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Prognosis by four-layer quasi-geostrophic model

T. K. MUKERJI and R. K. DATTA

Northern Hemisphere Analysis Centre, New Delhi

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ABSTRACT. Results of experiments in forecasting the heights of 850, 700, 500 and 300-mb levels, 24 hours in advance, using a four-layer quasi-geostrophic model are presented here. As a first step, the vertical velocities of surface and 200-mb level have been assumed to be zero. The static stability parameter is computed from known values of temperatures and pressures. The area covered for the study has been made large, so as to avoid the effect of constant values at the boundaries on the forecast for Indian areas.

1. Introduction

Preparation of computerised forecasts by the divergent barotropic model is being done at Northern Hemisphere Analysis Centre, New Delhi on a regular basis. Details of the working of this model are available (Mukerji and Datta 1971). This model leads to a forecast for 500-mb level. As is to be expected from a barotropic model, development or otherwise, of weather systems is not predicted by it. Moreover, forecasts for other levels of the atmosphere are also required to be made. As such, the problem of prognosis by a multi-layer quasi-geostrophic model was taken in hand.

In India, a number of authors have numerically solved the quasi-geostrophic ω -equation to obtain the diagnostic values of vertical velocity. Thus Das *et al.* (1971) used a ten-layer model for computation of ω and correlated the same to the cloud amounts. Rao and Rajamani (1967) obtained vertical velocities in the field of a cyclonic storm by using a five-layer model. Recently, Shukla (1971) has obtained solutions for the quasi-geostrophic diabatic ω -equation and investigated the diagnostic vertical velocities in a western disturbance over northern parts of India. There is as yet no published work in India regarding the prognostic values of ω and contour heights.

Multi-layer models for solution of the ω -equation and prognostication have been developed by a number of workers for middle and higher latitudes. Aubert (1957) and Knighting (1960) have come

up with models having different number of layers. It becomes clear from their results that, increasing the number of levels increases the details in the flow field and also increases the magnitude of the centres of positive and negative vertical velocities. However, the vertical motions from a 10-layer model were no better related to the observed weather pattern than were those from a two-layer model (Sanders *et al.* 1960). In making a compromise between the 'detail in the flow fields', the utility of the output products and the availability of the computer facilities, it was decided to work on a four-layer model, which would lead to forecasts for four levels of the atmosphere and consume relatively less amount of computer time.

In the present study, not only the diagnostic values of ω have been computed, they have been used to obtain the height tendencies for different atmospheric levels and the forecast values of ω upto a period of 24 hours.

2. Basic equations

Following Thomson (1965), we can write the equations of baroclinic flow as,

$$\nabla^2 \frac{\partial Z}{\partial t} + \mathbf{J}(Z, \zeta + f) - (f^2/g) \frac{\partial \omega}{\partial p} = 0 \quad (1)$$

$$\sigma \nabla^2 \omega - (f^2/g) \frac{\partial^2 \omega}{\partial p^2} = (g/f) \nabla^2 \mathbf{J} \left(Z, \frac{\partial Z}{\partial p} \right) - \frac{\partial}{\partial p} \mathbf{J}(Z, \zeta + f) \quad (2)$$

where the symbols have their meanings as given

in Appendix. σ , the static stability parameter, has been treated as a constant for a particular level of the atmosphere. A solution of Eq. (2) gives the diagnostic ω values. The prognostic elements are contained in Eq. (1) and this is solved by substitution of the diagnostic values obtained from Eq. (2). For this study, the above two equations have been solved numerically by three-dimensional Liebmann relaxation scheme.

3. Finite difference forms

To solve Eqs. (1) and (2) by numerical methods, it is first necessary to convert them into finite difference forms. Following standard methods, we define a network of points whose horizontal coordinates on a map are $x=id$ and $y=jd$. In the vertical direction, the position of these points are given by $K\Delta p$, where Δp is a constant increment of pressure and K has the values 0, $\frac{1}{2}$, 1, etc. The finite difference form of Eqs. (1) and (2) then become,

$$\nabla^2 \left(\frac{\partial Z}{\partial t} \right)_{k+\frac{1}{2}} + \frac{1}{4} \mathbf{J} \left(Z_{k+\frac{1}{2}}, \zeta_{k+\frac{1}{2}} + f \right) - \frac{f^2 d^2}{gm^2 \Delta p} \left(\omega_{k+1} - \omega_k \right) = 0 \quad (3)$$

