551.579.1(54)

Representativeness of average rainfall from smaller number of stations over Damodar and Barakar catchments

K. C. MAJUMDAR and M. GANGOPADHYAYA

Regional Meteorological Centre, Calcutta

(*Received 5 April 1954*)

ABSTRACT. Over the Damodar and Barakar catchments the weighted averages of rainfall calculated after Thiessen model is statistically found not to be significantly different from the simple arithmetic averages. The simple average calculated from a randomly chosen smaller number of stations is found to be statistically representative of the average rainfall calculated by using all stations. The confidence limits to the true mean have also been calculated.

1. Introduction

An estimate of the daily average rainfall over the Damodar and Barakar catchments is required by the Damodar Valley Corporation (D.V.C.) authorities for the purposes of their daily reservoir control operations at the two principal flood control dams, namely, Maithon and Panchet Hill. For convenience of work and maintenance of communication systems inter-linking the raingague stations with the headworks control section, the D.V.C. wants the estimate of the average daily rainfall in inches over these two catchments separately based on a much smaller number of raingauge stations.

In this paper an attempt has, therefore, been made to test if the average rainfall over particular catchment worked out from a randomly selected network of smaller number of stations, is representative of the average based on all available stations in the catchment. Stations have been selected by attaching random sampling numbers (published by Indian Statistical Institute) to all the stations in succession and then selecting in usual way (Kendall 1948a). Only that sample has been considered which contained stations within the catchment. The numbers of the stations in the selected networks were chosen practically in the ratio of the areas of the respective catchments, after giving due considerations to the numbers suggested by D.V.C. It has also been tested whether the weighted average calculated after Thiessen (Foster 1949) or the simple arithmetic mean is more preferable. A procedure is also laid down for calculating the confidence limits to the true average rainfall from the date of selected stations, as these might be useful in deciding control operations when reservoir is either full or nearly empty.

Statistical tests have been applied for fifteen individual days during the rainy season months spread over three years covering weak, moderate and active monsoon days and also for five individual months from June to October for one year. In applying these tests daily or monthly rainfall have been assumed to be normally distributed in space. Though the exact form of this distribution of rainfall is not known, yet in such statistical studies assumption of normality will not lead to much error (Mather 1949, Fisher 1948 a).

The missing data were estimated by drawing isohyetal maps for an area comprising of Damodar and Barakar catchments and their surrounding districts by taking into account all D.V.C. and State raingauge stations.

