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A mathematical model for fluxes associated with internal gravity waves excited by a corner mountain

PRASANTA DAS and SOMENATH DUTTA*

Department of Mathematics, Ramananda College, Bishnupur, Bankura – 722 122, West Bengal, India

**India Meteorological Department (IMD), MoES, Pune – 411 005, India*

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e mails : pdas.math1986@gmail.com

सार – इस मॉडल में एक बैरोक्लिनिक माध्य प्रवाह में कॉर्नर माउंटेन हिल्स (CMH) द्वारा उत्तेजित आंतरिक गुरुत्वाकर्षण तरंगों (IGW) से जुड़े ऊर्ध्वाधर ऊर्जा प्रवाह (E_z) और गति प्रवाह (τ_{zx} और τ_{zy}) के क्षैतिज घटकों को मानकीकृत करने का प्रयास किया गया है। भारत के उत्तर-पूर्वी क्षेत्र में, खासी-जयंतिया हिल्स (KJH) मोटे तौर पर पूर्व-पश्चिम उन्मुख हैं जबकि असम-बर्मा हिल्स (ABH) मोटे तौर पर उत्तर-दक्षिण उन्मुख हैं और वे लगभग समकोण पर मिलते हैं जिससे एक CMH बनता है। इस अध्ययन में, उछाल आवृत्ति (N) के यथार्थवादी ऊर्ध्वाधर भिन्नता और मूल प्रवाह के दो घटकों यू, वी पर विचार किया गया है। सर्दी के मौसम के दौरान दो चुनिंदा मामले हैं और बरसात के मौसम पर चर्चा की गई है। दो मामलों का अध्ययन किया गया है और सभी मामलों में विभिन्न स्तरों पर ऊर्जा प्रवाह और गति प्रवाह की गणना की गई है और पहले के जांचकर्ताओं द्वारा प्राप्त परिणामों के साथ तुलना की गई है।

ABSTRACT. In this model an attempt has been made to parameterize the vertical energy flux (E_z) and the horizontal components of momentum flux (τ_{zx} and τ_{zy}) associated with internal gravity waves (IGW) excited by the Corner Mountain Hills (CMH) in a baroclinic mean flow. In the north-eastern region of India, the Khasi-Jayantia Hills (KJH) is broadly east-west oriented whereas the Assam-Burma Hills (ABH) is broadly north-south oriented and they meet approximately at right angle forming a CMH. In this study, realistic vertical variation of the Buoyancy frequency (N) and two components U, V of the basic flow have been considered. There are two selective cases during winter season and rainy season have been discussed. Two cases have been studied and in all cases the energy flux and momentum fluxes at different levels have been computed and compared with the results obtained by earlier investigators.

Key words – Corner mountain hills, Energy flux, Momentum flux.

1. Introduction

It is a well-established fact that, when a stably stratified air-stream flows across an orographic barrier, gravity waves are excited to propagate upward direction under certain conditions of thermal stability and airflow stratification. Now, these gravity waves are known as internal gravity waves (IGW). These IGW can propagate vertically to a great distance carrying energy and momentum to higher levels in the atmosphere. Sometimes, they are associated with the formation of clear air turbulence (CAT). The information about standing waves, which under favourable meteorological conditions form on the lee side of the mountain barrier, is very important for the safety of aviation. Many aircraft accidents reported in mountainous areas are often attributed to the vertical velocities of large magnitude associated with the lee waves. Hence the studies on lee waves associated with air flow across an orographic barrier have an important bearing on the safety of aviation.

Sawyer (1959) first pointed out the relative importance of this momentum loss in the mean flow due to continuous extraction of momentum by the orographic gravity waves. He considered a 2-D bell-shaped obstacle with half with (a = 2 km) and height (b = 300 m) and determined the typical value of wave momentum flux of the order 1-10 dynes/cm^2. Eliassen and Palm (1961) showed that in case of 2-D linear gravity waves, the vertical flux of horizontal momentum due to gravity waves is independent of height when the waves are steady and non-dissipative. Blumen (1965) showed that the magnitude of the waves drag is sensible to the vertical wavelength. He also noted that the maximum value of the drag is attained when the vertical wavelength is twice the maximum height of the momentum. Bertherton (1969) reviewed the theories concerning the propagation of internal gravity waves (IGW) in a horizontal uniform shear flow. His computations showed that for a 19 m/s gradient wind over hilly terrain in north Welsh, the wave drag amounted to 4 dyne/cm^2 of which 3 dyne/cm^2

probably acted on the atmosphere above 20 km. Lilly (1972) reported that a conclusive evidence of the importance of the wave drag obtained from the data collected over the Front Range of the Colorado Rockies by instrumented aircraft during field experiments.

