Seasonal Oscillations of air temperature in India and neighbourhood*

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ABSTRACT. The annual and the half-yearly oscillations in the mean temperature of air at 4 feet above ground level at 167 meteorological stations in India and neighbourhood have been separated. The dependence of the components of the vectors of the different oscillations on the location of the stations have been determined. Regression equations for representing the components of the oscillations as a linear function of latitude, longitude and elevation have been derived. The fit of these representations has been found to be fairly good, the correlation
between the actual and the calculated values being of the order of 0.8 to 0.9 generally. The significance gradients with respect to the positional co-ordinates have been discussed.

1. Introduction

It is well known that next to the availability of water, the temperature of the air governs the important phytophases of the crop like the germination of seeds etc. Most plants will grow only within narrow ranges of temperature: and for each species and variety there is a minimum below which growth is not possible, an optimum at which growth is most rapid, and a maximum beyond which growth stops. These critical temperatures may vary from crop to crop (Lundegårdh 1931). Also, the seasonal course of air temperature is a characteristic of the location of the station and is a major factor in defining the climate as 'temperate' or 'severe'. 'maritime' or 'continental' etc. A study of the annual course of air temperature is important therefore for such and other purposes. The annual march of air temperature is primarily dependent upon the variation in the amount of heat received from the sun. The daily rate of solar radiation received at any point on earth's surface depends upon the quantity of radiation that emanates from the sun, the duration of the sun's presence above the horizon and the meridian altitude of the sun. There are, however, other factors also, such as, the distribution of land and water, in the neighbourhood of the station, its elevation, the atmospheric and oceanic circulations etc. which disturb to a

marked extent this picture of the annual march of temperature as determined by the sun.

2. Scope of the Study

The march of temperature of the air at any particular station is to a large extent a systematic oscillation repeating practically in a similar manner year after year. In Fig. 1 the variation of the mean temperature(see footnote on p.156) in respect of a few stations are shown graphically. These curves have certain general features common to all and certain distinguishing features peculiar to a few. For example. the temperature is lowest during the winter months and increases during the hot weather season. However, the slight variation in the course of the year at the coastal places like Cochin and Pamban distinguish themselves from the high ranges in the interior stations like Lucknow, Jacobabad etc. The highest mean temperature at places like Dras, Srinagar and Quetta occur in July-August. while at the other stations we have a double wave with the primary maximum occurring May or June and a secondary during maximum about October. The character of the variation in the air temperature during the course of the year, and the relationship between these distinguishing characters and the location of the stations form the subject matter of study in this paper.

Fig. 1. Mean air temperature

For this study, the mean temperature has been taken as $\frac{1}{2}$ (mean daily maximum temperature+mean daily minimum temperature)*. The mean temperatures of air over 167 meteorological stations distributed throughout India and neighbourhood have been used as the material for analysis. The stations selected are the same as those dealt with by the author (1948) for the study of the distribution of mean temperature over India and the diurnal range of temperature over India. The actual list of stations, their latitude, longitude and elevation above sea level as well as the mean temperature during the different months are given in Tables 1 and 2 of the paper cited above. The stations have been grouped into four regions. The specification of the regions and the location of the

centroid in the four regions are given in Table 1+.

3. Graduation of seasonal temperature variation

A suitable mathematical model for studyoscillation of temperature ing the annual appears to be a set of 'harmonics' with the fundamental period as the year. Symbolically, we aim at expressing the temperature T_t at any time of the year t in the form:

$$
T_t = \overline{T} + \sum_{r=1}^{r=n} a_r \cdot \sin\left(\frac{2\pi rt}{P} + \phi_r\right) \qquad (1)
$$

where.

 \overline{T} = mean temperature over the entire period of length P (here the year)

^{*} The temperature of air is that indicated by thermometers installed inside Stevenson Screens at a height of 4 ft above ground level.

As has already been pointed out (Jagannathan 1948), the mean temperature of air cannot be considered completely represented as the mean of the maximum and minimum temperatures. A closer examination of this problem and attempts to represent the mean temperature as a function of the mean of maximum and minimum temperatures as well as the range is in progress and will be discussed separately.

