally less than 1500 ft (500 metres).

The charts refer to departures of means from the assumed value but as the highest values occur in the monsoon months when temperature and humidity conditions are nearly uniform all over the country, these may be taken as generally representative of daily conditions. The charts for both 0800 and 1700 IST are nearly similar.

Figs. 2(a) and 2(b) give the values of vapour pressure used in the preparation of tables *minus* lowest of the monthly averages of vapour pressure at 0800 and 1700 IST.

Except for areas in the west over Saurashtra and Cutch and parts of Rajasthan and over northeast India, the differences are less than 10 mb. Below 20°N differences are less than 7 mb and over the Deccan plateau area 3 to 6 mb.

The maximum differences on individual days may not exceed 10 mb except over the plain areas of Saurashtra and Cutch, Rajasthan and northeast India. The elevations of these areas are less than 500 metres. Hence such differences will not be significant from the point of view of sea level corrections.

The above analysis shows that excepting on a few occasions, mainly in the monsoon months, differences in sea level pressure due to daily variations in surface vapour pressure from the values used in the preparation of reduction tables would be small (·1 or ·2 mb) and negligible (*vide* Section 5).

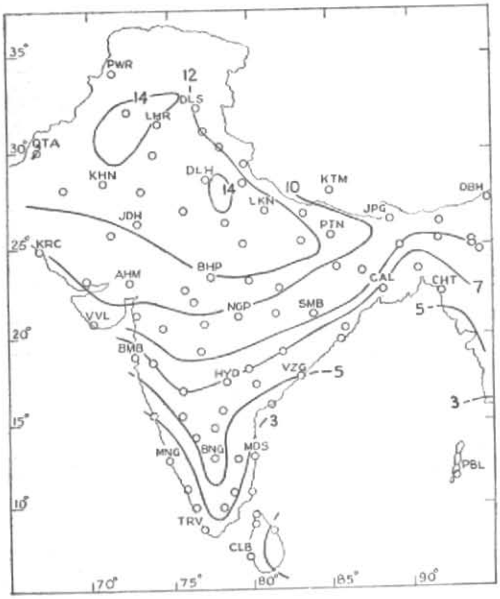

Fig. 1(a). Highest monthly average vapour pressure (in mb) at 0800 IST *minus* vapour pressure used in Barometer Reduction Tables (in mb)

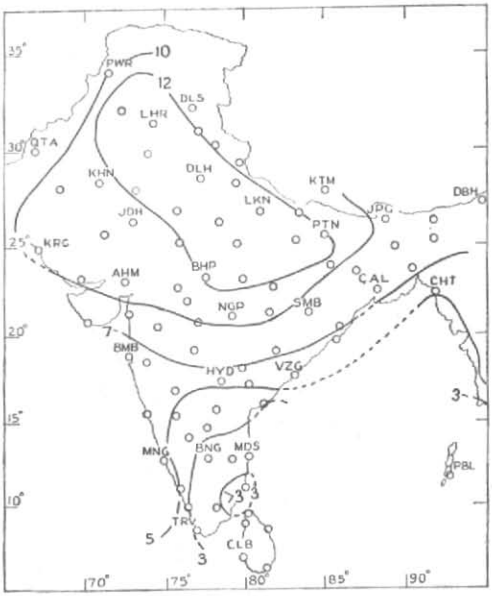

Fig. 1(b). Highest monthly average vapour pressure (in mb) at 1700 IST *minus* vapour pressure used in Barometer Reduction Tables (in mb)

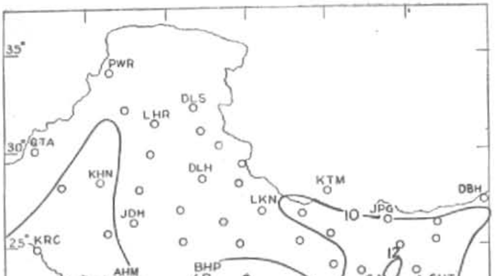

Fig. 2(a). Vapour pressure used in Barometer Reduction Tables *minus* lowest monthly average vapour pressure (in mb) at 0800 IST