Smith (1978) computed the pressure drag on the Blue-ridge Mountain in the Central Appalachains. During the first two weeks of January 1974, he observed several periods with significant wave drag with pressure differences typically of the order of 50 N/m^2 across the ridge. Blumen and Dietze (1981) considered a 3-D linear hydrostatic model of stationary mountain wave in a stably stratified air-stream. They took both the incoming basic flow buoyancy frequency to be independent of height, but lateral variation of incoming flow was incorporated by assuming a hyperbolic secant profile (U = *sechy*). Somieski (1981) considered a 3-D circular mountain for the stratified hydrostatic flow. He derived a second order wave equation from the primitive equation including constant rotation and vertical shear of the mean flow. He solved the equation numerically and showed that in case of non-shear and constant static stability, the nodal lines are parabolic for circular mountain of diameter 50 km. Palmer *et al*. (1986) addressed the general westerly bias of the global general circulation models (GCM). They showed that one way to reduce this general westerly bias is to incorporate the gravity wave drag parameterization scheme in the GCM. Gravity waves drag parameterization scheme proposed by Palmer *et al*. (1986) and McFarlane (1987) reduced the westerly bias mainly in stratosphere. Iwasaki *et al*. (1989) addressed a new type of gravity wave drag parameterization scheme to improve the tropospheric westerly bias by including the effects of these tropospheric-trapped lee waves.

Dutta (2001) studied the momentum and energy fluxes associated mountain wave across Mumbai-Pune section of the Western Ghats in an idealized air-stream. He showed that the momentum and energy flux both were independent of height and half width of the bell-shaped part of the barrier. Dutta and Naresh Kumar (2005) considered a 2-D mathematical model for an idealistic air-stream across the Assam-Burma Hills (ABH). They showed that fluxes were independent of height and also dependent the length of valley of ABH. Dutta (2007) addressed a linear dynamical model for air flow across the Western Ghats and Khasi-Jayantia Hills in a realistic air-stream. He also showed that the variation of momentum and energy flux both were not uniform with height. Das *et al.* (2016) developed a 3-D mathematical model for parameterization of momentum and energy flux for an realistic air flow across the Assam-Burma Hills (ABH). They showed that both momentum and energy fluxes vary in the vertical.

Das *et al.* (2017) used a mathematical model for the 3-D dynamics of lee wave across a meso-scale mountain corner. They studied the relation between the possible transverse and divergent lee wave numbers (k, l) and also discussed and mapped the updraft/downdraft regions associated with lee waves at different heights.

In some of the above studies, wind and stability were assumed to be either invariant with height or assumed to have some analytical behaviour with height. Solutions for such studies were essentially obtained by analytical method. In other studies, realistic vertical variation of wind and stability were considered and the solution obtained using quasi numerical or numerical method. In all the studies the barrier (2-D) or the major ridge axis (3-D) of the barrier has been assumed to be extended broadly either in the East-West (EW) direction or in North-South (NS) direction.

In India in the northeast region, the Khasi-Jayantia Hills (KJH) is broadly East-West oriented whereas the Assam-Burma Hills (ABH) is broadly North-South oriented and they meet at almost right angle forming a mountain corner to the northeast. It is believed that weather and climate in that region are neither controlled by KJH alone nor it is controlled by ABH alone, rather they may be controlled by their combined effect. To address the problem of this combined effect, one has to investigate the effect of the above mountain corner on airflow and rainfall in that region.

The objective of the present study is to propose a 3-D dynamical model for parameterizing energy flux and momentum flux associated with IGW across Corner Mountain Hills, which has not been addressed so far.

2. Data

Guwahati (26.19° N Latitude and 91.73° E Longitude) is the only Radio-Sonde station to the upstream of ABH. Accordingly, for the present study we have used the average of 0000 UTC and 1200 UTC RS/RW data of Guwahati for 8th January, 1967 and 18th July, 2004, have been obtained from Archive of India Meteorological Department (IMD), Pune, India.