$$\begin{aligned} \sigma_k \nabla^2 \omega_k - \frac{f^2 d^2}{gm^2 (\Delta p)^2} \left(\omega_{k-1} - 2\omega_k + \omega_{k+1} \right) &= \frac{g}{8fd^2 \Delta p} \nabla^2 m^2 \mathbf{J} \left(Z_{k+\frac{1}{2}} + Z_{k-\frac{1}{2}}, Z_{k+\frac{1}{2}} - Z_{k-\frac{1}{2}} \right) \\ &- \frac{1}{4\Delta p} \left[\mathbf{J} \left(Z_{k+\frac{1}{2}}, \zeta_{k+\frac{1}{2}} + f \right) - \mathbf{J} \left(Z_{k-\frac{1}{2}}, \zeta_{k-\frac{1}{2}} + f \right) \right] \end{aligned} \quad (4)$$

ζ in these equations is given by $(gm^2/fd^2) \cdot \nabla^2 Z$.

4. Construction of the grid

The grid, in three dimensions, has been constructed as follows: d , the grid length is 381 kms true at $22\frac{1}{2}^\circ$ N of a Mercator chart and is the same in x and y -directions. The number of grid points in x direction, i.e., i is 31 and in the y -direction, j is 16. In the vertical, the atmosphere is divided into layers, as shown in Fig. 1(b).

To solve equations (3) and (4), we adopted the boundary condition as discussed in the next section.

5. Boundary constraints and values of constants

- (i) In obtaining the diagnostic ω -value, the values of ω at 1000 mb and 200 should be known. It has been assumed to be zero at both these levels, as a first approximation, although methods are available for finding ω_{1000} (Das *et al.* 1970) as well as ω_{200} (Lateef 1968). These will be introduced in later modifications of this model,

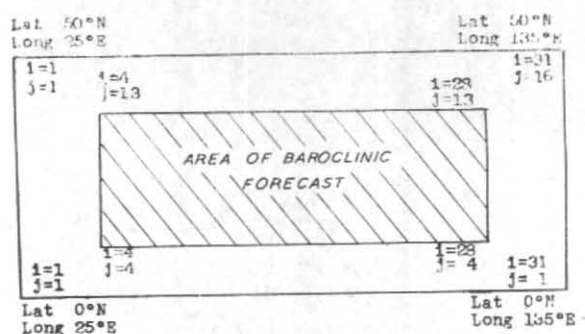


Fig. 1(a)

Horizontal grid

- (ii) The lateral boundaries are assumed such that the Z values do not change there and so, $\partial Z / \partial t$ is zero. This was decided upon after testing with floating boundary conditions, which did not produce any significant improvement in the forecasts.

- (iii) Values of static stability parameter σ have been worked out by averaging for each month, the values already available for all radiosonde stations in India (Mukerji *et al.* 1971). Thus for each of the levels, 400, 600, 800 mb there is a single value of σ for each month.

- (iv) The distance in the vertical, Δp , has been taken as 200 mb.

- (v) Except in the calculations for vorticity ζ , where actual values of f for respective latitudes have been used, a constant value for f has been utilised for all other computations. This value of f corresponds to Lat. 25° N.

6. Scheme of solution

Equation (4) is solved for $K = 2, 3$ and 4 to obtain ω for these levels (Fig. 1b). These ω values are substituted in equation (3) and it is solved for $K = 1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$ and $4\frac{1}{2}$ to obtain $\partial Z / \partial t$ at these

levels. From the initial values of Z at these levels and computed values of $\partial Z / \partial t$, forecast values of Z are obtained. A time step of 1-hr has been used. With the new values of Z the whole process is repeated, till a forecast for 24-hour is obtained. The area over which the computations are made is shown in Fig. 1 (a). The effective area for which the baroclinic forecast is made available is shown as the shaded area. This can however be extended to cover the whole area of Fig. 1(a) by standard 'cheating' process but has not been attempted here.

7. Discussion of the results

A number of case studies have been made in forecasting with the model discussed above. For reasons of economy, we have presented results of one case study, and the salient features of some others.