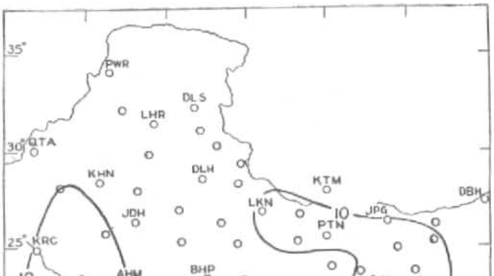

Fig. 2(b). Vapour pressure used in Barometer Reduction Tables *minus* lowest monthly average vapour pressure (in mb) at 1700 IST

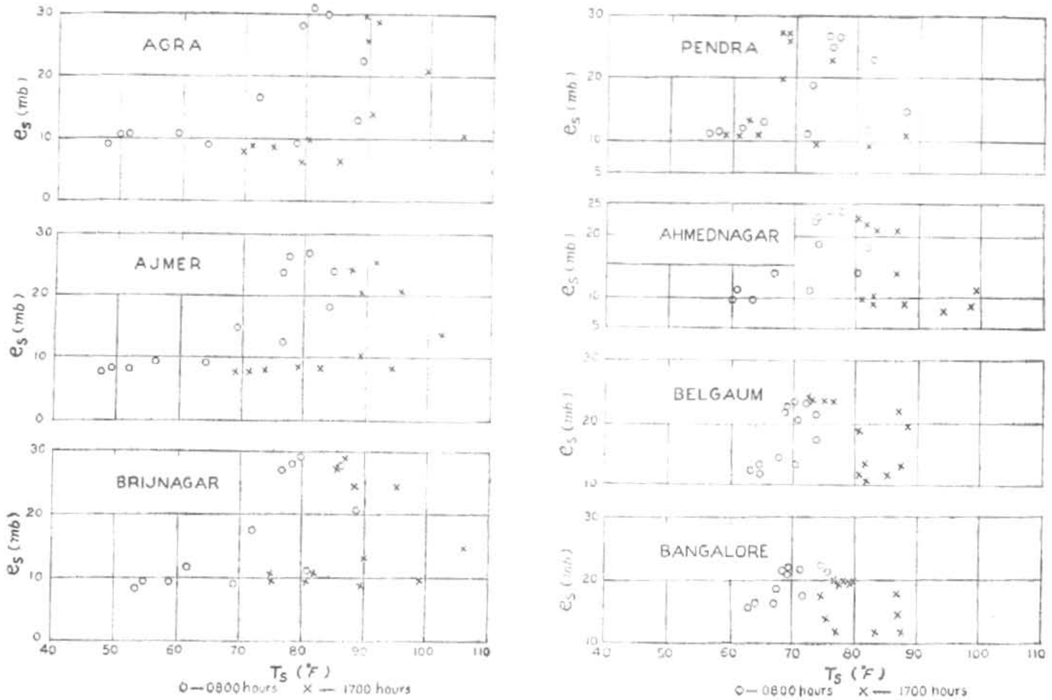

Fig. 3. Diagram of mean monthly values of T_s (°F) and e_s (mb) for 0800 and 1700 IST of selected stations

7. Feasibility of e_s, T_s curves

It has been suggested in the Technical Note (Sec. 2.11) that for practical purposes surface vapour pressure e_s may be treated as a function of T_s. Such a function may be secured by plotting monthly mean values of vapour pressure against monthly mean values of temperature at the station. A smooth curve is then to be constructed to obtain the best fit of the plotted points. This curve may be extended by using values from daily records. As suggested, attempts were made to draw curves of best fit for a few stations. A problem of no inconsiderable difficulty in our country will be that the monsoon values of e_s are very high at lower temperatures. The curve of best fit may be easily out by several millibars. The purpose in such a case of using a curve of best fit will not be served. The same curve will also not be valid for all hours of observations. e_s cannot, therefore, be regarded as a function of T_s. In view of these considerations e_s, T_s curves would not be suitable for use on a large scale in any country.