3. Methodology

The mathematical model has been used in this study for lee wave across a meso-scale mountain corner. The proposed model considers a steady state, adiabatic, inviscid, non-rotating and Boussines q mean flow across a meso-scale mountain corner. The realistic vertical variation of mean flow has been considered here. The basic flow consists of two components U and V along x

and y axis respectively. Under these assumptions the linearized governing equations are simplified to:

$$U\frac{\partial u'}{\partial x}+V\frac{\partial u'}{\partial y}+w'\frac{dU}{dz}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial x} \tag{1}$$

$$U\frac{\partial v'}{\partial x}+V\frac{\partial v'}{\partial y}+w'\frac{dU}{dz}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial y} \tag{2}$$

$$U\frac{\partial w'}{\partial x}+V\frac{\partial w'}{\partial y}=-\frac{1}{\rho_0}\frac{\partial p'}{\partial z}+g\frac{\theta'}{\theta_0} \tag{3}$$

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \tag{4}$$

$$U\frac{\partial \theta'}{\partial x}+V\frac{\partial \theta'}{\partial y}+w'\frac{d\theta_0}{dz}=0 \tag{5}$$

where, U, V, ρ_0, θ_0 are respectively zonal component, meridional component, density and potential temperature of basic flow and u', v', w', p', θ' are respectively the perturbation part of zonal wind, meridional wind, vertical wind, pressure, density and potential temperature. Since the perturbation quantities, u', v', w', p', θ' etc are all continuous functions of x, y, z hence their horizontal variation may be represented by a double Fourier integral, such as,

$$u'(x,y,z)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{u}(k,l,z)e^{i(kx+ly)}dkdl$$

where, $\hat{u}(k,l,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u'(x,y,z)e^{-i(kx+ly)}dxdy$ is the double Fourier transform of u' (x, y, z). Performing double Fourier transform from (1) to (5) we get

$$i(kU+lV)\hat{u}+\hat{w}\frac{dU}{dz}=-ik\frac{\hat{p}}{\rho_0} \tag{6}$$

$$i(kU+lV)\hat{v}+\hat{w}\frac{dV}{dz}=-il\frac{\hat{p}}{\rho_0} \tag{7}$$

$$i(kU+lV)\hat{w}=-\frac{1}{\rho_0}\frac{\partial\hat{p}}{\partial z}+g\frac{\hat{\theta}}{\theta_0} \tag{8}$$

$$ik\hat{u}+il\hat{v}+\frac{\partial\hat{w}}{\partial z}=0 \tag{9}$$

$$i(kU+lV)\hat{\theta}+\hat{w}\frac{d\theta_0}{dz}=0 \tag{10}$$

where, $\hat{u},\hat{v},\hat{w},\hat{p},\hat{\theta}$ are double Fourier transformation of u',v',w',p',θ.

Eliminating $\hat{u},\hat{v},\hat{p},\hat{\theta}$ from the equations (6) to (10) we get

$$\frac{\partial^2\hat{w}}{\partial z^2}+\frac{1}{\rho_0}\frac{d\rho_0}{dz}\frac{\partial\hat{w}}{\partial z}+\left\{\frac{N^2\left(k^2+l^2\right)}{\left(kU+lV\right)^2}-\left(\frac{k\dfrac{dU}{dz}+l\dfrac{dV}{dz}}{kU+lV}\right)\frac{1}{\rho_0}\frac{d\rho_0}{dz}-\left(\frac{k\dfrac{d^2U}{dz^2}+l\dfrac{d^2V}{dz^2}}{kU+lV}\right)-\left(k^2+l^2\right)\right\}\hat{w}=0 \tag{11}$$

where, $N=\sqrt{\dfrac{g}{\theta_0}\dfrac{d\theta_0}{dz}}$ is the Brunt-Vaisala frequency.