This case study is for a situation on 00 GMT of 1 July 1972. A depression had moved inland across the Orissa coast and a low pressure area was moving into the Arabian Sea across Maharashtra coast. The 'initial' or 'input chart' refers to 00 GMT of 1 July 1972 (Fig. 2). The 'actual chart' on 00 GMT of 2 July 1972 has been called the realised chart (Fig. 4). The 'forecast chart' refers to the forecast prepared on the basis of the multilayer prognostic model and valid for 00 GMT of 2 July 1972 (Fig. 3).

850-mb level — The input chart shows a depression, with a closed contour line of 1380 gpm, centred southeast of Nagpur. Another similar low pressure system is situated with centre approximately 2.7° south of Karachi. In the forecast chart, the former has been completely filled up and the latter has been moved west by 2.7° , weakening it at the same time. A comparison with the realised chart shows that the depression southeast of Nagpur did really weaken and was centred near Ahmadabad. But, since this 'low' now is of very small scale, almost single grid system, it is not brought out in the forecast. The other 'low' has its centre very close to the predicted centre.

A low pressure system south of Caspian Sea has been filled up in the forecast and agrees well with the realised chart. Another deep trough extending from Lat. 25°N and Long. 99.0°E to 11°N and 113.8°E has also been filled up in the forecast.

Low pressure systems, on the realised charts, with centres near Lat. 40°N , Long. 105°E and Lat. 28°N , Long. 106°E have not been predicted by the model, presumably because they are single-grid systems and are smoothed by the mild smoother used in the present study.

| | | |
|------------------|---------------------|---------|
| $K=1$ | $\omega=0$ (input) | 200 mb |
| $K=1\frac{1}{2}$ | Z (input) | 300 mb |
| $K=2$ | ω (computed) | 400 mb |
| $K=2\frac{1}{2}$ | Z (input) | 500 mb |
| $K=3$ | ω (computed) | 600 mb |
| $K=3\frac{1}{2}$ | Z (input) | 700 mb |
| $K=4$ | ω (computed) | 800 mb |
| $K=4\frac{1}{2}$ | Z (input) | 900 mb |
| $K=5$ | $\omega=0$ (input) | 1000 mb |

Fig. 1(b)
Vertical grid

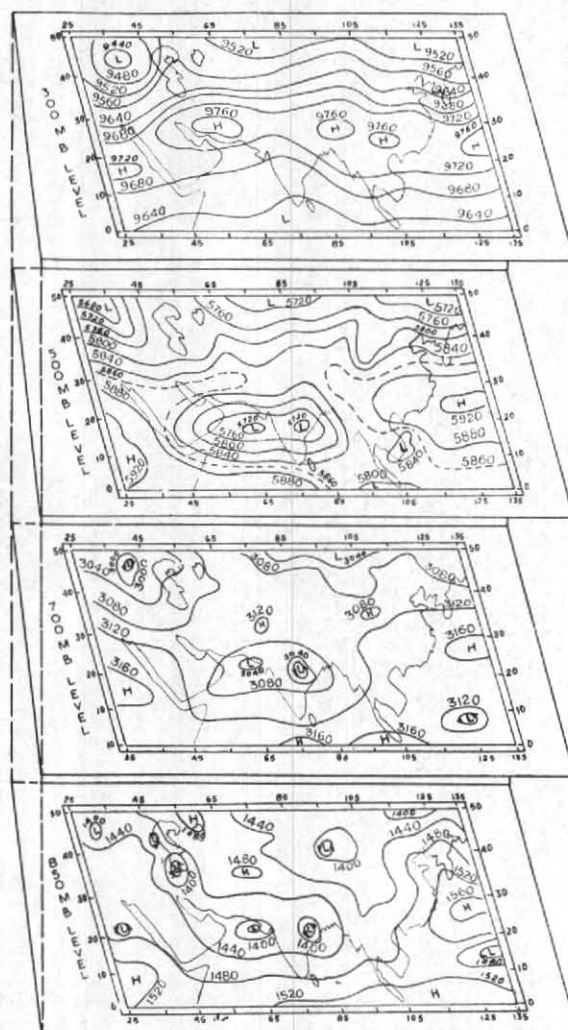


Fig. 2

Input contours (gpm) of 1 July 1972 (00 GMT)

