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Mean effective tailwinds along some Indian air-routes

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1. Introduction

A study has been made in this paper to find out the contribution to cruising speeds from the upper winds for certain air-routes, *viz.*, Delhi—Calcutta, Bombay—Calcutta, Bombay—Hyderabad and Hyderabad—Calcutta. The civil aviation traffic in India is mostly confined to the height slab 2-3 km and as such the mean effective tailwinds along the specified routes have been calculated for the heights 2 km and 3 km only. In a previous publication by Air Ministry, London (1950), the equivalent headwinds on some principal air-routes of the world including Delhi—Calcutta and Bombay—Calcutta have been published for heights 10,000 ft and above. Some of the values of the mean effective tailwinds along the Delhi—Calcutta and Bombay—Calcutta routes for 3 km as presented now are found to be significantly different from what was given earlier in London Air Ministry publications for 10,000 ft.

2. Basic theory

Mean effective tailwind (or headwind) may be defined as that average component of the prevailing wind which acting along the whole route all the time, produces the same effect as the actual system of winds at different points along the route. In effect, this becomes the difference between the air speed and the ground speed averaged for the flight for any particular route. The principles underlying the calculation of these winds may be stated as follows. Consider a point O on an air-route OA (Fig. 1). If the actual mean wind at O be represented by OB its resolved components u along OA and v along OC perpendicular to OA, both affect the actual ground speed of the aircraft proceeding along OA. The effect of u which may be called the track component, is directly additive or diminutive to the aircraft's speed. The component v acts perpendicular to the aircraft's course and has to be compensated by proper heading of the aircraft. When the effects of both u and v are taken into account, the effective tailwind along OA at the point O is given by the expression (Sawyer 1950)

$$w = u + \frac{v^2}{2A}$$

where A is the aircraft speed. Taking into account the individual variations of the wind from the mean at any point in the free air, and assuming that these variations are distributed normally about the mean vector wind, the actual contribution of the cross wind to the effective tailwind at O has been shown to be equal to $\sigma^2/4A + \bar{v}^2/2A$ by Crossley (1946). In this expression $\bar{v}$ indicates a mean value of v taken over along period of time and σ is the standard deviation of the individual vector winds from the mean. Thus the expression for w becomes

$$w = \bar{u} + \frac{\bar{v}^2}{2A} + \frac{\sigma^2}{4A}$$

Here $\bar{u}$ and $\bar{v}$ can be calculated by taking the appropriate resolved parts of the mean vector wind. However calculations of σ presents some difficulty. The usual expression for σ is

$$\sigma = \left[\frac{\Sigma V^2}{n} - V_r\right]^{\frac{1}{2}}$$

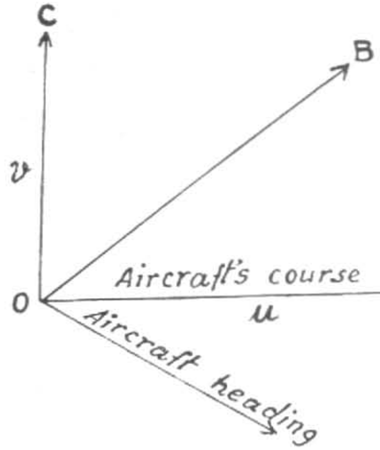

Fig. 1

where V denotes the individual wind velocity, n the number of observations of V and V_r the mean vector resultant wind. This involves laborious calculations and recourse has, therefore, to be taken to a different method. In this method we introduce a new factor q called the Constancy Factor and defined as $q = 100V_r/V_s$ where V_s is the mean wind speed irrespective of direction and V_r is the mean vectorial wind. Brooks and others (1946) have shown that q is related to σ as follows—

$$\frac{100}{q} = \frac{2}{\sigma^2 V_r} \int_0^{\infty} \left(1 - \frac{m}{10}\right) \left(V^2 + v^2\right)^{\frac{1}{2}} \times v\, e^{-v^2/\sigma^2} dv$$

where $m = v/V_r$. They have also calculated the values of q for different values of σ/V_r. These values have been given in Table 1 and utilised for calculating σ.