Now putting $\hat{w}(k,l,z)=\sqrt{\dfrac{\rho_0(0)}{\rho_0(z)}}\hat{w}_1(k,l,z)$ in equation (11), we get vertical structure equation

$$\frac{\partial^2\hat{w}_1}{\partial z^2}+\left[f(k,l,z)-K^2\right]\hat{w}_1=0 \tag{12}$$

where,

$$f(k,l,z)=\frac{N^2\left(k^2+l^2\right)}{(kU+lV)^2}-\left[\frac{k\dfrac{dU}{dz}+l\dfrac{dV}{dz}}{kU+lV}\right]\frac{1}{\rho_0}\frac{d\rho_0}{dz}-\left[\frac{k\dfrac{d^2U}{dz^2}+l\dfrac{d^2V}{dz^2}}{kU+lV}\right]+\frac{1}{4\rho_0^2}\left(\frac{d\rho_0}{dz}\right)^2-\frac{1}{2\rho_0}\frac{d^2\rho_0}{dz^2}$$

and $K^2=k^2+l^2$

It is very complicated to solve the equation (12) analytically. So, the equation (12) is solved quasi-numerically for the given wave number vectors (k, l) of all vertical levels. The direction of the zonal wind changes

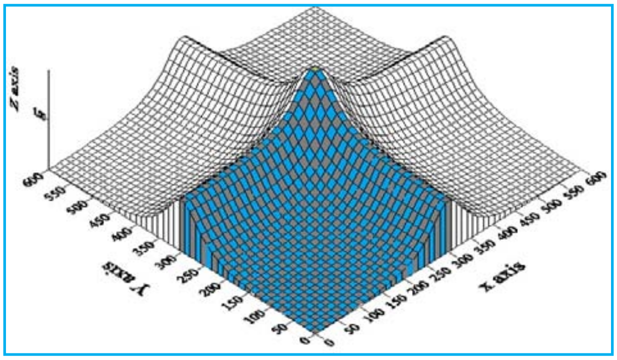


Fig. 1. The profile of Corner Mountain Hills

from north to south during winter season at all levels (De, 1973). The solution of (12) is strictly indeterminate unless the values of $f(k, l, z)$ is specified at great height. Therefore it is assumed that above the upper boundary $f(k, l, z)$ is constant. For simplicity, it is also assumed that above the upper boundary $f(k, l, z) = 0$, which is similar to Sarker (1967), Dutta (2005, 2007), Das *et al*. (2016) etc.

Therefore the approximate solution of the equation (12) in the region $f(k, l, z) = 0$ is of the form

$$\hat{w}_1(k,l,z) = Ce^{-Kz} \tag{13}$$

where, 'C' is an arbitrary constant. Since the pressure and vertical velocity are continuous function of z. So, $\hat{w}_1, \frac{\partial^2 \hat{w}_1}{\partial z^2}$ are also continuous function of z in the region $f(k, l, z) = 0$. Hence

$$\frac{\partial \hat{w}_1}{\partial z} = -K\hat{w}_1 \tag{14}$$

here, the equations (13) and (14) are the upper boundary condition of the equation (12). Now at the surface the airflow follows the contour of the corner mountain, the profile is given by :

$$h(x,y) = \frac{H}{2}\left[\frac{a^2}{a^2+(x-x_0)^2} + \frac{b^2}{b^2+(y-y_0)^2}\right] \tag{15}$$

Profile of (15) is given by Fig. 1**.** The chess colour of this figure is the corner mountain, which we are interested to study. Now the double Fourier transform of (15) is given by :

$$\hat{h}(k,l) = \frac{iH}{2}\left[\frac{a}{l}e^{-ak-ikx_0} + \frac{b}{k}e^{-bk-ily_0}\right] \tag{16}$$

Now the linearized lower boundary condition (at $z = 0$) for w_1' is given by

$$w'_1(x,y,0) = w'(x,y,0) = U(0)\frac{\partial h(x,y,0)}{\partial x} + V(0)\frac{\partial h(x,y,0)}{\partial y}$$

Therefore,

$$\hat{w}_1(k,l,z) = i[kU(0) + lV(0)]\hat{h}(k,l) \tag{17}$$

Using the above boundary conditions and following Das *et al.* (2016) the equation (12) has been solved quasi-numerically. Therefore the solution for $\hat{w}_1(k,l,z)$ is given by :

$$\hat{w}_1(k,l,z) = i[kU(0) + lV(0)]\hat{h}(k,l)\frac{\Psi(k,l,z)}{\Psi(k,l,0)} \tag{18}$$