These were not given in the previous paper and are being added here at Miss Carruther's suggestionvide Met. Mag., 79, 934 (Apr 1950)

Region No.	Specification	Position of centroid			
		Latitude (N)	Longitude (E)	Elevation (f _t)	
Ι	42 stations in the western half of the Penin- sula, bounded in the north by the tropic of Cancer and in the east by 78°E meridian	17° 4'	75° 2' ÷	$1461 \cdot 3$	
H	40 stations in the eastern half of the Peninsula, boun- ded in the north by the tropic of Cancer and in the west by 78°E meridian	$18^{\circ} 40'$	83° Ω'	545.6	
Ш	45 stations in the northeast India (including East- ern Pakistan) to the north of the tropic of Can- cer and to the east of 78°E meridian	$25^{\circ} 53'$	85° $34'$	$953 \cdot 0$	
IV	40 stations in the northwest India (including West) Pakistan) to the north of tropic of Cancer and to the west of 78°E meridian	$30^{\circ} 19'$	73° 24'	$2903 - 9$	

TABLE 1

$$
a_r \sin\left(\frac{2\pi rt}{P} + \phi_r\right)
$$
 represents the *r*th har-
monic of period P/r

 a_r =amplitude, *i.e.*, half the range between the maximum and the minimum in the oscillation, and

 ϕ_r =phase angle of the rth harmonic.

The evaluation of the components in the 'harmonics' has been made as follows-

The entire length of the period is marked out by N equal intervals of time and labelled $t=0, 1, 2, \ldots (N-1)$. The observed values of the variate at these N points of time be T_{0} , $T_1, T_2, \ldots, T_{(N-1)}$. We may write equation (1) above in the form

$$
T_t = \overline{T} + \sum_{r} p_r \cos\left(\frac{2\pi rt}{N}\right) + q_r \sin\left(\frac{2\pi rt}{N}\right) (2)
$$

where, $p_r = a_r \sin \phi_r$ and $q_r = a_r \cos \phi_r$

If the values of \mathcal{T}_t at the N points of time $t = 0, 1, \ldots N-1$ are substituted in eq. (2) , we get N equations, which can be solved for the $2r (< N)$ constants by the method of least squares.

For a fuller discussion of the method of analysis reference may be made t_{0}

Pollak (1947), where synthesis schedules for the quick calculation of the harmonic components have also been given.

The maximum in the oscillation of the rth order will occur when

$$
\sin \left[(2\pi r t/N) + \phi_r \right] = 1, \quad (3)
$$

i.e., when $(2\pi rt/N) + \phi_r = \pi/2$ or $5\pi/2$

Similarly the minimum will occur when $\sin [(2\pi rt/N) + \phi_r] = -1$ (4)

For example the date of occurrence of the maximum in the oscillation can be calculated from the phase angle from considerations of the fact that the unit of time is a twelfth of the year $= 30.4375$ days and that the origin of the time axis corresponds to January 16th.

The date of maximum in the annual oscillation is given by $-$

 $D_x = t_x 30.4375 + 16$ days from 1 Jan (5) and those in the half-yearly oscillation by-

 $d_x = t_x 30.4375 + 16$ and

 t_x 30.4375+198.625 days

from $1 Jan(6)$

where t_x are as obtained from eq. (3) above with $r = 1$ and 2 respectively.

Assuming that the mean monthly air temperatures are representative of the middle

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Fig. 2. Amplitude of annual oscillation of temperature (${}^{\circ}F$)

Fig. 3. Date of maximum of annual oscillation of temperature

of the months and that the 12 points of time are equally separated*, the harmonic analysis has been carried out. In Table 2 are given the mean annual temperatures and the amplitudes and phase angles for the annual and half-yearly oscillations, for the 167 stations grouped under the four regions.

4. Annual oscillation

In Figs. 2 and 3 are plotted the amplitudes of the annual oscillation of temperature and the dates of occurrence of the maximum in the annual oscillation and isopleths demarcating different levels are drawn. These figures bring out the following-

- The amplitude is lowest at barely (i) 2-3°F on the West Coast of the Peninsula. $12-15^{\circ}$ F along the Gangetic plains and over 20°F in the extreme northwest. Amplitudes generally increase northwards except in the sub Himalayan regions.
- The lowest values are obtained (i_3) in the coastal regions and the highest in the interior.
- The date of occurrence of the (iii) maximum in the annual oscillation is delayed in the coastal regions as well as towards the north.
- The peak occurs by 1 April over (iv) the west coast (south). By the middle of June, the entire Peninsula except for the east coastal regions have passed the maximum stage. By July first, the plains of North India south of the Ganges have been covered and by 15 July the northern parts of Assam and the foot of the Himalayas have attained their maximum.