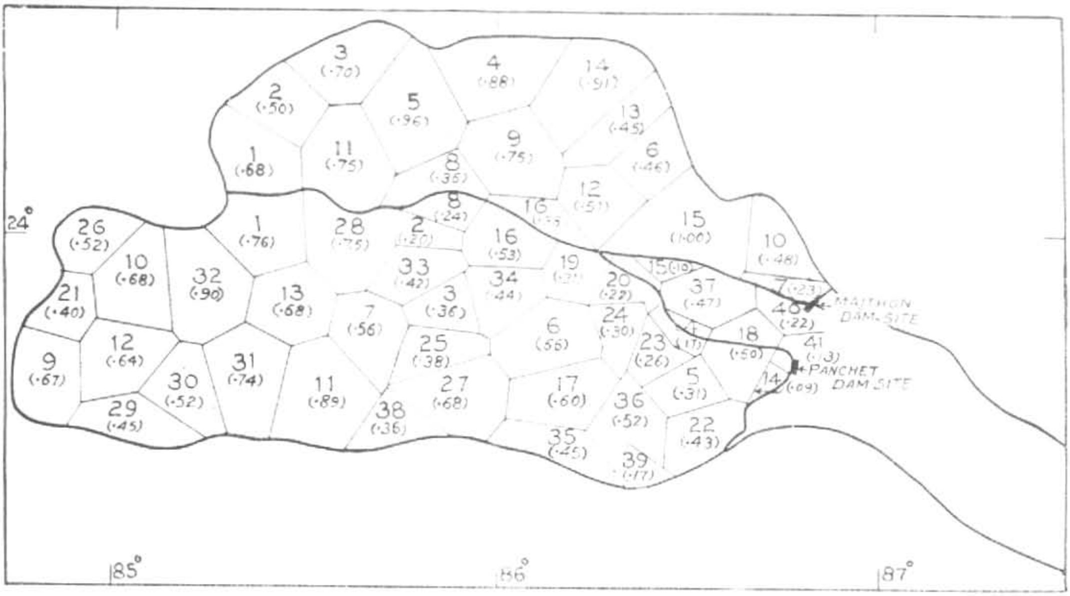

Fig. 1. Map of the catchments showing the raingauge stations and Thiessen construction

The respective weight factors are shown in brackets

Stations in Barakar—(1) Padma, (2) Barhi, (3) Kodarma, (4) Dhanwar, (5) Parasabad, (6) Giridih, (7) Maithon, (8) Bagodar, (9) Tuladih, (10) Jamtara, (11) Barakatha, (12) Barakar, (13) Pachamba, (14) Jamua, (15) Tundi, (16) Dumri.

The first 9 stations constitute the selected network

Stations in Damodar—(1) Hazaribagh, (2) Bishungarh, (3) Bokaro, (4) Dhanbad, (5) Sudamadih, (6) Pupunki, (7) Danea, (8) Bagodar, (9) Chandwa, (10) Tandwa, (11) Ramgarh, (12) Khalari, (13) Mandu, (14) Panchet, (15) Tundi, (16) Dumri, (17) Chas, (18) Nirsa, (19) Topchanchi, (20) Rajdaha, (21) Balumath, (22) Para, (23) Bagmara, (24) Katras, (25) Aiyre, (26) Silaichak, (27) Peterbar, (28) Daroo, (29) Mandar, (30) Burmu, (31) Bhurkunda, (32) Barkagaon, (33) Konar, (34) Nawadih, (35) Jaipur, (36) Chandankiari, (37) Govindpur, (38) Gola, (39) Purulia, (40) Maithon, (41) Kulti

The first 14 stations constitute the selected network

2. Weighted and simple average

Before attacking the main problem it is necessary to investigate whether weighted or the simple arithmetic mean is more preferable. In investigating this the two kinds of averages have been considered by taking all stations available in the two catchments. The Thiessen method has been applied in deriving the weighted average on account of its practical advantages. The distribution of the raingauge stations together with their respective weight factor obtained by Thiessen method for the Damodar and Barakar catchments are shown in Fig. 1. In this study three or four stations just outside the catchments have also been taken into account for obtaining a better network for estimating the true average rainfall.

Let $x_1, x_2, \ldots \ldots x_N$ be the rainfall at N stations and $w_1, w_2, \ldots \ldots w_N$ be the corresponding weight factors. Then the simple and weighted averages are respectively given by:

$$m = \frac{1}{N}(x_1+x_2+\ldots\ldots x_N) = \frac{1}{N}\Sigma x_i \qquad (1)$$

$$\text{and } m_w = \frac{w_1 x_1 + w_2 x_2 + \ldots\ldots w_N\ x_N}{w_1+w_2+\ldots\ldots w_N}$$

$$= \frac{\Sigma\ w_i\ x_i}{N\bar{w}} \qquad (2)$$

$$\text{where } \bar{w} = \frac{1}{N}\ \Sigma\ w_i$$

The daily simple and the weighted averages have been calculated for a number of different occasions and also for five rainy months of a year for the two catchments. The results are given in columns 2 and 3 of Table 1(a). It is seen from this table that the simple average can both be greater or less than the weighted average. Whether the difference between the simple and the weighted averages is at all significant can be inferred by testing whether the difference $(m_w - m)$ is significant or not. But, $m_w - m$ can be shown to be equal to

$$r \times \frac{\sigma_x\ \sigma_w}{\bar{w}}$$

where r is the correlation coefficient between w and x. Therefore testing $m_w - m$ for significance is the same as testing the significance of r. The significance of r is tested by using Student's t by the following relationship—

$$t = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}} \qquad (3)$$

which is a Student's t with N—2 degrees of freedom. The values of r and corresponding t calculated from equation (3) are shown in columns 6 and 7 of Table 1(a). It is seen from this table that at 5 per cent level of significance the values of t have come out to be insignificant in 90 per cent cases in case of Damodar catchment and in 80 per cent cases in case of Barakar catchment, while in the remaining cases it is not so.