Thus the effective tailwind (or headwind) $\bar{w}$ along an air-route would normally be the average of the different values of w calculated for different points along the route. However these winds w are mutually related. Assuming again that these values of w are normally distributed about the mean, the standard deviation σ_T for the whole route has been calculated by Sawyer (1950) as per following expression—

$$\sigma^2{}_T = \frac{\sigma^2}{S^2} \left\{1 + \tfrac{1}{4}\frac{\sigma^2}{A^2} + \frac{\bar{v}^2}{A^2}\right\} \int_0^S \int_0^s R\,(x) dx\, ds$$

Here S is the length of the route and $R\,(x)$ is the stretch coefficient. Durst (1954) has given the relevant curve for $R\,(x)$ against S for the tropics and sub-tropics. The appropriate values of $R\,(x)$ were obtained from this curve for different values of S. The double integral was then evaluated after expanding it according to Simpson's Rule, and then substituting corresponding values of $R(x)$ and S. Table 2 was prepared for σ_T/σ for different values of S, for the tropical and sub-tropical regions.

3. Computation of the effective tailwinds

To calculate $\bar{w}$ for the different routes in question three pilot balloon stations were taken on each route. Two of these were the terminal stations and one was an intermediate station approximately at the middle of the air-route. The monthly normals of the upper winds at 2 km and 3 km for these stations were calculated using all available data upto 1950. The resultant normal winds were resolved along and perpendicular to the direction of the air-routes in question. It was noticed that a difference of ten degrees in bearing would not normally affect the value of the resolved components except in the first place of decimals. The calculation of the resolved components at the intermediate station could, therefore, be done along a mean bearing when possible. For calculating the standard deviation σ, the mean scalar wind for each station was taken month by month and the constancy factor $q = 100V_r/V_s$ was calculated. The appropriate values of σ were then determined with reference to Table 1. Table 3 gives the standard deviation σ for each station. With these individual values of σ,

TABLE 1

Values of σ/V_r in terms of q

(extracted from *Quart. J. R. met. Soc.*, **72**, p. 63)

q	=	10	15	20	25	30	35	40	45	50	55	60	65
σ/V_r	=	11·3	7·6	5·6	4·4	3·7	3·1	2·6	2·3	2·0	1·8	1·6	1·4
q	=	70	75	80	85	90	$92\frac{1}{2}$	95	$97\frac{1}{2}$	100			
σ/V_r	=	1·2	1·1	0·9	0·8	0·6	0·5	0·5	0·4	0·0			

TABLE 2

σ_T/σ Calculated for Tropical and Sub-tropical regions

Length of air-route S (nautical miles)	0	100	200	300	400	500	600	700	800	900	1000
σ_T/σ	0·71	0·67	0·65	0·63	0·60	0·59	0·57	0·55	0·54	0·53	0·52