5. Relationship with latitude, longitude and elevation

With a view to determine the nature of the dependence of the components of the annual oscillation on the position of the stations, the values of the amplitude and the dates of occurrence† of the maximum in the oscillation have been correlated with the latitude, longitude and elevation of the stations. The correlation coefficients are given in Table 3 (p. 164). The high correlation between the amplitude and the latitude suggests a close relationship between the two. The correlation coefficients with the longitude are small except in the case of Region III. where a steep east-to-west gradient is indicated. Further there does not appear to exist much dependence with elevation except in Regions II and IV, where a slight positive relationship is indicated. In regard to date of maximum, a high correlation with latitude in Regions I and IV, with longitude in all but IV and with elevation in II and IV have been obtained.

Table 3 indicates the relationships between the components of the oscillation with the latitude, longitude and elevation but a proper appreciation of the dependence cannot be made unless the inter-relationship between the independent variates are eliminated. Regression equations connecting the latitude. longitude and elevation with the components of the annual oscillation have been calculated and the regression coefficients, their standard errors and the multiple correlation coefficients are given in Table 4 (p. 164). For this purpose the matrix of multipliers given on page 88 of the previous paper by the author referred to have been utilised. The origin of co-ordinates has been taken at 8°N, 60°E and mean sea level. The units for latitude and longitude are minutes and that for height feet. Amplitudes are in ${}^{\circ}\mathrm{F}$ and D' in months.

^{*} This assumption, it is realised, is not quite correct but the error introduced due to the slightly unequal lengths of the months, will it is believed, not vitiate the results and conclusions arrived at here.

The correlations were actually made with $D'=(D_x-31\cdot 2187)/m$, where D_x is the date of maximum and m is the mean length of the month in days $(=30.4375)$

TABLE 2

Seasonal Oscillation of Temperature

REGION 1

TABLE 2 (contd) REGION II

TABLE 2 (contd)

REGION III

TABLE 2 (contd)

 $\overrightarrow{\text{REGION}}$ IV

TABLE 5 Correlation coefficients between the components

of the half-yearly oscillation of temperature with

latitude, longitude and elevation

Correlation coefficients between the components of the annual oscillation of temperature with latitude, longitude and elevation

TABLE 3

 $d' = (d_x - 23.6094)/15.2187$

TABLE 4

Regression coefficients of components of annual oscillation of temperature on latitude, longitude and elevation

Compo- nent	Region	Partial regression coefficients and standard errors Latitude Longitude			Elevation		Multiple correlation coefficient	
		R.C.	S.E.	R.C.	S.E.	R.C.	S.E.	
		.0108	0009	.0072	.0017	$+00003$	-0001	.90
	П	(0087)	.0011	\rightarrow (10.4.5)	-0013	-00018	-0003	.88
a_1	\rm{III}	0068	.0077	$\rightarrow (00.59)$.0024	-00043	.0013	.82
	1V	.0195	(0026)	-0031	-0023	$\rightarrow 00011$	-0002	.81
		.0019	-0004	-0018	0008	-00003	.0006	75
D'	П	-0001	-11005	-0008	000.5	-00018	.00013	$.36*$
	ПI	(10122)	-0003	-0009	-0001	-00001	.00001	.87
	IV	.0011	-10001	-0004	.0001	.00002	-00001	-88

*Insignificant at the 5 per cent level

The important features brought out by the regression coefficients are-

- (i) Elevation has no effect on the amplitude as well as the date of occurrence of the maximum in the oscillation.
- (ii) The amplitudes increase northwards at the rate of about 7°F for every 10° of latitude. The low insignificant value in Region III can be readily seen to be due to the peculiar nature of the orography in this region.
- (iii) The amplitudes increase eastwards in Region I which comprise of the west coast of the Peninsula, in Regions II and III decrease eastwards and in Region IV there appears to be no dependence with longitude.
- (iv) The date of occurrence of the maximum is delayed in Regions I, III and IV northwards and in the Regions I. II and III eastwards and in Region IV westwards.

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TABLE 6

Partial regression coefficients of the components of the half-yearly oscillation on latitude, longitude and elevation

*Insignificant at the 5 per cent level

6. Half-yearly oscillation

In Figs. 4 and 5 are plotted the amplitudes and the dates of occurrence of the maxima in the half-yearly oscillation. The amplitudes of the half-yearly oscillation are much smaller in magnitude than those of the annual oscillation. The highest values of the order of 5°F are obtained in the central parts of the country between 20° and 27°N latitudes. In the extreme south of the Peninsula they are of the order of 2°F. It may also be seen that the values are smaller near the coasts and the hills. The maxima in the half-yearly waves occur towards the end of April and October in the central parts of the country between 20° and 33°N latitudes and 70° and 80°E longitudes. In the extreme south of the Peninsula they occur by the first week of April and October and in Kashmir early in March and September.