Thus in a large number of cases the hypothesis H (ρ=0) is true, so that in these cases there does not exist any significant correlation between the rainfall and the corresponding weights and hence any real differences between weighted and simple averages. Since, however, in the remaining fewer cases this is not true, a combination test (Kendall 1948 b, Rao 1952) of all cases has been made separately for each catchment to see whether on an aggregate the differences $(m_w - m)$ can be considered as insignificant. For conducting such test the exact probability p_k of 't' exceeding the calculated value has been found out and the statistics:

$$\chi^2 = 2\sum_{k=1}^{\nu} \log_e\ p_k \qquad (4)$$

with 2ν degrees of freedom, where ν is the number of cases considered, has been worked out. Since in the present problem 2ν is greater than 30, use is made of another statistic

$$z = \sqrt{2\chi^2} - \sqrt{2\nu-1} \qquad (5)$$

which is a normal deviate with zero mean and unit variance (Fisher 1948). The values of χ^2 and corresponding z are given in Table 1(b). From a comparison with the table of normal distribution it is seen that the values

TABLE 1 (a)

Comparison of simple and weighted average rainfall

Date	Averages rainfall (inches)		Variance of			
	Simple m	Weighted m_w	Simple average $V(m)$	Weighted average $V(m_w)$	r*	t†
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			DAMODAR CATCHMENT			
11-6-49	1·523	1·587	0·021	0·026	+0·148	+0·93
12-6-49	1·408	0·915	0·037	0·044	—0·235	—1·51
25-6-49	0·472	0·510	0·007	0·009	+0·153	+0·97
11-7-49	0·641	0·617	0·008	0·010	—0·089	—0·56
21-7-49	1·354	1·389	0·014	0·017	+0·100	+0·63
16-8-49	1·130	1·108	0·025	0·031	—0·047	—0·29
27-8-49	1·163	1·105	0·019	0·023	—0·141	—0·89
11-6-50	1·767	1·533	0·022	0·027	—0·541	—3·99
10-7-50	0·577	0·487	0·010	0·012	—0·306	—2·00
20-7-50	0·490	0·481	0·008	0·009	—0·035	—0·22
22-8-50	0·303	0·344	0·003	0·004	+0·234	+1·50
30-6-51	2·080	1·982	0·021	0·026	—0·227	—1·45
1-7-51	1·069	0·874	0·020	0·024	—0·470	—3·33
29-7-51	0·933	0·901	0·036	0·044	—0·055	—0·34
17-8-51	1·255	1·297	0·014	0·016	+0·122	+0·77
Jun 52	6·287	6·066	0·069	0·084	—0·284	—1·85
Jul 52	14·199	13·817	0·214	0·260	—0·278	—1·81
Aug 52	11·596	11·836	0·428	0·519	+0·124	+0·78
Sep 52	9·556	9·711	0·202	0·245	+0·116	+0·73
Oct 52	3·290	3·088	0·081	0·098	—0·239	—1·54

* r=Correlation coefficient between rainfall and weights

† For Damodar catchment—D. F. for t = 39; t at 5 % level (Fisher and Yates 1943) = ±2·02; t at 1 % level = ±2·71

