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Radiation climate of New Delhi
Part II : Longwave radiation and energy budget

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ABSTRACT. Monthly budgets of longwave, shortwave, and net radiation, separate for air-space and air-soil interfaces, and also for the atmospheric column, are presented for the years 1964 and 1965 at New Delhi. A combination of direct radiation measurements at Lodi Rd. Observatory, New Delhi, evaluations of satellite based scannings of the region, and climatonomical theories (as developed by H. Lettau), yields the monthly budgets of latent and sensible heat. Radiative shortwave heating and longwave cooling rates for the local atmospheric column are summarised for the seasons, resulting in a mean annual net cooling rate of 0·4°C in both years.

In this climate where the seasonal maximum of precipitation coincides closely with that of insolation, the moisture budget at the air-soil interface shows that about 80 per cent of the annual evaporation occurs from July to September in both years. Surface runoff appears to be restricted to the monsoon season and amounts to less than evapotranspiration even in the wettest month. Calculated values of exchangeable soil moisture were about twice as high in 1964 than in 1965 due to the intensive monsoon. The carry-over into the following dry season was considerable. Net moisture import (horizontal advection) into the atmospheric column over New Delhi was more than five times higher during the summer of 1964 than of 1965. The change-over from moisture import to moisture export occurs abruptly with the greatest drop during October in both years.

The calculation of a complete energy budget for the soil-air interface shows that due to the great demand of energy for evapotranspiration, monthly means of sensible heatflux, normally directed from the ground into the air during summer, can be reversed resulting in 'psychrometric cooling' of the ground. In 1964, such reversal occurred from July to September, but in 1965 it was weak and restricted to the month of September. Accordingly, the monthly mean surface temperature dropped from June to July by 10°C in 1964, but only by 6°C in 1965. Flux of sensible heat from and into the soil reaches extremes in December and April, respectively, but turned out to be small in comparison with other constituents of the monthly heat budget.

With the aid of the 'climatonomical' analysis method it is possible to establish quantitatively the constituent terms of the energy budget. For the atmospheric column, however, the energy supplied by advection cannot be separated from the contribution by subsidence. Considering the importance attributed to subsidence for maintaining arid conditions over northwestern India, a supplementary synoptic-aerological analysis of the atmospheric circulation over the region appears to be desirable in order to separate the effects of the two processes in the course of the year.

1. Introduction

In Part I of the paper (Lettau 1970) a budgetary method was introduced which permits us to appraise the various processes which attenuate insolation in the atmospheric column and explain climatic averages of global radiation, including diffuse sky radiation over a region. Within the framework of upper and prescribed lower boundary conditions, the total shortwave energy budget was determined quantitatively. Seasonal heating rates (in °C/day) were calculated for New Delhi and prorated into constituent contributions due to water vapour, other gases, aerosol, and cloudiness for the year 1959.

In the present paper cooling rates due to emitted longwave radiation and a complete energy budget for the atmospheric column over the New Delhi area is presented for the years 1964 and 1965. It would seem logical to employ in this paper also data of 1959, but the important information on radiation fluxes measured outside the atmosphere by orbiting satellites is not available before 1964. Considering the importance attributed to subsidence for maintaining aridity in parts of northwestern India, it seems desirable to present a method which permits us to estimate the combined effects not only of shortwave heating and longwave cooling, but also of conduction and condensation. Therefore, a complete shortwave radiation budget was calculated for 1964 and 1965 by the same methods discussed in earlier paper, based on measured climatic data as listed in Table 5 under 'Basic Data'. Under the heading 'Derived Data', shortwave attenuation parameters due to aerosol absorption (a_{ae}) and scattering (σ_{ae})

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TABLE 1

Radiation budgets (all terms in ly/day) for New Delhi region, 1964 and 1965 separate for air-space and air-soil interface

LW↑ from satellite data according to Vander Haar (1969) : SW↑ with top albedo a^{**} from Part I of 'Radiation Climate of New Delhi' (1970) ; effective radiation terms from Lodi Rd. Observatory, New Delhi

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
	Air-Space Interface : Shortwave (down) + Shortwave (up) + Longwave (up) = Top Net Radiation													
1964	SW↓	530	646	774	886	962	988	972	925	825	694	564	497	772
	SW↑	—149	—217	—189	—196	—274	—284	—288	—264	—177	—114	—149	—114	—201
	LW↑	—502	—512	—515	—526	—480	—438	—416	—432	—450	—405	—461	—458	—471
	Top Net	—121	—83	70	164	208	266	268	229	198	175	—46	—75	100
1965	SW↓	530	646	774	886	962	988	972	925	825	694	564	497	772
	SW↑	—117	—202	—170	—186	—269	—316	—273	—206	—167	—136	—134	—114	—190
	LW↑	—455	—451	—440	—444	—446	—454	—460	—468	—490	—504	—518	—475	—466
	Top Net	—42	7	164	256	247	218	239	251	168	54	—88	—92	107
	Air-Soil Interface : Effective SW + Effective LW = Ground Net Radiation													
1964	Eff. SW	254	291	355	396	423	381	333	367	381	400	314	245	345
	Eff. LW	—165	—168	—169	—172	—155	—107	—66	—67	—105	—172	—173	—155	—139
	Ground Net	89	123	186	224	268	274	267	300	276	228	141	90	206
1965	Eff. SW	257	304	371	409	426	387	357	405	380	354	291	244	349
	Eff. LW	—152	—155	—168	—160	—156	—121	—75	—84	—111	—135	—148	—154	—135
	Ground Net	105	149	203	249	270	266	282	321	269	219	143	90	214

TABLE 2

Seasonal heating and cooling rates (°C/day) in the atmospheric column over the New Delhi Region, 1964 and 1965

	D—J—F		M—A—M		J—J—A		S—O—N		Annual	
	1964	1965	1964	1965	1964	1965	1964	1965	1964	1965
Eff. SW	0·5	0·6	1·1	1·1	1·3	1·3	0·8	0·8	0·9	0·9
Eff. LW	—1·3	—1·2	—1·4	—1·2	—1·4	—1·5	—1·2	—1·5	—1·3	—1·3
Net Rad.	—0·8	—0·6	—0·3	—0·1	—0·1	—0·2	—0·4	—0·7	—0·4	—0·4
Sens. *plus* Latent heat	0·4	0·4	0·8	0·9	2·4	1·4	0·8	0·9	1·1	0·9
Advection *plus* Subsid.	0·4	0·2	—0·5	—0·7	—2·4	—1·2	—0·5	—0·3	—0·7	—0·5
Storing	0·0	—0·0	0·0	0·1	—0·1	0·0	—0·1	—0·1	0·0	0·0

are included for comparison with those of 1959 shown in Part I.