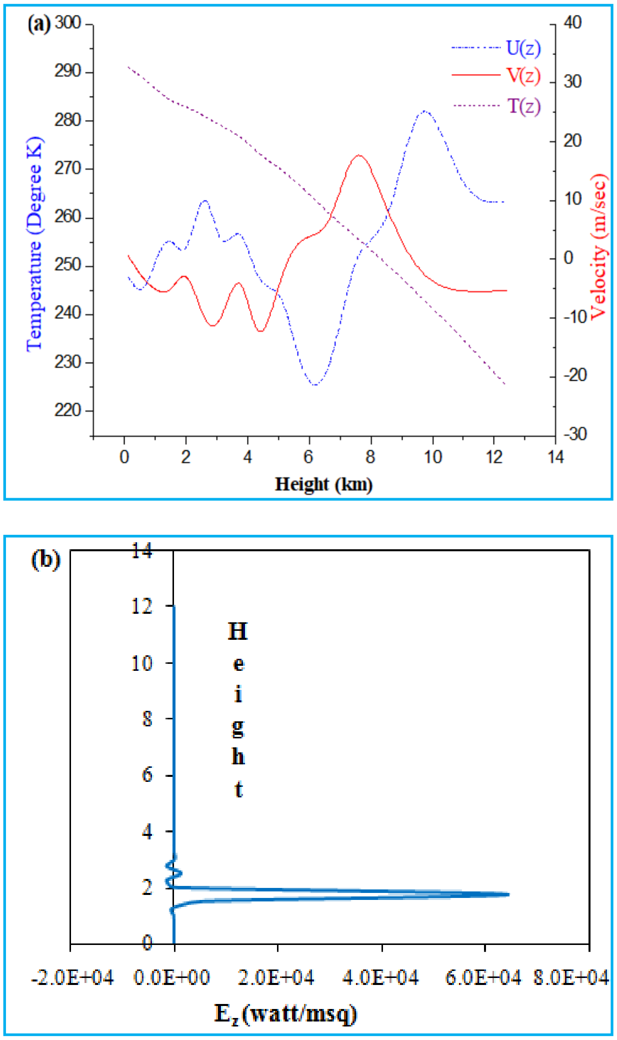
Hence,

$$\hat{w}(k,l,z) = \sqrt{\frac{\rho_0(0)}{\rho_0(z)}}\, i\left[kU(0) + lV(0)\right]\hat{h}(k,l)\frac{\Psi(k,l,z)}{\Psi(k,l,0)} \tag{19}$$

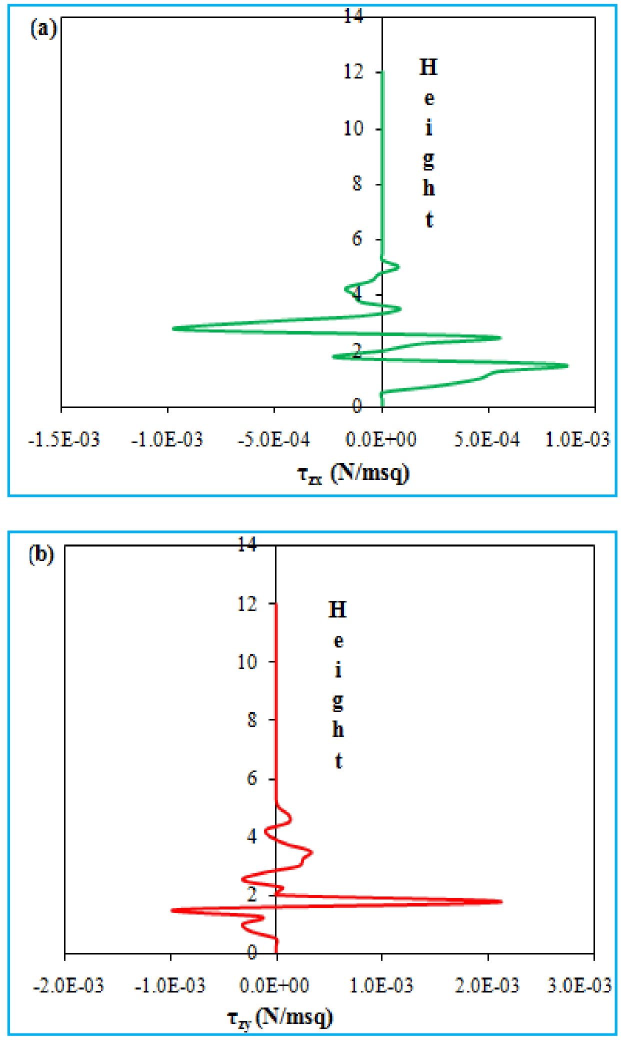
where, $\Psi(k,l,z)$ is an arbitrary function, which satisfying equations (12), (13) and (14) and its value at above the upper boundary is 1. Following Das *et al.* (2016) $\Psi(k,l,z)$ has been computed numerically at different vertical levels and different vertical grid points, at intervals of d = 0.25 km, for a given wave number vector (k, l). Now we obtain $\hat{p}, \hat{u}$ and $\hat{v}$ from equations (6), (7), (8) and (10) by following Das *et al*. (2016).