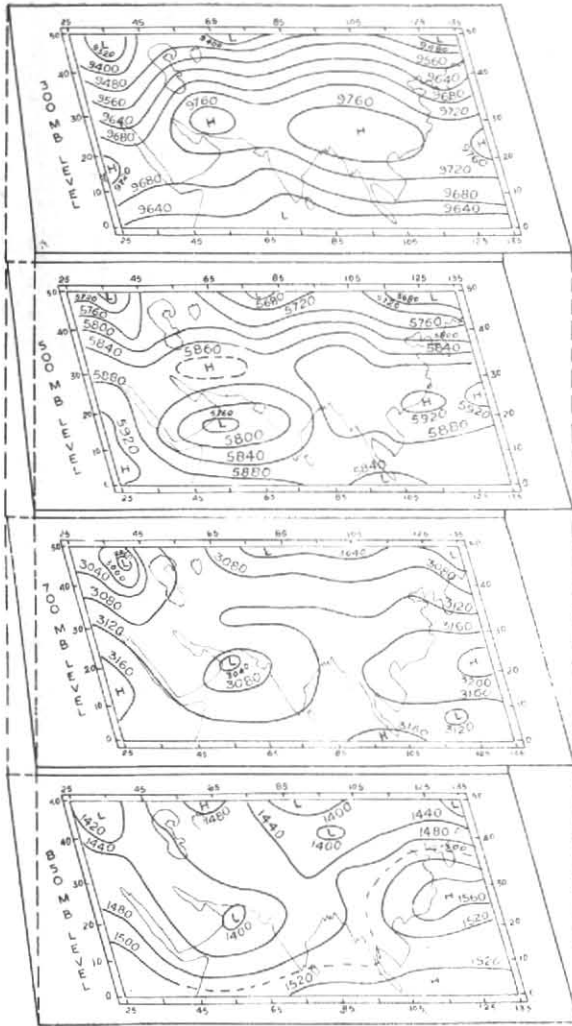


Fig. 3

Predicted contours (gpm) valid for 2 July 1972 (00 GMT)

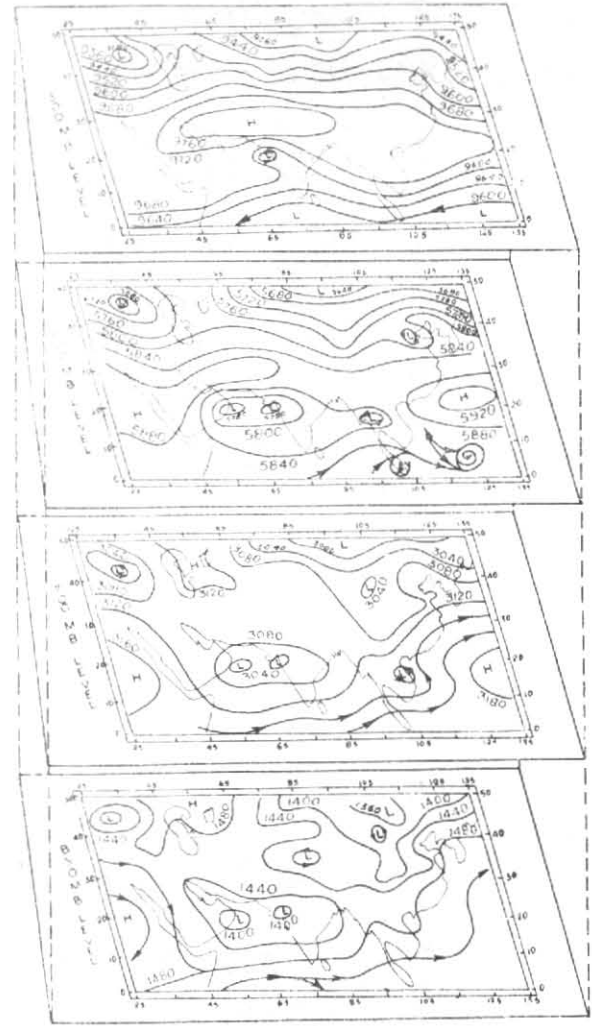


Fig. 4

Actual contours (gpm) of 2 July 1972 (00 GMT)

700-mb level—In the input chart we have a depression located SE of Nagpur and a low pressure area off the Katiawar coast. In the forecast chart the former system has completely dissipated, but in the realised chart it is having a 3040 gpm closed contour, with centre near Ahmadabad. The

other 'low' has been moved west in the forecast and forecast centre agrees well with the realised centre. The low pressure areas centred near 16°N and 106°E and 39°N and 105°E have not been brought out by the forecast because of the smallness of their scale.