As has been shown earlier, differences in sea level pressure even due to extreme variations of vapour pressure are small and may be neglected for all practical purposes. If, however, one desires to use the observed values of vapour pressure, a small table showing corrections due to variations in e_s or dew point temperature T_d should meet the requirements. As dew point temperatures are to be calculated at each hour of observation, it will not be necessary to calculate values of e_s. A sample table is given in Appendix I.

Diagrams showing plots of monthly mean values of T_s, e_s for a few stations are given in Fig. 3.

8. Summary

(1) The ratio e_m/e_s assuming Hann's law of variation of vapour pressure is shown to be equal to $(\exp Z/\lambda - 1)/(Z/\lambda)$ and approximately $1 + Z/2\lambda + Z^2/6\lambda^2$. Variations in the ratio $0 \cdot 378\ T_m/p_m$ with height contribute very little to T_{mv} even for high values of surface vapour pressure (30 mb).

(2) An increase of 1°C in T_{mv} (mean virtual temperature) decreases pressure reduced to sea level by 0·3 mb for stations at elevations of 800 gpm and proportionately for other elevations.

(3) The variation of vapour pressure with height contributes less than 0·2 mb to the total correction even in the case of stations at 800 gpm and when surface vapour pressure is 30 mb. For most of our stations whose heights are less than 500 gpm and surface vapour pressure is of the order of 20 to 25 mb, the effect of the variation of surface pressure with height may be neglected for all practical purposes.

(4) The decrease in sea level pressure if the surface vapour pressure value differed by 15 mb from the value assumed for the preparation of reduction tables is 0·5 mb for a station at a height of 800 gpm and proportionately for other elevations.

(5) The differences between the values of vapour pressure used in preparing barometer reduction tables and their highest and lowest values may not generally differ by more than 5 to 10 mb. The correction, therefore, due to assuming the annual mean value in preparing reduction tables introduces generally a negligible error. A table of corrections for variations in vapour pressure from the value assumed in preparing barometer reduction tables may supplement the existing tables, if more accurate values are required.

(6) For Indian data, e_s cannot be regarded as a function of T_s. Drawing of e_s, T_s curves is, therefore, not feasible.

(7) As the main variation in reduced sea level pressure is due more to the actual values of e_s and most of our stations where very high values of e_s occur are at elevations of less than 500 gpm, the existing practice ensures satisfactory values and does not need a change. The proposed formula and particularly the method of drawing e_s, T_s curves do not appear to be an improvement over the existing methods.

REFERENCE

World Meteorological Organisation 1954 Reduction of Atmospheric Pressure (Preliminary Report on Problems involved), Tech. Note No. 7 (W.M.O. No. 36 TP. 12)

APPENDIX I

Table for reduction of pressure to sea level

Each station with elevation less than 800 gpm is provided with a table for reducing station level pressure (corrected for temperature and gravity) to sea level. The table is detailed and gives values to be added to surface pressure for ranges of pressure and temperature likely to be experienced at the station. In preparing this table, only one value of vapour pressure and uniform lapse rate are assumed. For stations at elevations above 230 metres the values to be added to surface pressure are given for pressure intervals of 2 mb and of temperature intervals of 2°F.

For taking account of variations in surface vapour pressure a small additional table to the main one may be supplied to observing stations. This table will give corrections to be applied for the difference between the actual observed vapour pressure and the assumed value in preparing the main table. This correction table may be in terms of dew point temperature (T_d) or in terms of vapour pressure.

Table giving corrections due to variations from the assumed values of e_s

(For this table assumed value of e_s is 17 mb, H_p = 558 gpm)

Observed values of T_d		Correction
(°C)	(°F)	(mb)
0	32·0	+0·2
2·5	36·5	+0·2
5·0	41·0	+0·2
7·5	45·5	+0·1
10·0	50·0	+0·1
12·5	54·5	+0·1
15·0	59·0	0
17·5	63·5	−0·1
20·0	68·0	−0·1
22·5	72·5	−0·2
25·0	77·0	−0·3

NOTE—Table in terms of vapour pressure will be similar