TABLE 1(a)—*contd*

Date	Averages rainfall (inches)		Variance of			
	Simple m	Weighted m_w	Simple average $V(m)$	Weighted average $V(m_w)$	r*	t†
(1)	(2)	(3)	(4)	(5)	(6)	(7)
			BARAKAR CATCHMENT			
11-6-49	2·039	1·891	0·159	0·180	—0·255	—·99
25-6-49	0·419	0·368	0·015	0·017	—0·287	—1·12
11-7-49	0·575	0·542	0·027	0·030	—0·139	—0.53
20-7-49	1·442	1·505	0·097	0·110	+0·139	+0 53
22-7-49	1·620	1·520	0·031	0·035	—0·389	—1·58
18-8-49	1·206	1·049	0·061	0·069	—0·436	—1·81
16-9-49	1·260	1·095	0·066	0·075	—0·441	—1·84
17-7-50	0·966	0·964	0·086	0·098	—0·005	—0·02
28-7-50	1·278	1·440	0·042	0·048	+0·543	+2·42
12-8-50	1·027	1·075	0·020	0·023	+0·233	+0·90
30-6-51	2·985	2·890	0·119	0·135	—0·189	—0·72
12-7-51	0·904	0·859	0·065	0·073	—0·122	—0·46
29-7-51	1·941	2·129	0·204	0·231	+0·286	+1·12
17-8-51	0·487	0·443	0·016	0·018	—0·240	—0·93
4-7-52	0·984	1·007	0·036	0·040	+0·084	+0·32
Jun 52	7·261	7·544	0·310	0·351	+0·350	+1·40
Jul 52	12·280	12·795	0·490	0·555	+0·505	+2·19
Aug 52	11·126	11·700	0·490	0·555	+0·564	+2·56
Sep 52	11·850	12·519	0·680	0·770	+0·557	+2·51
Oct 52	2·156	2·275	0·094	0·107	+0·266	+1·03

* r=Correlation coefficient between rainfall and weights

† For Barakar catchment— D.F. for t = 14; t at 5% level (Fisher and Yates 1943) = ± 2·15; t at 1% level=±2·98

TABLE 1 (b)

Combination test

Catchment	$\chi^2 = -\dfrac{2\sum_{k=1}^{\nu}\log_{10} p_k}{\log_{10} e}$	Degrees of freedom (D.F. =2ν)	$z=\sqrt{2\chi^2}-\sqrt{2\nu-1}$	z at 5% level (Fisher 1948b)	z at 1% level (Fisher 1948b)
(1)	(2)	(3)	(4)	(5)	(6)
Damodar	28·45	40	—1·35	±1·96	±2·58
Barakar	58·62	40	+1·94		

of z for both the catchments are insignificant at 5 per cent level of significance. Thus at this usually accepted level of significance, on an aggregate, there exists no correlation between rainfall and the corresponding weights and so no real differences between the weighted and simple averages.

It is, therefore, decided to use in this investigation arithmetic average of daily rainfall instead of weighted average, because the simple average has the advantages of easy calculation.

3. Average rainfall from smaller network

The simple average calculated from the smaller network of selected stations, as shown in column 2 of Table 2 will hereafter be denoted by $\bar{x}$. The problem that next arises is whether the mean rainfall $\bar{x}$ based on the selected stations represents the true mean within the acceptable limits of error. In the absence of exact idea of the hypothetically true mean, the mean m of the complete network of all available stations in each catchment, is taken to represent the true mean. The problem then reduces to testing the hypothesis $H\ (\bar{x} = m)$.

Before testing this hypothesis the variance of the rainfall of selected network has been tested against the variance of the rainfall of the complete network by applying χ^2 test using

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

which has $n-1$ degrees of freedom. The values of variance and χ^2 are shown in columns 4 and 5 of Table 2. It is seen that the values of χ^2 are all insignificant at 5 per cent level of significance, thus showing that the variances of the rainfall of the selected and the complete network of stations are not significantly different. Even then use of the variance from complete network of stations should not be made in testing the hypothesis $H(\bar{x} = m)$ as that would involve an additional assumption that this variance represents the true variance. Hence Student's t test has been applied in testing the hypothesis $H(\bar{x} = m)$, in which the exact knowledge of the true variance is not required.

Student's t has been calculated for each of the occasions considered in the previous section and shown in column 6 of Table 2. It is seen that none of these values of t exceeds the corresponding value of t at 5 per cent level of significance. That is to say, the hypothesis $H\ (\bar{x} = m)$ is true at 5 per cent level of significance for all cases. In other words, $\bar{x}$, the simple average of rainfall from selected stations, can be considered to be equal to the true mean, m, for all practical purposes. Thus mean rainfall of the selected network of stations can be taken within reasonable degree of accuracy to be the repesentative value of the true mean.