$$\hat{p}(k,l,z) = \frac{i\left[\left(k\frac{dU}{dz} + l\frac{dV}{dz}\right)\hat{w}(k,l,z) - (kU + lV)\frac{\partial \hat{w}}{\partial z}\right]\rho_0(z)}{K^2} \tag{20}$$

$$\hat{u}(k,l,z) = \frac{i\left[\hat{w}(k,l,z)\frac{dU}{dz} + \frac{k}{K^2}\left\{\left(k\frac{dU}{dz} + l\frac{dV}{dz}\right)\hat{w}(k,l,z) - (kU + lV)\frac{\partial \hat{w}}{\partial z}\right\}\right]}{(kU + lV)} \tag{21}$$



Figs. 2(a&b). (a) Vertical profile of $U(z)$, $V(z)$, $T(z)$ and (b) Vertical profile of energy flux, on 8th January, 1967



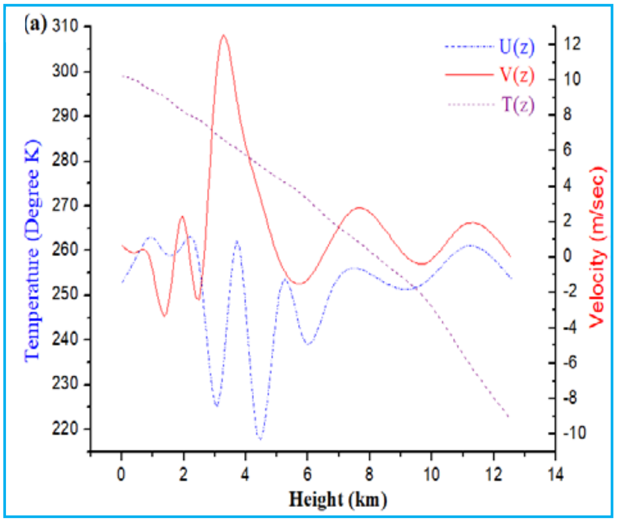
Figs. 3(a&b). (a) Vertical profile of momentum flux (τ_{zx}) along x-axis and (b) Vertical profile of momentum flux (τ_{zy}) along y-axis on 8th January, 1967

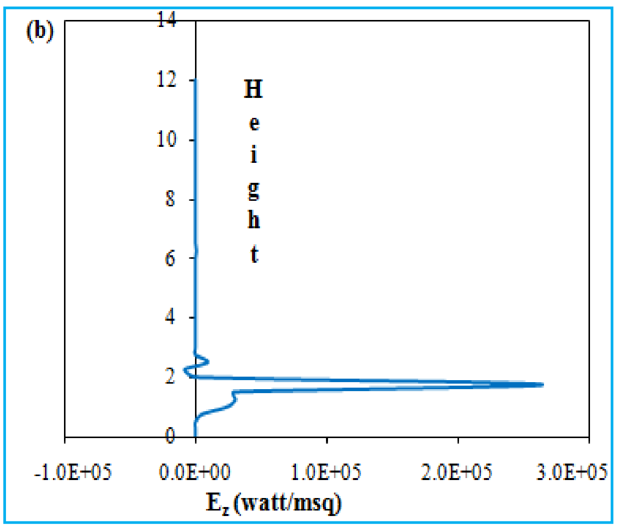
$$\hat{v}(k,l,z) = \frac{i\left[\hat{w}(k,l,z)\dfrac{dV}{dz} + \dfrac{k}{K^2}\left\{\begin{array}{r}\left(k\dfrac{dU}{dz} + l\dfrac{dV}{dz}\right)\hat{w}(k,l,z) \\ -(kU+lV)\dfrac{\partial \hat{w}}{\partial z}\end{array}\right\}\right]}{(kU+lV)} \tag{22}$$