7. Relationship of the components of the half-yearly oscillation with latitude, longitude and elevation

The correlation coefficients of the components of the half-yearly oscillation with latitude, longitude and elevation are given in Table 5.

Partial regression coefficients of the components of the half-yearly oscillation on

 $d' = (dx - 23.6094)/15.2187$

latitude, longitude and elevation, their standard errors and the multiple correlation coefficients are given in Table 6.

Here also as in the annual oscillation, the elevation has no effect on the components except in Region IV, where the amplitude decreases upwards at the rate of 0.2° F per 1000 ft. The amplitudes increase at the rate of about 3°F for every 10° latitude northwards over the Peninsula, while over North India they decrease, the decrease being of the order of 5°F in Region III and 2°F in Region IV. The amplitudes increase towards east in the western half of the country and towards west on the eastern half at the rate of about 2°F. As regards the dates of occurrence of the maximum oscillation the multiple correlation for Region III only is significant, being of the order of .93, where the maxima occur earlier castwards and northwards.

We have so far fitted the harmonics for the annual and the half-yearly oscillations. It would be worthwhile seeing how much of the variation in the actual pattern of the mean temperature curve have been accounted for by these two oscillations.

As the harmonics are orthogonal to each other, the variance accounted by each of the terms directly reduces the variance

Fig. 4. Amplitude of half-yearly oscillation of temperature (°F)

Fig. 5. Date of maximum of half-yearly oscillation of temperature

Figures (without brackets) at the end of lines refer to the date of occurrence of the maximum in the half yearly
oscillation in April and October and those within brackets indicate the dates in March and September

Fig. 6. Annual range of temperature

in the original series and the residual variance at any stage can be calculated by subtracting $\frac{1}{2}$ a² from the residual variance upto that stage. Thus

$$
\sigma^{'2} = \sigma^2 - \frac{1}{2}(a_1^2 + a_2^2 + \ldots) \tag{7}
$$

where σ^2 is the variance of the twelve monthly values subjected to the analysis, a_1, a_2, \ldots are as defined previously the amplitudes of the first, second etc harmonics and σ'^2 is the residual variance. This actually gives a criterion as to whether or not it pays to calculate further terms of the Fourier Series (Brunt 1937). The values of a_1 and a_2 are given in Table 2. The residual variances have been calculated and have been found to be generally of the order of 10 per cent of the variance in the original series, except in a few stray cases, e.g., Sambalpur-50 per cent, Karachi-40

per cent, Delhi-30 per cent, where the variations in the temperature are probably further controlled by higher order harmonics and other sources of variations. It is not proposed to go into these cases in greater detail here.

8. Annual range of temperature

Before closing this discussion, it would be worthwhile examining the distribution of the 'Annual Range of Temperature', i.e., the range between the highest and lowest mean temperatures of the vear. In Fig. 6 are shown the isopleths of Annual Ranges*. The partial regression coefficients of the annual range of temperature ou latitude, longitude and elevation are given in Table 7.

The decrease of the annual range at the rate of about a degree for 1000 ft of elevation over the Eastern Himalayas is

*It should be remembered that the amplitudes mentioned in the previous sections are half the ranges in the particular oscillation

TABLE 7

the important feature brought out here. The range increases at the rate of about 17°F for every 10° of latitude northwards in all regions except III. The east-west gradient is about a degree for every degree of longitude eastwards over the west Peninsula and westwards over the eastern half and slightly less than a degree in Region III.

A reference to the map of the 'Forest Wealth in India' (Lorenzo 1948) reveals a striking resemblance with the pattern of annual ranges of temperature. The concentrated over the 'evergreeps' are Western Ghats and to some extent over the Eastern Ghats, where the annual ranges are not more than 15°F; over the subHimalayan regions and over the Deccan Tableland, with ranges of 20-30°F are associated the 'Deciduous Monsoon types' and over northwest Himalayas, with higher annual ranges, the 'Mixed Himalayan'. The occurrence of the different forest types are of course further linked up with the soil, rainfall and other local factors and fer a detailed study of the plant geography the combined effect of all the factors should be taken into consideration.

letitude longitude and elevation

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REFERENCES