Numerical values of moisture and heat budget terms are the result of combining local observations with climatonomical considerations as formulated by Lettau (1969). It must be emphasized that the main purpose of this study is to present new methods for the determination of moisture and heat budget components rather than to compile representative climatic values, something that, obviously, cannot be done from only two years of observations. A similar study based on long-term climatic data from the desert area of northwestern India is planned as a contribution to the Rajasthan Desert Project, the joint program of the India Meteorological Service and Meteorology Department of the University of Wisconsin.

2. Symbols and Definitions

Observed climatic elements

T_s (°C)	air temperature (station data read at screen height)
e_s (mb)	vapour pressure
c	fraction of sky covered
G (ly/day)	global radiation
g (cm/sec)	gravity
D (ly/day)	diffuse radiation
p_0 (mb)	stations pressure
P (mm/mo)	precipitation
c_p (cal g^{-1} K^{-1})	specific heat of air

Derived Data (representative for region)

T_o (°C)	surface temperature
w (mm)	precipitable water in atomospheric column
m (mm)	exchangeable soil moisture
a^*	surface albedo of the region
a^{**}	albedo at top of atmosphere
A^*	Angstrom ratio at surface
A^{**}	Angstrom ratio at top of atmosphere
α_{ae}	efficiency of aerosol as SW absorber
σ_{ae}	efficiency of aerosol as SW scatterer

Parameters

L (cal/g H_2O)	heat of vaporization
e^*	evaporivity of the region
t^* (month)	residence time of soil moisture
P^* (mm)	threshold value of immediate runoff
n^*	ratio of immediate runoff to $(P-P^*)$

Budget Terms at Air-Soil Interface

E' (mm/mo)	immediate evapotranspiration
E'' (mm/mo)	delayed evapotranspiration
$E=E'+E''$	actual evapotranspiration
N' (mm/mo)	immediate runoff
N'' (mm/mo)	delayed runoff
$N=N'+N''$	actual runoff
$SW\updownarrow=(1-a^*)G$ $=F(t)$ (ly/day)	effective shortwave radiation or forcing function
$LW\uparrow=\epsilon\sigma T_o^4$ (ly/day)	emitted longwave radiation
$LW\updownarrow$ (ly/day)	effective longwave radiation
E_0 (ly/day)	flux density of latent heat
S_0 (ly/day)	soil heat flux into the ground
Q_0 (ly/day)	flux density of sensible heat into the air
$R_0=SW\updownarrow-LW\updownarrow$	net radiation
$C^*=\overline{N}/\overline{P}$	runoff ratio from annual means
$B^*=\overline{E}_0/\overline{Q}_0$	Bowen ratio from annual means
$D^*=\overline{R}_0/\overline{LP}_0$	Budyko (dryness) ratio from annual means

3. Longwave and Net Radiation Budgets

3.1. *Air-Space Interface*

Monthly values of shortwave, longwave, and net radiation at the top of the atmosphere are listed in Table 1. Extra-terrestrial irradiation is interpolated from tables by Bolsenga (1964) for 28·5°N latitude based on a solar constant of 1·98 ly/min. Reflected shortwave radiation is computed with albedo values a^{**} (*see* Table 5) derived by budgetary methods discussed in Part I. Outgoing longwave radiation is measured by satellites during orbits over NW India. Thomas Vander Haar of the Space Science and Engineering Center, University of Wisconsin, evaluated and kindly supplied the data which represent averages for an area of about 500 miles square centred at 28°N and 79°E. The resulting top net radiation shows a relatively weak heat loss from November to February and a strong gain in summer. The annual mean value is positive which is to be expected in sub-tropical latitudes. It is slightly larger during the drier year 1965 due to higher effective albedo than during the stronger monsoon year of 1964.

3.2. *Air-Soil Interface*

Monthly means of surface radiation fluxes (global, diffuse, and effective longwave, also counter radiation from the night sky), as measured at Lodi Rd. Observatory in New Delhi, were kindly provided by Miss Mani, Deputy Director General of Observatories in India. Effective shortwave radiation at ground level, or SW $\updownarrow = G\,(1-a^*)$ (*see* the preceding list of symbols for notations) is derived from measured global radiation and albedo values for the region (a^*) as listed in Table 5.

With the aid of effective longwave and counter radiation from the sky, measured at 0530 and 2030 IST, an effective Angstrom ratio can be determined empirically. Defined as $A^* =$ LW $\updownarrow$ /LW $\uparrow$, or $1-A^*=$LW $\downarrow$ /(LW $\updownarrow$ +LW $\downarrow$). A^* is assumed to be also representative for daytime. Consequently monthly means of effective LW radiation are calculated as $A^*\sigma T_{\circ}^{4}$, with black-body emission $\sigma T_{\circ}^{4}$ taken from Smithsonian Meteorological Tables (1963). Since A^* is obtained from actual measurements, the possibility that emissivity can depart from unity is assumed to be absorbed in the loca-value of A^*. Because the representative surface temperature T_0 is obtained from the complete energy budget (as shall be shown in Section 4·3), the tabulated values $A^*\sigma T_{\circ}^{4}$ differ slightly (within ± 5 per cent) from the effective outgoing radiation as observed at New Delhi.