4. Confidence limits

As in the absence of the exact value of the true mean confidence limits are of some practical importance, these limits, m_1 and m_2, have been calculated using the relations

$$m_1 = \bar{x} + \frac{s}{\sqrt{n}}\ t_{5\%} \text{ and}$$

$$m_2 = \bar{x} - \frac{s}{\sqrt{n}}\ t_{5\%} \qquad (6)$$

where $t_{5\%}$ is the value of t at the probability level 0·05. The probability level chosen here being 5 per cent the chance of the true mean being less than or equal to m_1, or being greater than or equal to m_2 is 97·5 per cent. The values of m_1 and m_2 for each occasion are shown in columns 7 and 8 of Table 2.

An Example

Various steps of the procedure described above have been illustrated in the example given below :—

Name of catchment: Barakar

Date: 12-8-50

S. No.	Name of station	Thiessen weight w_i	Rainfall x_i
1	Padma	0·66	0·39
2	Barhi	0·52	1·09
3	Kodarma	0·70	1·50
4	Dhanwar	0·88	1·51
5	Parasabad	0·96	1·60*
6	Giridih	0·46	0·40
7	Maithon	0·23	0
8	Bagodar	0·36	1·57
9	Tuladih	0·75	1·00
10	Jamtara	0·48	0·37
11	Barakatha	0·75	1·35
12	Barakar	0·51	1·18
13	Pachamba	0·45	0·62
14	Jamua	0·91	1·30
15	Tundi	1·00	0·47
16	Dumri	0·38	2·08

* Estimated from isoheytal map

Total number of stations $N = 16$

Number of selected stations $n = 9$ (first nine stations of above list)

Averages

$$m_w = 1 \cdot 075'', \; m = 1 \cdot 027''$$

$$\bar{x} = 1 \cdot 007'', \; \bar{w} = 0 \cdot 625$$

Corrected sum of squares

All stations : 5·1530,
Selected stations : 2·9288

Variances (all stations)

Individual rainfall : $\sigma^2 = 0 \cdot 322$

Simple average : $V(m) = 0 \cdot 020$,
Weighted average : $V(m_w) = 0 \cdot 023$

Sample estimate of variance from selected stations : $s^2 = 0 \cdot 366$

Correlation coefficient between rainfall and weights

$r = 0 \cdot 233$

$t = 0 \cdot 90$ with 14 D.F., which is not significant at 5% level of significance

$p_k =$ Prob. $\{ t \geqslant 0 \cdot 90 \} = 0 \cdot 192$ (for combination test)

Chi-square for testing variances of selected stations

$\chi^2 = 9 \cdot 09$ with 8 D.F., which is not significant at 5% level of significance

Testing of significance of average based on selected stations

$t = -0 \cdot 10$ with 8 D.F. which is not significant at 5 % level of significance

Confidence limits

$t_{5\%}$ (8 D. F.) $= \pm 2 \cdot 306$
Upper confidence limit = 1·472
Lower confidence limit = 0·542

5. Conclusion

The difference between the weighted averages calculated by Thiessen method and the simple arithmetic averages has been statistically found on an aggregate to be insignificant. The simple average has, therefore, been considered for further study in this paper, as it has the advantage of easy calculation.

The simple averages obtained from the randomly selected network of 14 stations in the Damodar catchment and 9 stations in the Barakar catchment, as indicated in Fig. 1, are all found to be statistically representative of the true averages. In each case the variability of the rainfall of the selected stations is found not to be significantly different from the variability of the rainfall of the complete network. Confidence limits to the true averages with 97·5 per cent degree of confidence have also been

TABLE 2

Simple average rainfall in inches for selected stations, its test of significance and confidence limits