500-mb level — At this level, the situation is similar to that at 700-mb level. Thus there are two 'lows' southeast of Nagpur and south of Karachi. In the forecast, the former system has been filled up, whereas the latter has been moved westwards. The forecast centre agrees well with the realised one. The contour value has been increased from 5720 gpm to 5760 gpm in the forecast, but the actual values is 5780 gpm. Also, another 5780 gpm 'low' just north of Bombay, as seen in the realised chart, is not seen in the forecast chart. However, the 5780 gpm line in the forecast, does extend upto the Bombay coast. A separate 'low' is not brought out as it is of small size and falls within a single grid distance.

A westerly trough, in the input, just west of Srinagar, has been moved eastwards by 2.7° longitude in the forecast and agrees with the actually realised position.

Two low pressure areas, roughly 3 degrees on either side of Long. 105°E and Lat. 35°N , present in the input chart, are not seen in the forecast chart, although according to the realised chart they combined and intensified into a low pressure area of 5760 gpm value. The reason for this may be found in the fact that, both the lows in the input are single-grid systems and on the application of smoother, will lose their identity and become a trough. In the forecast, this trough has been moved eastwards.

300-mb level — On this level, an easterly wave situated roughly along Long. 78°E and south of Madras, has been moved in the forecast to Long. 73.1°E . In the realised chart, the easterly wave seems to have moved very little. Other salient features are the highs along Lat. 25°N and has been brought out well in the forecast. A low pressure area near Bombay on the realised chart, is of single grid length in dimension and as such, has not been brought out in the forecast.

In a case study for the synoptic situation of 3 October 1972 (12 GMT), it was felt that the situation was barotropic. As such, the baroclinic 500 mb forecast and the barotropic 500 mb forecast, as prepared at NHAC by divergent barotropic model, were compared to see if there was any agreement. It was found that they were very similar. This points out, in an indirect way, that the working of the multi-layer model proposed in this study, is sound.

For another case study relating to the chart of 9 August 1972 (00 GMT), a new low pressure area was generated in the forecast for 850-mb level. A comparison with the actual 850 mb chart of 10 August

1972 morning, brought out the fact that, a low had really formed at the place indicated by the forecast.

Thus we have tested this model for generation, dissipation and no change conditions of low pressure areas and found that it is capable of forecasting all the three types correctly.

8. The vertical velocity charts

In Figs. 5 and 6 are reproduced the diagnostic and prognostic vertical velocities ω based on data of 1 July 1972. The layers for which ω is calculated are 800, 600 and 400 mb and are shown in the top three diagrams. They are followed by the isohyetal analysis for 00 GMT of 1 July 1972 in Fig. 5 and for 00 GMT of 2 July 1972 in Fig. 6.

Diagnostic ω values for 800 mb show an area of upward vertical velocity extending from Bombay coast to the U.P. hills. Keeping in mind the position of the 850 mb low (Fig. 2), we find that this area of negative ω is situated to the west of the centre and not above it. This type of result has been found by some earlier workers also. Corresponding to the low pressure area in the north Arabian Sea, the ω field is again shifted west of the centre. Similar area of upward vertical velocities is also seen for the low over south Caspian Sea. Other areas of upward and downward ω are also fitting well with trough and ridge systems available on 850 mb chart.

We now compare the 600 mb ω chart with the 500 mb analysis (Fig. 2a). Over central Indian region, there is a centre of upward ω with two areas of downward ω on each side of it. The magnitude of ω is in agreement with those obtained by other methods (Hawkins 1972). Corresponding to the low at 12°N , 107°E and the low pressure system between longitudes 105° to 110°E and latitudes 32° to 38°N , the areas of negative vertical velocities are in agreement. The area of downward ω corresponding to the ridge, west of the low pressure system mentioned above, also agrees well in position. A trough over the Caspian Sea, shows up as an area of upward ω in this chart.

As the 400-mb level, the patterns of the vertical velocities over the Indian area are similar to the ones at 600-mb level. Centres of strong upward motion are seen over Caspian Sea and at Lat. 33°N , Long. 115°E .