Monthly means of effective SW, effective LW, and net radiation are listed for both years in Table 1. Net values are positive in each month and exhibit a pronounced annual trend with maximum in August and minimum in December or January. Compared to 1964, the January to April net values of 1965 are considerably higher due to increased SW $\updownarrow$ and decreased LW $\updownarrow$ radiation. (A flux is negative when directed upwards). The annual mean is also slightly higher during the drier year of 1965, at the surface as well as at the top of the atmosphere.

3.3. *Atmospheric column*

The differences between top and ground net radiation represent absorption within the atmospheric column over the region. Using the same simplified approach as in Part I, the columnar heat capacity of the entire atmosphere is taken as $c_p p_0/g$, or 247 ly/deg C (using a mean surface pressure of 1008 mb for the two years considered). Upon multiplication by $g/c_p p_o$, radiation absorption (in ly/day) is transformed into heating rates (in deg C/day). Summarizing monthly means to a seasonal means because of small changes from month-to-month radiative heating and cooling rates are listed in Table 2; also contributions due to latent *plus* sensible heat, advection *plus* subsidence, and storing, which are derived from values listed in Table 4. Whereas heating by insolation shows a pronounced annual trend, cooling by longwave emission remains fairly constant. Results of the two-years' analysis suggest an annual variation of net radiative cooling of about 1°C in winter and close to 0°C in summer, with an annual mean of 0·4°C. Godbole and Kelkar (1969) reported a longwave radiative cooling rate of—1·1 to—1·2°C for July 1962 in the surface to 300 mb layer over the New Delhi region. With the methods described above, July cooling rates of—1·4°C and —1·5°C were found for 1964 and 1965, respectively.

4. Energy Budget of a Region

According to H. Lettau's model of climatonomy, solar radiation which is absorbed by the ground represents the primary 'forcing function' of the thermal climate. This radiation supplies the energy for maintaining effective longwave radiation of the earth's surface, for the sensible heat fluxes into the air and ground, for latent heat fluxes as in evapotranspiration or in photosynthetic processes. Neglecting the latter, the balance equation for a terrestrial surface can be written as—

$$F = \epsilon\,\sigma\,T_0^4 - \text{LW}\downarrow + E_0 + S_0 + Q_0$$
$$= \text{LW}\updownarrow + E_0 + S_0 + Q_0 \qquad (1)$$

All terms of the right-hand side are directly or indirectly dependent on surface temperature T_0. This fact is fundamental to Lettau's concept of climatonomy and serves to answer the question : Given $F(t)$, what is $T_o(t)$?

The terms E_o, S_o and Q_o of equation (1) will be calculated either directly or by parameterization and climatonomical iteration. The latent heat term E_o will be computed by utilizing Lettau's model of 'Evapotranspiration Climatonomy' (1969) and regionally available hydrological data on precipitation and runoff. Sensible heat flux into the ground S_o will be determined by climatonomical methods and basic heat conduction theories, using T_s as a first order approach to T_o. With the SW and LW radiation terms known from observations, sensible heat flux into the air can then be obtained as the remainder of the balance equation (1). Its value together with an estimate of eddy diffusivity of the region permits us to assess the difference between conventionally measured air temperature and representative surface temperature at a climatic station. Consequently, all right-hand side terms of equation (1), except E_0, must be re-calculated employing T_0 instead of T_s. An improved

TABLE 3

Moisture Budget (all terms in mm/mo) for New Delhi area, in 1964 and 1965

Evapotranspiration, runoff, and soil storing from H. Lettau's 'Evapotranspiration Climatonomy' (1969) with $e^{*}=0 \cdot 70$ and surface runoff (immediate runoff) $N'=0 \cdot 53$ $(P-137)$ for $P>137$ only

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
1964		Soil-Air Interface : (Prec=Evap.+Runoff+Soil storing)												
	Precip.	0	1	1	5	16	28	538	446	181	0	0	16	103
	Evap.	6	5	4	6	16	23	237	244	159	30	20	22	64
	Runoff	2	1	1	1	0	1	216	173	32	7	5	4	37
	Storing	—8	—5	—4	—2	0	4	85	29	—10	—37	—25	—10	2
1965														
	Precip	9	9	2	14	6	3	168	185	197	0	0	0	49
	Evap.	16	14	8	17	9	5	117	145	144	18	12	8	43
	Runoff	3	2	2	1	1	1	18	29	37	5	3	2	8
	Storing	—10	—7	—8	—4	—4	—3	33	11	16	—23	—15	—10	—2
1964		Atmospheric column : (Evap.—Precip+Advection=Air Storing)												
	E-P	6	4	3	1	0	—5	—301	—202	—22	30	20	6	—39
	Advection	—7	—1	2	1	4	19	315	203	5	—47	—25	—9	39
	Storing	—1	3	5	2	4	14	14	1	—17	—17	—5	—3	0
1965														
	E-P	7	5	6	3	3	2	—51	—40	—53	18	12	8	—6
	Advection	—7	—4	—3	—1	1	14	61	35	43	—28	—19	—10	6
	Storing	0	1	3	2	4	16	10	—5	—10	—10	—7	—2	0

Q_0 will yield an improved T_0 and, in a true iteration the process will be continued until the Q_0 used to generate T_o agrees with the Q_o generated.

4.1. *Latent heat*

Evapotranspiration E includes all vertical fluxes of water vapour into the air from vegetation, bare ground, and bodies of water (including puddles after rainfall). Because $LE = E_0$ is an important constitutent of the energy budget at the soil-air interface, many attempts have been made to determine E_0 satisfactorily, either by experiment or theory. For surfaces with an unlimited water supply, the energy balance method has been successful because, in this case, the ratio between sensible and latent heat flux into the air is only dependent on temperature T_0. The method is less satisfactory for vegetation-covered or bare surfaces, especially in arid climates [*see*, for example, the discussion by Chang (1968)]. For monthly and yearly moisture budget considerations over land areas, an alternate approach is to estimate the actual evaporation as a portion of a calculated potential evapotranspiration using Thornthwaite's or Penman's formulae which are based mainly on air temperature. The disadvantage of this method lies in the uncertainty of transforming potential into actual evapotranspiration. To avoid problems inherent in the use of semi-empirical regression methods Lettau (1969) proposed a new numerical model which is based on the integration of the hydrological balance equation for a continental area (watershed). The model takes into account that evapotranspiration is dependent on available moisture as supplied by precipitation of preceding months as well as available external energy as supplied by solar radiation absorbed at the ground.