TABLE 3

Standard deviation σ for each station in metres per sec

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Delhi	2km	5·5	7·1	6·7	6·1	5·0	5·8	7·8	5·9	6·7	5·5	5·0	3·8
	3km	5·6	6·4	8·5	6·4	6·1	5·9	5·1	5·2	5·5	6·2	5·0	4·8
Allahabad	2km	6·6	5·3	5·2	4·5	5·5	7·3	9·1	6·6	7·9	6·6	11·9	5·7
	3km	6·4	8·5	6·7	6·0	5·6	9·0	7·1	3·3	7·7	6·2	6·2	5·6
Calcutta	2km	5·4	4·9	5·4	5·4	4·3	6·8	6·2	6·8	5·9	6·8	5·3	5·1
	3km	6·3	7·0	5·4	6·4	5·4	7·6	5·7	6·0	5·9	6·0	6·1	6·2
Bombay	2km	5·6	6·0	7·0	6·7	6·0	6·3	4·4	5·0	6·0	4·7	4·4	5·1
	3km	7·0	7·2	6·2	6·8	5·6	5·5	5·8	4·0	5·5	5·2	5·6	7·0
Nagpur	2km	6·0	6·1	6·1	7·3	4·8	4·6	6·1	5·2	6·4	4·7	5·3	5·5
	3km	3·3	6·6	6·1	5·6	5·8	5·8	5·2	5·1	5·2	5·6	5·2	5·6
Poona	2km	5·6	5·2	3·3	6·5	6·7	6·5	6·4	5·5	7·5	5·3	4·4	5·2
	3km	7·0	6·8	6·4	5·9	6·1	3·3	6·8	5·8	4·4	4·9	5·6	6·1
Hyderabad	2km	6·5	7·5	6·6	4·4	7·3	6·6	5·2	6·9	7·0	6·2	5·2	5·6
	3km	6·1	6·8	6·2	5·4	5·3	6·0	4·6	5·4	6·7	5·8	5·0	6·2
Jagdalpur	2km	4·2	5·9	5·6	4·5	6·2	6·2	5·4	6·3	8·3	5·3	5·0	5.6
	3km	6·1	6·7	6·6	5·2	6·0	6·9	4·6	5 4	7·4	5·6	5·3	5·4

average values of σ for the route were calculated and then referring to Table 2, the values of σ_T for the routes were evaluated, by taking into account the length of the route. The two terms of the cross-wind component, namely, $\bar{v}^2/2A$ and $\sigma^2/4A$ when calculated, assuming the aircraft velocity A to be 160 mph, turn out to be negligible in the cases under study, and therefore, have not been reproduced. However, it should be remembered that the effect of the cross-wind $\bar{v}$ is independent of the sense of the route and when its value is not negligible the effective tailwind values for the direct and return flights along a route will differ by

$$\frac{\bar{v}^2}{A} + \frac{\sigma^2}{2A}$$

The variation and the different characteristics of these mean monthly values of the Track Wind are represented in Figs. 2 to 5.

4. Discussion

4.1. *Delhi–Allahabad–Calcutta route* (Fig. 2)

The average values for the 3 km effective tailwinds are more than that of 2 km during the period October to middle of June and *vice versa* during the rest of the period. The maximum effective tailwinds occur in the month of February for both levels — 21 mph at 3 km and 17 mph at 2 km. The minimum is also reached simultaneously during the month of August (–2 mph at 3 km and –1 mph at 2 km). It would appear from the graph that choosing a flight level at 3 km for onward flight from Delhi to Calcutta, and 2 km for return flight from Calcutta to Delhi would be definitely advantageous during the month October to June.

4.2. *Bombay–Nagpur–Calcutta route* (Fig. 3)

The mean effective tailwinds at 3 km are significantly more than that at 2 km for the period of October to April and *vice versa* during the remaining period of the year. The effective tailwinds during the months of August and September are very nearly identical for both the levels. The mean maximum for the 3 km level is reached in the month of February, but for the 2 km, it occurs in the month of July. There is a subsidiary maximum for the 2 km winds in the month of February and for the 3 km winds in the month of August. Here again, choosing a flight level at 3 km for onward flight from Bombay to Calcutta, would be preferable during the period October to April.

4.3. *Bombay–Poona–Hyderabad route* (Fig. 4)

Significantly enough the 2 km tailwinds dominate the route for best part of the year from March onwards up to September. The maximum value for the winds at both levels is reached during the month of July, it being 25 mph for 2 km and 15 mph for 3 km. The minimum values are also reached simultaneously for both the levels during the month of October. The 3 km tailwinds are generally stronger than 2 km winds during the months of January, February, October, November and December.

4.4. *Hyderabad–Jagdalpur–Calcutta route* (Fig. 5)

The effective tailwinds at 3 km are stronger than at 2 km during October to April and are generally weaker than 2 km wind for rest of the months. The maximum value for the 2 and 3 km winds is attained during July, the 2 km winds being stronger than those at 3 km. The months of April-May and September-October appear to be transition months.