Date	Simple average	Corrected sum of squares	Variance	χ^2*	t*	Confidence limits Lower	Confidence limits Upper
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			DAMODAR CATCHMENT				
11-6-49	1·523 *1·631*	35·85 *6·91*	0·874 *0·531*	7·90	+0·55	1·210	2·052
12-6-49	1·048 *0·838*	61·37 *14·44*	1·497 *1·111*	9·65	−0·75	0·230	1·446
25-6-49	0·472 *0·686*	11·86 *5·70*	0·289 *0·438*	19·69	+1·23	0·304	1·068
11-7-49	0·641 *0·596*	13·94 *4·99*	0·340 *0·383*	14·66	−0·27	0·239	0·953
21-7-49	1·354 *1·454*	23·63 *7·05*	0·576 *0·542*	12·23	+0·51	1·029	1·879
16-8-49	1·130 *1·186*	42·51 *18·75*	1·037 *1·442*	18·08	+0·19	0·493	1·879
27-8-49	1·163 *1·211*	32·47 *8·89*	0·792 *0·684*	11·23	+0·22	0·734	1·688
11-6-50	1·767 *1·491*	37·04 *7·09*	0·903 *0·545*	7·85	−1·40	1·065	1·917
10-7-50	0·577 *0·541*	16·54 *9·00*	0·403 *0·692*	22·31	−0·16	0·061	1·021
20-7-50	0·490 *0·261*	12·72 *2·57*	0·310 *0·198*	8·29	−1·93	0·004	0·518
22-8-50	0·303 *0·286*	5·88 *1·92*	0·143 *0·148*	13·39	−0·17	0·064	0·508
30-6-51	2·080 *2·281*	35·76 *12·50*	0·872 *0·962*	14·34	+0·77	1·715	2·847
1-7-51	1·069 *1·088*	32·87 *11·50*	0·802 *0·885*	14·35	+0·08	0·545	1·631
29-7-51	0·933 *1·334*	60·42 *25·10*	1·474 *1·931*	17·03	+1·08	0·532	2·136
17-8-51	1·255 *1·519*	22·81 *7·91*	0·556 *0·609*	14·22	+1·26	1·068	1·970
Jun 52	6·287 *6·913*	115·99 *28·84*	2·829 *2·218*	10·19	+1·57	6·053	7·773
Jul 52	14·199 *13·236*	360·33 *108·72*	8·788 *8·363*	12·37	−1·27	11·567	14·905
Aug 52	11·596 *12·114*	718·93 *376·59*	17·535 *28·968*	21·48	+0·36	9·007	15·221
Sep 52	9·556 *10·276*	338·99 *104·58*	8·268 *8·015*	12·65	+0·95	8·639	11·913
Oct 52	3·290 *3·444*	136·30 *67·29*	3·324 *5·176*	20·24	+0·30	2·131	4·757

*For Damodar catchment—D.F. for both t and χ^2 = 13 ; t at 5% level (Fisher and Yates 1943) = ±2·160 ; t at 1% level = ±3·012; χ^2 at 5% level (Fisher and Yates 1943) = 22·36 ; χ^2 at 1% level = 27·69

Figures in first row of columns 2, 3 and 4 against each date refer to all raingauge stations, while figures in second row to selected stations