It is necessary to describe the method briefly since certain improvements concerning the variability of parameters are used in the following application to the New Delhi area which are not included in Lettau's original publication (1969).

4·1·1· *Summary of Evapotranspiration Climatonomy* — The water balance equation of a land surface is written as —

$$P=E+N+\triangle m/\triangle t \qquad (2)$$

The following assumptions and parameterizations are characteristic of the new model :

1. Runoff N as well as evaporation E are considered to be sums of two terms, $N=N'+N''$ and $E=E'+E''$ where the symbols with primes denote

TABLE 4

Heat Budgets (all terms in ly/day) for New Delhi Region, 1964 and 1965, separate for air-soil interface and atmospheric column

Effective radiation terms from Lodi Rd. Observatory, New Delhi; Latent heat from evapotranspiration data of Table 3; soil heat from climatonomic iteration.—Storing in column from changes in vertically averaged air temperature; advection *plus* subsidence as remainder of the budget

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
		Air-Soil Interface : Eff. SW + Eff. LW = Latent Heat + Sens. Heat ↑ + Sens. Heat ↓												
1964	Eff. SW	254	291	355	396	423	381	333	367	381	400	314	245	345
	Eff. LW	—165	—168	—169	—172	—155	—107	—66	—67	—105	—172	—173	—155	—139
	Lat. Heat ↑	11	10	8	11	31	44	453	464	306	57	38	42	123
	Sens. Heat ↑	92	114	166	195	223	224	—187	—165	—30	178	117	67	83
	Sens. Heat ↓	—14	—1	12	18	14	6	1	1	0	—6	—14	—19	0
1965	Eff. SW	257	304	371	409	426	387	357	405	380	354	291	244	349
	Eff. LW	—152	—155	—168	—160	—156	—121	—75	—84	—111	—135	—148	—154	—135
	Lat. Heat ↑	31	27	15	33	17	10	224	278	276	34	23	15	82
	Sens. Heat ↑	88	125	178	198	237	247	56	44	—4	191	132	91	132
	Sens. Heat ↓	—14	—3	10	18	16	9	2	—1	—3	—6	—12	—16	0
Atmospheric column. Heating by (SW ↕ + LW ↕ + Conduction + Condensation + Advection + Subsidence = Heat storing in column)														
1964	Eff. SW	127	138	230	294	265	323	351	294	267	180	101	138	226
	Eff. LW	—337	—344	—346	—354	—325	—331	—350	—365	—345	—233	—288	—303	—332
	Conduct.	92	114	166	195	223	224	—187	—165	—30	177	117	67	83
	Condens.	0	2	2	10	13	53	1022	849	344	0	0	30	197
	Advect. *plus* subsidence	135	117	—22	—139	—206	—292	—848	—605	—236	—138	48	64	—174
	Storing	17	27	30	6	—13	—23	—12	8	0	—14	—22	—4	0
1965	Eff. SW	156	140	233	291	267	285	342	314	278	204	139	139	233
	Eff. LW	—303	—296	—272	—284	—290	—333	—385	—384	—379	—369	—370	—321	—331
	Conduct.	88	125	178	198	237	247	56	44	—4	191	132	91	132
	Condens.	17	17	4	23	11	6	321	353	376	0	0	0	94
	Advect. *plus* subsidence	40	22	—127	—208	—203	—201	—342	—325	—269	—34	69	66	—128
	Storing	—2	8	16	20	22	4	—8	2	2	—8	—30	—25	0

'immediate', and the symbols with double primes 'delayed' processes. Since the discussion in this study is restricted to time increments $\triangle t$ of one month, the 'immediate' processes are defined as those which involve, precipitation fallen during the same month; 'delayed' processes involve water with a residence time of more than one month in the ground.

2. Immediate runoff N' will only be significant if a certain threshold value P^* of the monthly precipitation P is surpassed. Let $N'=n^*\ (P-P^*)$ for $P>P^*$ and $N'=0$ for $P<P^*$. The fraction n^*, a numerical parameter, varies with different watersheds. According to definition, $N'=0$ in a month without precipitation. The delayed portion N'', however, will continue to be positive for one or several months depending on the residence time t^*, which is a local parameter.

3. It is a main feature of the new model that the immediate evapotranspiration of a given month is proportional to concurrent monthly means of both, precipitation (*minus* immediate runoff) and solar energy absorbed by the ground. Let $E'=e^*(P-N')\,F/\bar{F}$, where $F=(1-a^*)\,G$ and $\bar{F}$=annual

mean of F. The numerical coefficient e^* is a new parameter called 'evaporivity' by Lettau (1969) who defines e^* as a "non-dimensional measure of the capacity of a given land surface to utilise a portion of the solar energy (absorbed during one month) for the evaporation of precipitation that has been received during the same month (or in any specified interval)". It was tentatively established that e^* will normally lie between 0·4 and 0·8 depending on such local conditions as vegetation-cover, porosity of the soil, occurrence of frozen ground, etc. It may vary with the seasons and must be looked upon as an empirical local parameter comparable to the albedo of land areas.

4. According to definition, delayed processes, N'' and E'', involve a residence time t^* of rainwater in the ground which must exceed the time interval for which the immediate processes were defined (in this case longer than one month). The residence time also depends on local conditions and is another new parameter of the model. A basic model assumption is that the delayed processes N'' and E'' are directly proportional to the monthly values of total exchangeable soil moisture m; this implies that $N'' = m\overline{N''} / (\overline{N''} + \overline{E''})\, t^*$, and $E'' = m\overline{E''}/(\overline{N''}+\overline{E''})\, t^*$, whereupon $N''+E'' = m/t^*$.