The different values for the mean effective tailwinds in this note have been calculated from the normals of the morning pibal observations only. On examination of the afternoon normal upper winds at 2 km and 3 km it was observed that differences from the corresponding values for the morning winds were small. Actual calculations for one route, namely, Delhi—Allahabad—Calcutta showed that the effective tailwind values differed by less than 2 mph on 79 per cent of the cases and were between 2 mph and 3 mph for the remainder. The calculations for the other routes

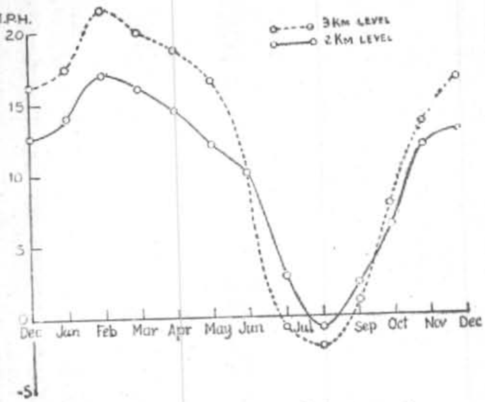

Fig. 2. Mean effective tailwind along the Delhi—Allahabad—Calcutta route

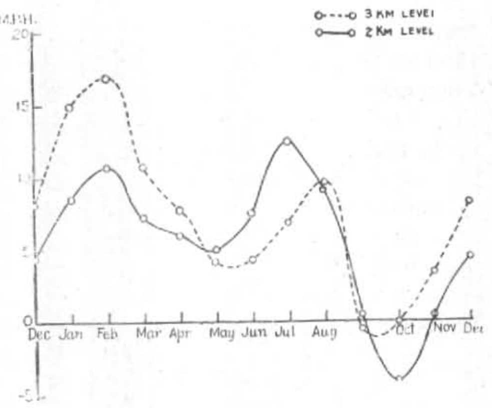

Fig 3. Mean effective tailwind along the Bombay – Nagpur – Calcutta route

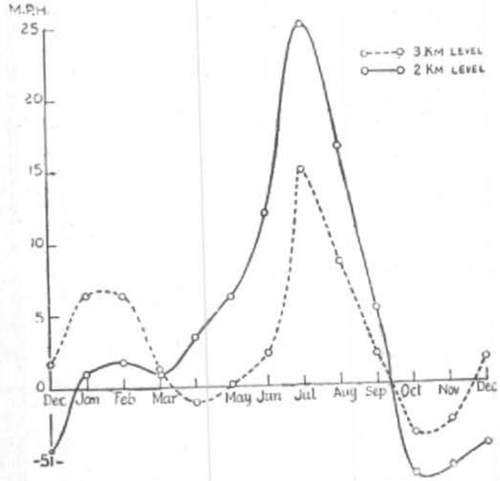

Fig. 4. Mean effective tailwind along the Bombay—Poona—Hyderabad route

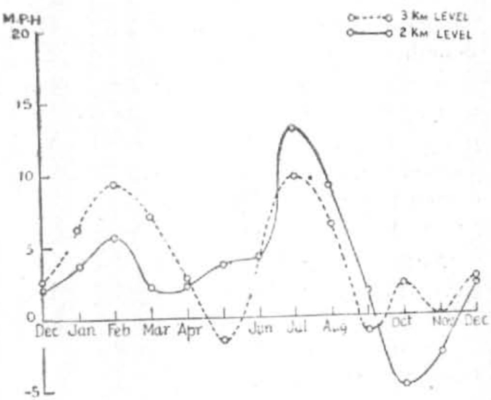

Fig. 5. Mean effective tailwind along the Hyderabad—Jagdalpur—Calcutta route

TABLE 4

Values of track wind components and σ_T for different routes

		Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Delhi–Allahabad–Calcutta route (length 675 nautical miles, bearing 125°–120°)													
Track wind (mph)	2 km	14·1	17·1	16·1	14·5	12·1	9·9	2·7	—0·7	2·3	6·3	11·6	12·6
	3 km	17·4	21·4	20·1	18·7	16·6	10·0	—0·7	—2·1	1·1	7·6	13·4	16·3
Cross component		negligible											
σ_T (mph)	2 km	7·0	7·0	7·0	6·4	5·9	7·9	9·2	7·7	8·1	7·7	9·0	5·7
	3 km	7·5	8·8	8·1	7·5	6·8	9·0	7·0	5·7	7·7	7·5	7·0	6·6
Bombay–Nagpur–Calcutta route (length 853 nautical miles, bearing 75°–80°)													
Track wind (mph)	2 km	8·5	10·6	7·1	6·0	5·1	7·6	12·5	9·6	0·6	—4·1	0·4	4·4
	3 km	15·0	16·9	10·8	7·9	4·3	4·5	6·8	9·8	—0·6	0·1	3·6	8·2
Cross component		negligible											
σ_T (mph)	2 km	6·6	6·6	7·0	7·4	5·9	6·8	6·6	6·6	7·0	6·4	5·9	6·2
	3 km	6·4	8·1	6·8	7·3	6·6	7·3	6·4	5·9	6·4	6·6	6·6	7·3
Bombay–Poona–Hyderabad route (length 327 nautical miles, bearing 105°)													
Track wind (mph)	2 km	1·1	1·8	0·9	3·4	6·2	12·1	24·6	16·5	5·2	—6·3	—5·9	—4·3
	3 km	6·5	6·4	1·2	—1·0	0·2	2·3	14·9	8·5	2·1	—3·4	—2·6	1·7