In the prognostic ω charts, at 800-mb level, the upward vertical velocities over India are replaced by downward motion. All the other downward and upward ω centres are reduced in magnitude. The 600 mb chart shows that

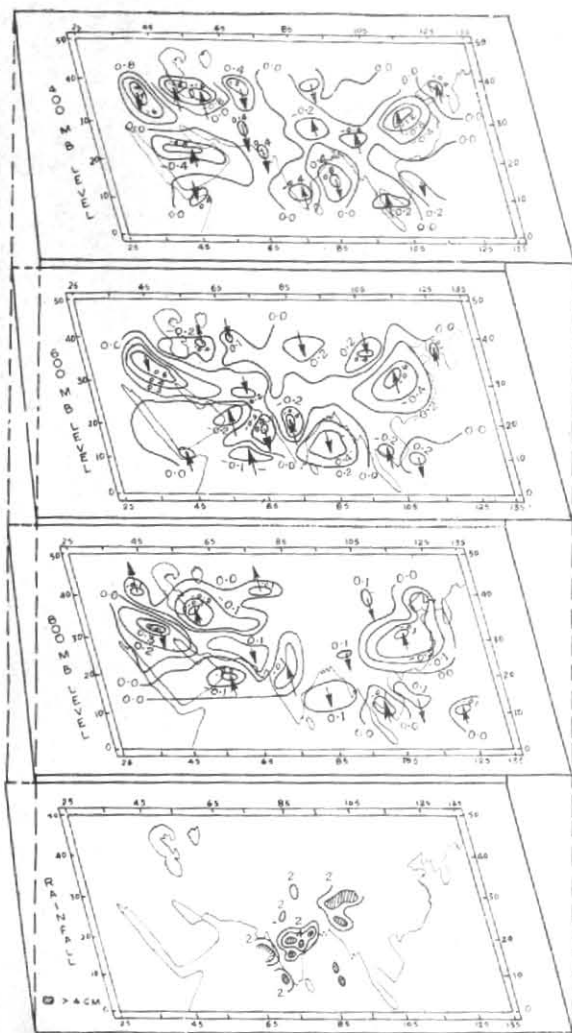


Fig. 5

Diagnostic vertical velocity (ω) in cm/sec for 1 July 1972
(00 GMT)

Lowest diagram gives the isohyetal analysis (cm) for
03 GMT of 1 July 1972

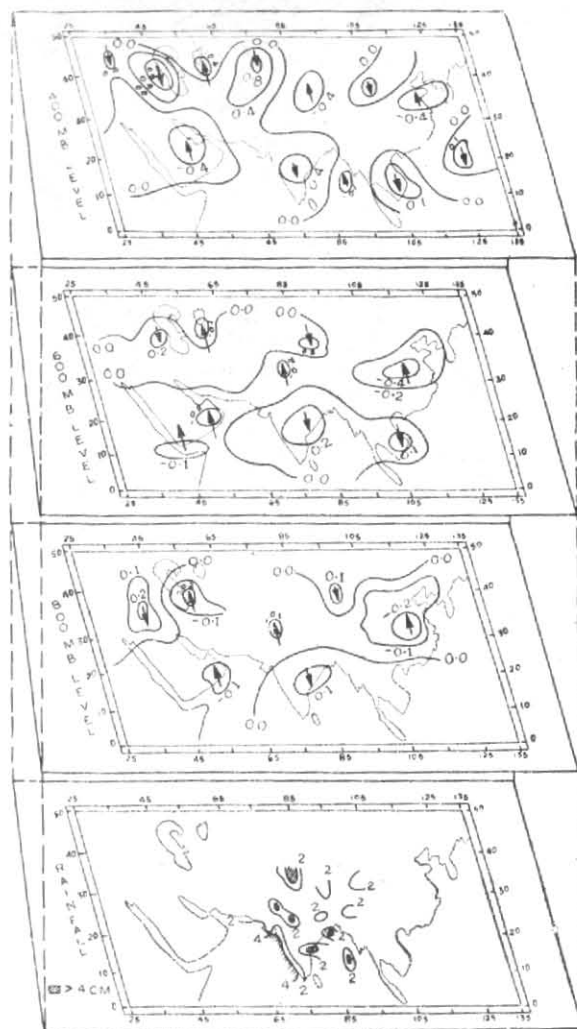


Fig. 6

Prognostic vertical velocity (ω) in cm/sec for 2 July 1972
(00 GMT)

Lowest diagram gives the isohyetal analysis (cm) for
03 GMT of 2 July 1972

the upward vertical velocities over India and Indo-China areas, seen in the diagnostic chart have been replaced by downward motion. Other features are similar to the 800 mb prognostic ω chart. The same trend is maintained in the chart of 400-mb also. The only difference here is

that of reduced magnitudes of vertical velocities.