5. Subtraction of immediate runoff and evapotranspiration from precipitation produces the 'effective' precipitation $P' = P - E' - N'$. With the above assumptions and parameters, Eq. (2) transforms into the ordinary first order differential equation —

$$dm/dt + m/t^* = P' \tag{3}$$

The residence time t^* can vary from month-to-month. Let a non-dimensional time scale be defined by —

$$\tau = \int_0^t dt/t^*, \text{ or, } d\tau = dt/t^* \tag{4}$$

which, in case of $t^* = \bar{t}^* = \text{const.}$, reduces to $\tau = t/\bar{t}^*$.

For variable or constant t^*, Eq. (3) is solved by,

$$m = e^{-\tau}\left[\, m_0 + \int_0^\tau t^* P' e^{\tau}\, d\tau \,\right] \approx e^{-\tau}\left[\, m_0 + \overline{t^* P'} \int_0^\tau e^{\tau}\, d\tau \,\right] \tag{5}$$

where, m_0 is the initial value of m at the time $t = 0$.

6. In his original publication Lettau (1969) discussed solution of (3) only for a constant t^*. A follow-up publication is under preparation in which the case of a variable t^* is treated. With Lettau's permission the resulting formulae to calculate m_{n+1} from m_n of the preceding month n, with the aid of t^*_n, P'_n and t^*_{n+1}, P'_{n+1} are summarised here —

Let us define averages of values for consecutive months as —

$$(\overline{t^* P'})_{n,\, n+1} = (t^*_n P'_n + t^*_{n+1} P'_{n+1}) / 2, \text{ and}$$
$$\triangle\tau = \triangle t\, (1/t^*_n + 1/t^*_{n+1}) / 2$$

where $\triangle t$ is one month. It follows then from Eq. (5) considering that —

$$\int_0^{\Delta\tau} e^{\tau}\, d\tau = e^{\Delta\tau} - 1$$

$$m_{n+1} = (\overline{t^* P'})_{n,\, n+1} + [m_n - (\overline{t^* P'})_{n,\, n+1}]\, e^{-\Delta\tau} \tag{6}$$

To start the stepwise calculations, the initial m_0 can be arbitarily assumed. The correct m_0-value follows from the requirement that for the total length of calculation time $n\triangle t$ (which will normally be equal to the 'average' year, or a sequence of full years), the end-value of m must be equal to the correct m_0 if the climate is stable. If the first tentative choice of m_0 does not meet this requirement, the integration is repeated using the final m as the new m_0 until the value generated agrees with the value used to generate it.

After the time-series of m-values has been calculated for the desired number of months and years, N'' and E'' are obtained with the aid of t^* (according to point 4). With N' and E' established previously, N and E are determined, and also $\triangle m/\triangle t$. It can thus be seen that the new model of evapotranspiration climatonomy is not another semi-empirical evaporation formula but a numerical solution of the hydrological balance equation.

4.1.2. *Practical methods of parameter determination for the model* — Independent information includes time series of precipitation, global radiation, and albedo, in the form of monthly average, of at least one, better several full years, or multi-annual (climatic) monthly means. If the mean annual runoff (for a period when $\overline{dm/dt}=0$) is known also, $\bar{E}$ follows directly as $\bar{P}-\bar{N}$. If unknown, N' can be determined from energy budget data using the runoff ratio $C^* = \bar{N}/\bar{P}$, and the Budyko relation (as discussed by Lettau, 1969) with the aid of the dryness ratio D^* (which is Budyko's radiational index of dryness).

TABLE 5

Meteorological background data for New Delhi area in 1964 and 1965

Basic data from climatic records. Derived data : Surface temperature T_0 from climatonomical iteration; precipitable water w from Smith's formula; exchangeable soil moisture m from evapotranspiration climatonomy; surface albedo a^* from climatonomical iteration; albedo at top of atmosphere a^{**}, Angstrom ratio A^{**} at top of atmosphere, aerosol efficiency as SW absorber α_{ae} and a scatter σ_{ae} from 'Shortwave Radiation Climatonomy'. Angstrom ratio at surface A^* from observations.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual
							Basic Data 1964						
T_s (°C)	12·3	16·7	24·1	29·5	30·9	34·0	28·7	29·4	27·7	25·4	19·1	15·1	24·4
e_s (mb)	8·2	9·4	10·7	15·1	13·5	18·7	30·8	31·6	27·2	15·7	10·7	11·0	16·9
c (—)	0·20	0·34	0·24	0·24	0·24	0·36	0·74	0·61	0·46	0·06	0·14	0·30	0·33
G (ly/day)	355	417	518	588	632	562	421	429	442	475	389	314	462
D (ly/day)	104	123	144	224	258	319	252	227	182	97	93	106	177
							1965						
T_s (°C)	15·6	16·7	21·7	26·3	31·3	35·0	31·1	29·5	28·5	26·3	21·3	14·7	24·8
e_s (mb)	10·7	10·3	10·3	13·1	12·8	17·5	27·5	26·7	22·8	17·1	12·7	8·9	15·9
c (—)	0·30	0·21	0·21	0·44	0·36	0·26	0·61	0·49	0·26	0·14	0·15	2·15	0·29
G (ly/day)	339	412	516	582	616	569	490	518	473	445	379	331	473
D (ly/day)	99	135	158	203	268	330	277	245	173	134	103	91	185
							Derived Data 1964						
$T_\circ$ (°C)	13·2	17·8	26·0	31·7	33·4	36·5	26·6	27·6	27·4	27·3	20·3	15·8	25·3
w (mm)	12	14	17	25	22	33	49	61	51	27	18	17	29
m (mm)	20	13	9	6	5	7	51	107	117	92	62	45	45
a^*(—)	·28	·30	·32	·33	·33	·32	·21	·15	·14	·16	·19	·22	·25
a^{**}(—)	·28	·34	·24	·22	·29	·29	·30	·29	·21	·16	·26	·23	·26
α_{ae}(—)	·04	·03	·09	·11	·07	·09	·12	·06	·09	·06	·01	·07	·07
σ_{ae}(—)	·07	·05	·10	·13	·20	·18	·19	·19	·09	·07	·08	·09	·13
A^* (—)	·21	·20	·18	·17	·15	·10	·07	·07	·11	·18	·20	·19	·15
							1965						
$T_\circ$ (°C)	16·5	17·9	23·7	28·5	34·0	37·7	31·7	30·0	28·5	28·3	22·7	15·6	26·3
w (mm)	16	16	17	22	21	31	52	50	42	29	21	15	28
m (mm)	35	27	20	14	11	8	23	45	59	56	38	25	30
a^*(—)	·24	·26	·28	·30	·31	·32	·27	·22	·20	·20	·23	·26	·25
a^{**}(—)	·22	·31	·22	·21	·28	·32	·28	·22	·20	·20	·24	·23	·24
α_{ae} (—)	·09	·03	·10	·11	·09	·07	·07	·07	·10	·09	·05	·08	·08
σ_{ae} (—)	·07	·11	·10	·12	·17	·26	·19	·20	·16	·16	·07	·07	·14
A^* (—)	·19	·19	·19	·16	·15	·11	·08	·08	·11	·14	·16	·19	·15