Cross Component		negligible											
σ_T (mph)	2 km	8·1	9·5	7·9	8·1	7·3	8·8	7·3	8·1	9·5	7·5	6·4	7·3
	3 km	9·2	9·5	8·6	8·6	7·9	6·8	7·9	6·8	7·7	7·3	7·5	8·8
Hyderabad–Jagdalpur–Calcutta route (length 617 nautical miles, bearing 60°)													
Track wind (mph)	2 km	3·7	5·8	2·2	2·2	3·6	4·2	13·0	9·1	1·8	—5·0	—2·7	2·1
	3 km	6·3	9·4	7·1	2·6	—1·6	4·2	9·7	6·3	—1·0	2·2	0·0	2·6
Cross component		negligible											
σ_T (mph)	2 km	6·6	7·7	7·3	5·9	7·5	8·1	7·0	8·4	8·8	7·7	6·4	6·8
	3 km	7·7	8·6	7·5	7·0	6·8	8·6	6·2	7·0	8·4	7·3	6·8	7·5

TABLE 5

Mean effective tailwinds in mph at 10,000 ft (3 km)

	Dec—Feb			Mar—May			Jun—Aug			Sep—Nov		
	D	*R*	*S.D.*	*D*	*R*	*S.D.*	*D*	*R*	*S.D.*	*D*	*R*	*S.D.*
					Delhi-Calcutta route							
Air Ministry's values	10	—10	10	6	— 8	11	—1	1	11	5	—5	10
Author's values	18	—18	8	18	—18	8	2	—2	7	7	—7	7
					Bombay-Calcutta route							
Air Ministry's values	6	— 8	10	8	—8	11	5	—6	11	4	—4	11
Author's values	13	—13	7	8	—8	7	7	—7	7	1	—1	7

D—Tailwinds for direct flight, *R*—Tailwinds for return flight, *SD*—Standard deviation

were, therefore, not undertaken separately for the afternoons. Therefore, the mean monthly effective tailwinds given in Table 4, can be taken as generally valid, irrespective of the time of the day.

5. Comparison with the Air Ministry values

The values of the mean effective tailwinds along the Delhi—Calcutta and Bombay—Calcutta routes for 10,000 ft presented in the publication of the Air Ministry referred to in the introduction are seasonwise, for the four periods of the year December-February, March-May, June-August and September-November. The monthly data given in this note for 3 km (10,000 ft) have been computed for the same periods of the year and the two sets of values are given in Table 5.

It will be seen that the two sets of values are significantly different during the period December-February for both the routes, and for the period March-May for the Delhi—Calcutta route. The difference is presumably due to the larger amount of data utilised in the present study.

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