Using Eqn. (19), $\hat{w}$ can be found out at each level for a given wave number vector (k, l) and also using Eqn. (19) and vertical profile of basic state wind and temperature field, $\hat{p}, \hat{u}, \hat{v}$ can be found out. By performing inverse double Fourier transformation on equations (19)-(22) numerically, we obtain w', p', u', v' at each horizontal grid point (5 km apart) at each vertical level. Following Das *et al*. (2016), then the two horizontal components of the momentum flux vector, *viz*., $\tau_{zx} = \overline{\overline{\hat{u}\hat{w}}}$, $\tau_{zy} = \overline{\overline{\hat{v}\hat{w}}}$ and energy flux $E_z = \overline{\overline{\hat{p}\hat{w}}}$ at any vertical level and any horizontal grid point have been computed, where $\overline{\overline{()}}$ indicates the average surface area 'S' of the orographic barrier, where $S = \dfrac{H}{2}\left[y_0 \tan^{-1}\left(\dfrac{x_0}{a}\right) + x_0 \tan^{-1}\left(\dfrac{y_0}{b}\right)\right]$.

4. Results and discussion

Using the equations (19)-(22) we have computed the energy flux and the components of horizontal flux for two selected cases when air stream characteristics where favourable for occurrence of the mountain wave.





Figs. 4(a&b). (a) Vertical profile of $U(z)$, $V(z)$, $T(z)$ and (b) Vertical profile of energy flux (E_z), on 18th July, 2004

Computations are made allowing realistic vertical variation of wind and static stability.

Case 1 : *8th January, 1967 (Winter season)*

The vertical profiles of two components of basic flow $U(z)$, $V(z)$ and temperature $T(z)$ in the undisturbed flow shown in Fig. 2(a), which are based on the average of 0000 UTC and 1200 UTC RS/RW data of Guwahati for that date during winter season. This figure shows that the profile of temperature $T(z)$ is constant lapse rate with vertical. Using this profile the vertical energy flux (E_z) and two horizontal components of momentum flux τ_{zx} along x-axis and τ_{zy} along y-axis are computed at different levels.

The energy flux (E_z) is shown in Fig. 2(b). The figure shows that energy flux is invariant in the vertical above z = 3 km. It is seen that at the lower levels up to z = 1.2 km E_z is invariant and in the layer from z = 1.2 km to z = 2 km E_z is vertically upward and in the layer from z = 2 km to z = 3 km E_z is vertically alternatively downward and upward. Hence Fig. 2(b) also shows that the divergence/convergence of E_z in the layer up to z = 3 km.

The vertical profile of momentum flux (τ_{zx}) along x-axis is shown in Fig. 3(a). It is seen that the momentum flux τ_{zx} is invariant in the vertical above z = 5.3 km. It is seen that at the layer from z = 0.4 km to z = 1.7 km and from z = 2.1 km to z = 2.6 km the flux τ_{zx} is vertically upward. In the layer from z = 2.6 km to z = 3.4 km τ_{zx} is vertically downward.

The vertical profile of momentum flux (τ_{zy}) along y-axis is shown in Fig. 3(b). From this figure, it is seen that, the momentum flux (τ_{zy}) is invariant in the vertical above z = 5 km. It is also seen that at the layer from z = 1.7 km to z = 2 km τ_{zx} is vertically upward and in the layer from z = 0.5 km to z = 1.7 km the momentum flux (τ_{zy}) is vertically downward.