The parameters n^* and P^* will normally be available from hydrological studies of the region. As long as the model is new, the determination of empirical parameters e^* and t^* remains difficult. They may be selected without monthly variations until a set of pilot studies provides sufficient guidelines for an a priori estimate. It is known that the albedo of a region depends on amount and type of vegetation-cover which, in turn, depends on soil moisture. It will be assumed that a^* varies about its annual $\overline{a^*}$ inversely proportional to m *plus* a constant.

4.1.3. *Determination of parameters for the New Delhi Area* — A study by Dhir, Ahuja, and Majumdar (1958) was used to estimate the immediate runoff in the New Delhi area. In hydrological studies total runoff from a river basin (of area A) is usually given in volumetric total units, such as 10^6 acrefeet, while the area is measured in squaremiles, and precipitation in inches. Empirical formulae based on annual totals of observed runoff and precipitation for a sequence of year are unusually expressed in the following form,

$A N' \triangle t = -a + b P \triangle t$; (valid only for $A N' \triangle t \geqslant 0$).

(Note that N' was defined as runoff per month). The empirical constants of the regression line, a and b, are given in 10^6 acrefeet and 10^6 acrefeet/inch, respectively.

For the Chambal River basin (catchment area 8,700 squaremiles), the authors determined a regression formula for runoff as $AN' \triangle t = -4 \cdot 006 + 0 \cdot 247\ P \triangle t$. Since the relation is only valid for $AN' \triangle t > 0$, a threshold value $P^* \triangle t$ exists which is equal to $16 \cdot 2$ inches/$\triangle t$, or 411 mm/$\triangle t$. Noting that during the two years considered, almost all the rain fell within three months, $\triangle t$ is taken to be equal to 3 months; thus $P^* =$ 137 mm/month. With $P^* = a/b\ \triangle t$, the regression equation can be reformulated into $AN' = b\ (P-P^*)$, and upon division by A, the relation between runoff and precipitation for the Chambal River basin yields $N' = 0 \cdot 53\ (P\text{–}137)$ mm/month.

Pilot studies have indicated that the evaporivity e^* lies between $0 \cdot 4$ and $0 \cdot 8$ depending on vegetation-cover and soil conditions. It will be relatively high in a subtropical climate with summer rains. For the New Delhi area, e^* tentatively assumed to be $0 \cdot 7$, and constant for the entire year.

To keep seasonal albedo variations within reasonable limits under preservation of an annual mean of $0 \cdot 25$, an empirical interpolation formula with a^* proportional to $3\,\overline{m} / (2\,\overline{m} + m)$ was prescribed. Beginning with $a^* =$ const, first set of m-values is obtained which yields then a first set of variable monthly a^* values ; hence new F-values are obtained, and *via* E', an iterated set of m-values. It was found that this iteration converges sufficiently after one repeat. Results of the iterated a^*- values are listed in Table 5.

The last parameter which must be predetermined is t^*, the residence time of rainwater in the ground. Tentatively, it will be assumed to be constant and equal to $2 \cdot 5$ months which may be permissible in a climate where the ground is never frozen during the year.

This evapotranspiration model is applied to the 24-month period of January 1964 to December 1965, and the results are summarized in Table 3, and discussed in Section $5 \cdot 1$. It may be noted here that the calculated soil moisture (m) was significantly higher in 1964 than in 1965 during the monsoon season. It remained relatively high during the following winter and spring, replenished by occasional winter and spring rains. It will be interesting to compare these calculations with direct observations. The author would appreciate comments from readers.

4.2. *Sensible heat transfer into the ground*

4.2.1. *General Formulae* — Amplitudes as well as phase angles of the annual cycle of heat flux into the submedium ($S_\circ$) and surface temperature ($T_\circ$) are theoretically interrelated by means of physical soil parameters such as heat conductivity (λ) and heat capacity (c). Let the amplitude ratio for a given cycle of frequency i be denoted by $\triangle_i S / \triangle_i T = \Psi_i$ and the phase difference by ϕ_i. If Fourier analysis yields an equation for $T_\circ$ such as the following for the first and second harmonic of the annual cycle.

$$T_\circ = \overline{T}_\circ + \triangle_1 T_\circ \cos(\omega t - \delta_1^*) + \\ + \triangle_2 T_\circ \cos(2\,\omega t - \delta_2^*) \qquad (7)$$

then Fourier 'synthesis' yields the soil heat flux as–

$$S_\circ = \overline{S}_\circ + \Psi_1 \triangle_1 T_\circ \cos(\omega t - \delta_1^* - \phi_1) + \\ + \Psi_2 \triangle_2 T_\circ \cos(2\omega t - \delta_2^* - \phi_2) \quad (8)$$

For homogeneous soil, classical theory predicts that $\phi_1 = \phi_2 = 45°$ and if the climate is stable, the annual means $\overline{S}_\circ$ will be zero.