TABLE 2 *(contd)*

Date	Simple average	Corrected sum of squares	Variance	χ^2*	t*	Confidence limits	
						Lower	Upper
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			BARAKAR CATCHMENT				
11-6-49	2·039 *2·183*	40·74 *15·14*	2·546 *1·892*	5·95	+0·31	1·126	3·240
25-6-49	0·419 *0·560*	3·82 *2·92*	0·239 *0·365*	8·76	+0·70	0·96	1·024
11-7-49	0·575 *0·487*	6·78 *3·65*	0·424 *0·456*	8·60	—0·39	0·0	1·006
20-7-49	1·442 *1·204*	24·93 *15·20*	1·558 *1·900*	9·75	—0·52	0·144	2·264
22-7-49	1·620 *1·469*	7·99 *4·22*	0·499 *0·527*	8·44	—0·62	0·911	2·027
18-8-49	1·206 *1·328*	15·70 *11·27*	0·982 *1·409*	11·49	+0·33	0·416	2·240
16-9-49	1·260 *1·106*	16·94 *5·78*	1·059 *0·722*	5·46	—0·54	0·453	1·759
17-7-50	0·966 *0·801*	22·06 *11·77*	1·379 *1·471*	8·53	—0·41	0·0	1·733
28-7-50	1·278 *1·428*	10·79 *6·14*	0·674 *0·768*	9·11	+0·51	0·754	2·102
12-8-50	1·027 *1·007*	5·15 *2·93*	0·322 *0·366*	9·09	—0·10	0·542	1·472
30-6-51	2·985 *3·052*	30·55 *20·22*	1·909 *2·528*	10·59	—0·13	1·830	4·274
12-7-51	0·904 *0·836*	16·55 *5·53*	1·034 *0·692*	5·35	—0·25	0·197	1·475
29-7-51	1·941 *1·667*	52·12 *13·71*	3·258 *1·714*	11·88	—0·59	1·029	2·305
17-8-51	0·487 *0·530*	4·07 *2·82*	0·254 *0·353*	11·09	+0·22	0·073	0·987
4-7-52	0·984 *0·966*	9·11 *6·60*	0·569 *0·824*	11·59	—0·06	0·268	1·664
Jun 52	7·261 *7·376*	79·24 *41·85*	4·953 *5·232*	8·45	+0·15	5·618	9·134
Jul 52	12·280 *12·212*	125·56 *113·24*	7·847 *14·155*	14·43	—0·05	9·320	15·104
Aug 52	11·126 *11·564*	125·42 *75·55*	7·839 *9·443*	9·64	+0·43	9·202	·13·926
Sep 52	11·850 *12·073*	174·16 *120·74*	10·885 *15·092*	11·09	—0·17	9·087	15·059
Oct 52	2·156 *1·927*	24·18 *8·45*	1·511 *1·056*	5·59	—0·67	1·137	2·717

*For Barakar catchment—D.F. for both t and χ^2 is 8 ; t at 5% level $= \pm 2{\cdot}306$;
t at 1% level $= \pm 3{\cdot}355$; χ^2 at 5% level $= 15{\cdot}51$; χ^2 at 1% level $= 20{\cdot}09$

Figures in first row of columns 2, 3 and 4 against each date refer to all raingauge stations, while figures in second row to selected stations calculated for each case. The idea of confidence limit may be useful for reservoir operations, when the reservoir level is critical, that is to say towards the end of monsoon when the reservoir is full, the reservoir operating unit may like to consider the upper confidence limit of average for safety while in the dry season, it may like to have an idea of the lower limit.

For the purposes of daily reservoir operations of the two flood control dams it will thus be enough to estimate the average rainfall separately for each catchment, Damodar and Barakar, from the network of selected stations. Since the number of stations has been cut down greatly it is obvious that the network and method suggested will not only minimise labour but will also reduce permanent recurring organisational expenditure. The confidence limits to the true average in each individual case may also be calculated, if necessary, strictly in accordance with the procedure shown in the example. It should, however, be pointed out that the data from all the above mentioned selected stations in the respective catchments must be taken into account in calculating the average rainfall and the confidence limits for each of the catchments.

REFERENCES

Fisher, R. A.	1948 (a)	*Statistical Methods for Research Workers*, p. 114.
	1948 (b)	*Ibid.*, p. 81.
Fisher, R. A. and Yates, F.	1943	*Statistical Tables for Biological, Agricultural and Medical Research.*
Foster, E. E.	1949	*Rainfall and Runoff*, p. 153.
Kendall, M. G.	1948 (a)	*The Advanced Theory of Statistics*, **1**, Chap. 8, or any book of random sampling numbers.
	1948 (b)	*Ibid.*, **2**, p. 132.
Mather, K.	1949	*Statistical Analysis in Biology*, p. 57.
Rao, C. R.	1952	*Advanced Statistical Method in Biometric Research*, p. 44.
Yule, G. U. and Kendall, M. G.	1953	*An Introduction to the Theory of Statistics*, p. 334.