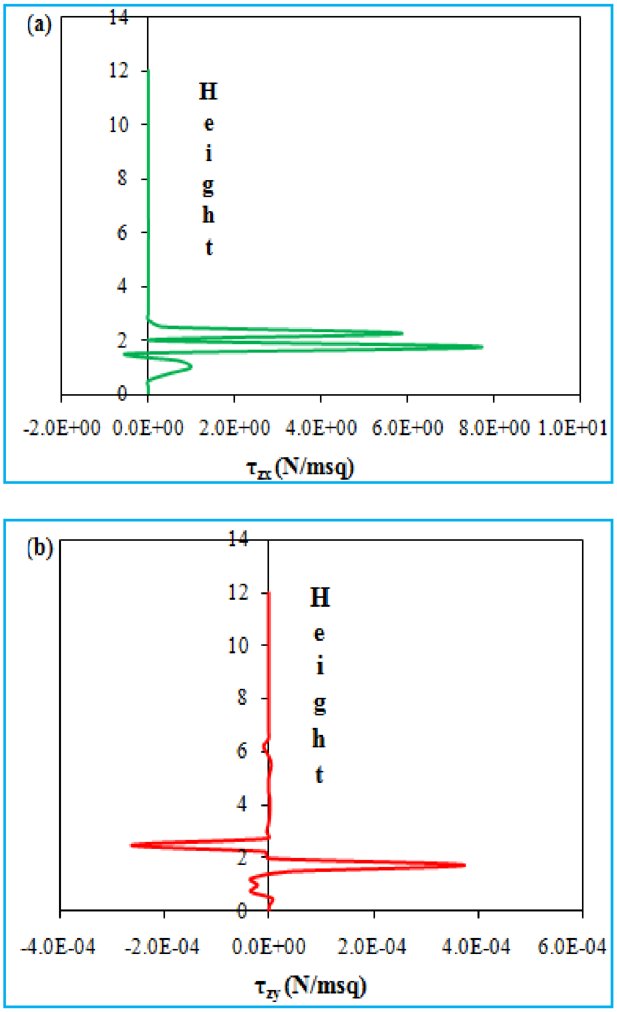
Case 2 : *18th July, 2004 (Rainy season)*

The vertical profiles of $U(z)$, $V(z)$ and $T(z)$ in the undisturbed flow are shown in Fig. 4 (a), which are based on the average of 0000 UTC and 1200 UTC RS/RW data of Guwahati of 18th July, 2004 during rainy season. The figure shows that the profile of temperature $T(z)$ is constant moist adiabatic lapse rate with vertical and hence following Sarker (1967) and De (1971), $T(z)$ is approximated by the pseudo-adiabatic line through the surface dry bulb temperature. Using this profile, the vertical energy flux (E_z) and the two horizontal components of the momentum flux τ_{zx} and τ_{zy} at different levels have been computed.

The vertical profile of the energy flux (E_z) has been shown in Fig. 4(b). It is seen that the energy flux (E_z) is invariant in vertical above z = 2.6 km. It is also shown that from z = 0.5 km to z = 2 km the energy flux (E_z) is upward.

The vertical profile of momentum flux (τ_{zx}) along x-axis has been shown in Fig. 5(a). It is seen that τ_{zx} is vertically upward in the layer from z = 1.7 km to z = 2.8 km and it is also invariant in vertical above z = 2.8 km.

The momentum flux (τ_{zy}) along y-axis is invariant with vertical above z = 2.8 km, which is shown in



Figs. 5(a&b). (a) Vertical profile of momentum flux (τ_{zx}) along *x* axis and (b) Vertical profile of momentum flux (τ_{zy}) along *y* axis on 18th July, 2004

Fig. 5(b). The Fig. 5(b) also shows that τ_{zy} is vertically upward in the layer from z = 1.6 km to z = 2 km and vertically downward in the layer from z = 2 km to z = 2.8 km. Dutta (2001) considered the profile of Mumbai-Pune section of Western-Ghats (WG) and there was only one ridge and a plateau. He showed that the plateau portion does not contribute to the fluxes of energy or momentum. In the present study, corner of the CMH has contributed to the fluxes of energy and momentum.

5. Conclusions

In this investigation, we have presented the wave momentum flux and energy flux for 3-D meso-scale lee wave across the CMH following a quasi-numerical approach. In the sequel, we have made some interesting observation. Moreover,

(*i*) From the study of above two cases, it is found that the fluxes vary in the vertical but the vertical variation is not uniform with height. In the some layers, fluxes are upward and somewhere fluxes are downward.

(*ii*) In the both cases, the effects of corner of the CMH have been observed. This makes the energy flux (E_z) or momentum fluxes (τ_{zx}, τ_{zy}) divergent / convergent in the vertical.

(*iii*) The above model may be used for any 3-D mountain profile to compute the energy flux (E_z) and two horizontal components of the momentum fluxes (τ_{zx} and τ_{zy}) at different levels.

(*iv*) Information revealed from this study about vertically upward fluxes of energy and momentum at lower levels appears to be important for aircraft operation.

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