Equations (7) and (8) yield—

$$S_\circ = \Psi_1 \triangle T_1 \cos(\delta_1^* - 45°) \cos \omega t + \\ + \Psi_1 \triangle T_1 \sin(\delta_1^* - 45°) \sin \omega t + \\ + \Psi_2 \triangle T_2 \cos(\delta_2^* - 45°) \cos 2\omega t + \\ + \Psi_2 \triangle T_2 \sin(\delta_2^* - 45°) \sin 2\,\omega t \qquad (9)$$

In a homogeneous conductor with caloric admittance $\mu = \sqrt{\lambda c}$ (in mly / $\sqrt{}$ sec/degree, independent of time), the amplitude ratio Ψ_i will be equal to $\mu\sqrt{i\omega}$. The term 'caloric admittance' used here for μ is alternative to other names [*See* for example Sellers (1967, p. 139)]. With Ψ_i determined, numerical solution of Eq. (9) yields the $S_\circ$-term of the energy budget equation (1).

4.2.2. *Determination of parameters for the New Delhi Area* — With the simplifying assumption of homogeneous soil, only the caloric admittance μ of the ground must be estimated. According to literature tabulations values of μ vary considerably; for example, from 14 for dry sand, 36 for sandy clay (with 15 per cent moisture) to about 56 for concrete. A mean value of 40 (mly/ $\sqrt{\text{sec}}$/deg) was tentatively used for numerical calculations in the New Delhi area. With $\omega = 2\,\pi / 365$ per day, the amplitude ratios are $\Psi_1 = 1 \cdot 56$ (ly/day deg), and $\Psi_2 = 2 \cdot 18$ (ly/day deg). Amplitudes $\triangle_1 T$ and $\triangle_2 T$, also phase angles δ_1^* and δ_2^* result from harmonic analysis of the monthly means of temperature T_s. By harmonic synthesis of Eq. (9) a first set of $S_\circ$-values is obtained. The final series can be determined by iteration after an effective surface temperature $T_{\circ}$ has been determined from the balance equation (1).

4.3. *Sensible Heat Flux into the Air*

4.3.1. *General Formulae* — With SW $\updownarrow$, LW $\updownarrow$ $E_{\circ}$, and $S_{\circ}$ determined previously, flux of sensible heat from the surface into the air can be obtained as the remainder of the balance equation (1). With the aid of the eddy conduction equation in the surface layer, the temperature difference between screen height (z_T) and actual surface can be estimated from—

$$T_s - T_{\circ} = -Q_{\circ}(\rho c_p k v^*)^{-1} \ln z_T / z_{\circ} \qquad (10)$$

where $k = 0{\cdot}42$, the Karman constant, v^* = friction velocity, $z_{\circ}$ = roughness length, and $Z_T = 180$ cm thermometer height. The original T_s and $Q_{\circ}$ series yields with the aid of Eq. (10) a first set of monthly mean surface temperatures T_{01}. Any heat budget constituent based previously on T_s is then recalculated with $T_{\circ 1}$ resulting in a revised $Q_{\circ}$. This iteration process is continued until the resulting temperature agrees with the input temperature. After one iteration the second temperature series $T_{\circ 2}$ differed only occasionally by one-tenth of a degree from the corresponding values of the $T_{\circ 1}$ series. No further iteration was made, and the $T_{\circ 2}$-values were entered as the final $T_{\circ}$-values in Table 5.

4.3.2. *Determination of parameters for the New Delhi Area* — The numerical solution of Eq. (10) requires that the friction velocity v^* for the area can be derived from a relationship between wind speed and surface stress. Although local climatic wind records are available at New Delhi, they are not used to calculate v^* because of possible local influences. A second reason for not using them is to show that climatonomical methods are applicable to areas where conventional climatic observations are lacking, or at least minimal. It is possible to calculate v^* for a region from the average horizontal pressure field. In this case the concept of geostrophic drag coefficient as a function of the surface Rossby number can be used (see Lettau 1962). The only other regional parameter to be employed is $z_{\circ}$, tentatively estimated as 12 cm for an urban area. Kung (1963) used this model and provided estimates of v^* and other geostrophic data for a great number of grid points on the northern hemisphere; tabulated below are values for 28°N latitude of the Asian continent separated for seasons according to Kung.

Geostrophic wind speed v_g (cm/sec), geostrophic drag coefficient C, friction velocity $v^* = v_g \cdot C$ (cm/sec), surface Rossby number $Ro_{\circ}$; also wind velocity v (cm/sec) computed for the 300-cm level with $z_{\circ} = 12$ cm as follows—

	v_g	C	v^*	Ro_0	v
Dec—Feb	870	0·034	29·6	1·07 10^6	390
Mar—May	730	0·035	25·5	0·91	335
Jun—Aug	730	0·036	26·2	0·91	346
Sep—Nov	780	0·035	27·3	0·95	358

The wind velocity v at the 300 cm level is listed only in order to show that with the deduction of v^* from a continent-wide pressure field, the resulting v-values are of the right order of magnitude.

5. Discussion of Results

5.1. *Moisture Budget at the Soil-Air Interface and for the Atmospheric Column*

According to World Meteorological Records, Safdarjung Airport at New Delhi received 1232 mm rainfall in 1964; that is 516 mm above normal. Monthly contributions to departures were –93 mm from January to June, +631 mm from July to September, and —21 from October to December. The year 1965 had less than normal rainfall with an annual deficit of 128 mm. All moisture terms were calculated for 24 consecutive months to include a complete moisture year from one monsoon season to the next. According to Agarwala (1961) rainfall at Safdarjung seems to be up to 20 per cent higher during the rainy season than at Lodi Rd. Observatory. Considering the natural degree of variability in the shower-type rainfall of the monsoon, the uncertainties of runoff estimates by using data from the Chambal River Basin, and the estimates of surface albedo for the region, an error tolerance of about $\pm$ 10 per cent should be realistic for the results listed in Table 3. In a region where high insolation coincides with high precipitation, climatonomy predicts relatively high immediate evapotranspiration rates. In both years, about 80 per cent of the annual evapotranspiration occurred between July and September. Surface runoff dominates during the rainy season, but delayed runoff decreases also quickly to almost zero during the premonsoon months. The remaining portion of precipitation which neither evaporates nor flows out of the region is listed under 'storing' in Table 3. Negative storing indicates withdrawal of soil moisture.