One interesting feature, which emerges from the study of the ω charts is that the ω values for low pressure systems keep on increasing with height. Since, in this model, ω is assumed to vanish at

the 1000-mb and 200-mb levels, it is expected that its value will gradually increase reaching a maximum at about 600-mb level. However, in the cases studied by us the maximum is reached only at 400-mb level. Similar results have been reported by Rex (1958) who studied a low latitude disturbance in the Pacific by kinematic methods. Yanai (1967) also found that ω tended to be strongest at higher elevations.

A comparison between the ω fields and the rainfall patterns bring out quite clearly that, in a qualitative way, these fields agree with each other. Particular mention may be made of the heavy rainfall over the central parts of India and the upward ω there on 1 July 1972. Similarly on 2 July 1972, the drastic reduction of the rainfall corresponds well with downward ω over the same area. However, since in this model, the vertical velocity generated by the ground has been neglected, the rainfall caused due to orography does not agree with the vertical velocity field. This is very prominently seen all along the west coast and hilly areas of Assam.

9. Conclusions

From the case studies made so far, we can arrive at the following tentative conclusions:

- (i) A beginning is being made in testing multi-layer model for the Indian region. Due to well known reasons, we do not expect that the present model will solve all or even most prediction problems. Nevertheless, the first few trials are quite encouraging.

(ii) Small scale weather systems which are smaller than 381 km in diameter (one grid length), get lost in the forecast. This can be improved by decreasing the grid length and will be tried in future modifications of this model.

(iii) This model disregards non-adiabatic process like release of latent heat etc, which is not correct when dealing with depressions. In addition, the vertical velocity, produced by ground contour has also been neglected. In spite of these approximations, the diagnostic and prognostic vertical velocities show a close resemblance with the rainfall pattern over the plains. Results of inclusion of non-adiabatic effects and ground contour effect will be reported in a separate paper.

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REFERENCES

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|---|------|---|
| Aubert, E. J. | 1957 | <i>J. Met.</i> , 14 , 527-542. |
| Charney, J. G. | 1948 | <i>Geophys. Publikasjoner</i> , 17 , 17. |
| Das, P. K., Datta, R. K. and Chhabra, B. M. | 1971 | <i>Indian J. Met. Geophys.</i> , 22 , pp. 331-336. |
| Hawkins, H. F. | 1972 | NOAA Tech. Mem., ERL NHRL-98. |
| Knighting, E. | 1960 | <i>Quart. J.R. Met. Soc.</i> , 86 , pp. 318-325. |
| Lateef, M. A. | 1968 | <i>Mon. Weath. Rev.</i> , 95 , pp. 778-790. |
| Mukerji, T. K. and Datta, R. K. | 1971 | Prepubl. Sci. Rep., 176, India met. Dep. |
| Mukerji, T. K., Shaik, Z. E. Gupta, R. N. | 1972 | <i>Indian J. Met. Geophys.</i> , 23 , p. 57. |
| Rao, K. V. and Rajamani, S. | 1967 | Prepublished Sci. Rep. 54 (India met. Dep.). |
| Rex, D. F. | 1958 | <i>Geophysics</i> , 6 , pp. 479-501. |
| Sanders, F., Wagner, A. J. and Carlson, T. N. | 1960 | <i>Sci. Rep. 1</i> , Dep. Met., M.I.T. |
| Shukla, J. | 1971 | <i>Indian J. Met. Geophys.</i> , 22 , 4 , pp. 33-46. |
| Thomson, P. D. | 1961 | <i>Numerical Weather Analysis and Prediction</i> , Macmillan and Co., New York. |
| Yanai, M. and Nitta, T. | 1967 | <i>J. met. Soc. Japan</i> , 45 , pp. 441-446. |