The annual trend of each moisture term follows obviously the annual trend of precipitation. How much of it evaporates during one month depends on the numerical value of the evaporivity e^*. The assumption of a constant e^* for the entire year must be considered as a tentative first approximation. Probably, e^* varies within similar seasonal limits as the albedo of a region. Additional studies for different climates will help to establish empirically realistic values of e^*, such as the work by Thaboub (1970) for the Jordan River Valley, where low insolation coincides with high precipitation in winter. The example of New Delhi is shown in Fig. 1.

A moisture budget for the atmospheric column is also included in Table 3. Storing was derived from changes of the precipitable water (w) from month-to-month. The w-values listed in Table 5 are calculated with the aid of a semi-empirical relationship derived by Smith (1966) based on surface dewpoint temperature. These values compare reasonably well with the five-year means (1956-1961) of precipitable water in the atmosphere over New Delhi as presented by Ananthakrishnan *et al.* (1965). Storing is negative when the water content of the column decreased from the previous month. Advection is obtained as the remainder of the balance equation. It was negative when more moisture leaves the column than moves into it during the month. At the end of the rainy season, the change-over from moisture import to moisture export occurs abruptly with the greatest drop in October in both years. Advection declines to almost zero, during the pre-monsoon season. The net import of moisture during the unusually heavy monsoon rains of 1964 was more than five times higher than during the same months of 1965. It seems desirable to compare these findings with results from aerological soundings.

5.2. *Energy Budget*

With all the terms of Eq. (1) finally determined, a complete energy budget for the air-soil interface is summarized in the top part of Table 4. The radiation terms are the same as listed in Table 1 ; latent heat is the evaporation of Table 3 expressed in ly/day ; sensible heat fluxes into the air and ground are obtained by the methods outlined in Sections 4·2 and 4·3.

The radiation components of the budget equation were already discussed in Section 3·2. Interesting is the reversal of sensible heat flux from July to September in 1964 which appears to be due to the great energy demand for evaporation resulting in 'psychrometric cooling' at the surface. In the following year when evaporation in July and August was about half of that of 1964, no such cooling occurred. But in September, with evaporation almost as high as in the previous year, a weak reversal of Q_o is apparent. As a consequence, the local Bowen ratio $B^* = \bar{Q}_o/\bar{E}_o$ (based on annual means) equals 0·67 when the annual rainfall is 516 mm above normal, and 1·61 when rainfall is 128 mm below normal. Before the monsoon, about 80 per cent of the net radiation at the ground is used to heat the air above. After the break more than 80 per cent is used for evaporation in 1965 but 170 per cent in 1964, with 70 per cent of the total being withdrawn from the air above. Accordingly, the decrease in surface temperature from June to July is 10° in 1964, but only 6° in 1965. Sensible heat flux from and into the soil seems to be small in comparison and reaches extremes in December and April, respectively.

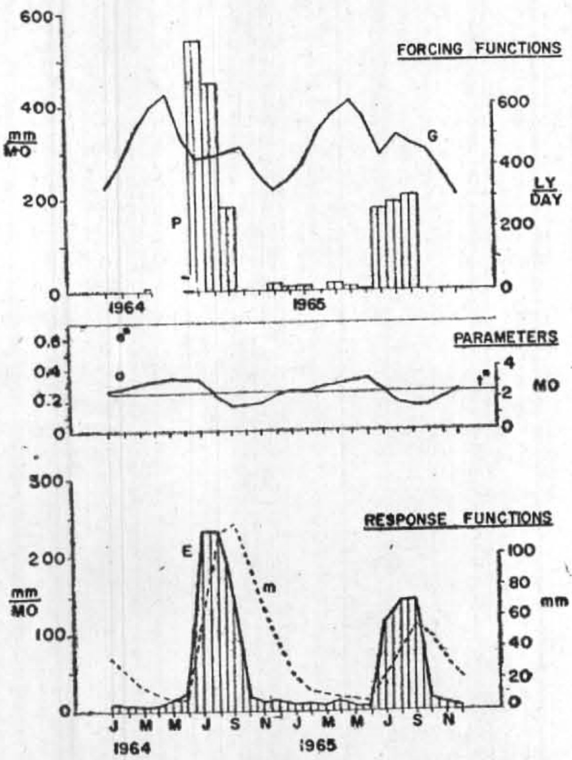

Fig. 1

Example of Evapotranspiration Climatonomy. Monsoon Climate—New Delhi, India 1964 & 1965. Forcing functions: Precipition (P), Global radiation (G); Response functions : Evapot anspiration (E), Exchangeable soil moisture (m) [Calculated according to Eq. (6)]; Parameters : Evaporivity (e^*) Ground albedo (a) and Residence time of soil moisture (t^*)

Based on data of Table 1 and the results discussed above for the air-soil interface, heat budget terms for the total atmospheric column are listed in the lower part of Table 4. Effective SW and effective LW radiation components are the differences between upper and lower boundary values ; conduction equals sensible heat and condensation is derived from precipitation of Table 3 expressed in ly/day. The 'storing' term is directly obtained from month-to-month changes of air temperature averaged over the entire column of the region. 'Advection *plus* subsidence' is then left as the remainder of the balance equation. Only a synoptic aerological analysis of the airflow over the region would permit separating these two proces es. But this is beyond the scope of this investigation. Independent determination of advection effects would clarify the contribution of subsidence which seems to play such an important role in maintaining the aridity of nearby desert areas.

The summary of seasonal heating and cooling rates (*see* Table 2) may suggest that in winter, with considerable heat loss by LW emission and relatively small gain by latent and sensible heat, subsidence may be a dominant factor in balancing the energy budget in the atmospheric column. In summer, however, with great amounts of latent energy set free by condensation, import of cool moist air is the dominant contribution to 'advection and subsidence'. There are characteristic differences in seasonal heating and cooling rates between the two years which are obviously related to the relative intensity of monsoon rainfall.

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