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Independence of rainfall over Indian subcontinent over time scale of less than a month during monsoon season

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ABSTRACT. Independence of rainfall during monsoon season at representative Indian stations over time scales of 5, 10, 15, 20 and 25 days has been investigated. Chi-square tests have been applied to test the null hypothesis of independence against the alternative hypothesis of non-independence or association. The study reveals that, (i) rainfall over time scales of 5 and 10 days is not pairwise independent for consecutive periods but is pairwise independent for non-consecutive periods, (ii) rainfall over time scales of 15, 20, 25 days is pairwise independent and, (iii) rainfall over all the time scales considered in this study is not tripletwise independent. The significance of these features for the combination of frequency distributions is discussed.

1. Introduction

In an earlier study of pentad rainfall over India during the SW monsoon season, Mooley and Appa Rao (1970) have shown that the pentad rainfall over India is Gamma distributed. They have mentioned that two or more pentads could be combined and that Gamma parameters for the rainfall distribution of the combined period could be obtained if pentad rainfall is independent and scale parameters for the rainfall distribution of the pentads are not different. The values of the scale parameters, ($\hat{\beta}$), tabulated by them have been tested for significant differences over successive pentads. Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be scale parameters for rainfall during two successive pentads and $\sigma\hat{\beta}_1$ and $\sigma\hat{\beta}_2$, the corresponding standard errors.

If $\hat{\beta}_1$ lies within the 95 per cent confidence limits $\hat{\beta}_2 \pm 1{\cdot}96\ \sigma\hat{\beta}_2$ or $\hat{\beta}_2$ lies within the limits $\hat{\beta}_1$, $\pm 1{\cdot}96\ \sigma\hat{\beta}_1$ then $\hat{\beta}_1$ and $\hat{\beta}_2$ are not significantly different. The requisite values of $\sigma\hat{\beta}$ were obtained from the values of $(\sigma\hat{\beta}/\hat{\beta})$ tabulated by them (1970). Except for the following few pairs, scale parameters for successive pentads are not significantly different.

Station	Successive pentads (denoted by the first pentad) with scale parameters significantly different
Ahmadabad	36 ; 42
Allahabad	Nil
Bombay	Nil
Calcutta	38; 39
Cochin	41; 46
Jaipur	49
Madras	47; 48; 49
Nagpur	52; 53
Port Blair	41; 43; 52
Simla	35
Visakhapatnam	33; 34; 42; 44; 46

A knowledge about the independence of variates can be utilised when two or more variates are to be combined. For example, when n variates $x_1, x_2, \ldots\ldots\ldots, x_n$ are known to be pairwise independent then we can deduce the variance of the sum of these from the variance of each of these n variates. If the distribution functions of two or more variates are known, then the distribution function of the sum of two or more variates can be derived from the characteristic function of the sum of the variates as mentioned by Kendall and Stuart (1963) if the variates are collectively independent. From this point of view it is essential to find out whether the variates we are concerned with are independent.

It is also necessary to know if any relationship exists between two or more variates and if it exists, the extent of this relationship. For this purpose, the first logical step is to explore the areas over which relationsihp can be expected. The second step would be to obtain the extent of relationship over these explored areas.

In view of the above considerations it is proposed to investigate into independence of rainfall over different time scales over India during monsoon season. The time scales considered are 5, 10, 15, 20 and 25 days. It is, however, not proposed to investigate in this study the extent

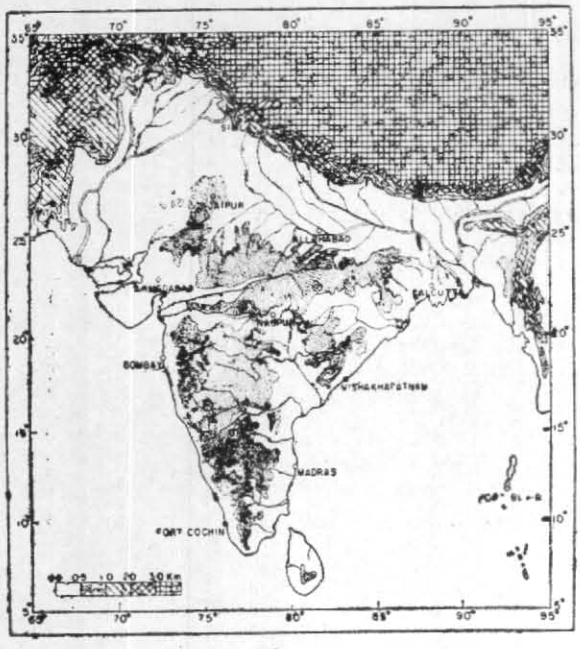

Fig. 1

of relationship over areas where relationship is expected to exist; this is a topic for a separate study.

2. Independence of rainfall

If $x_1, x_2 \ldots\ldots\ldots\ldots x_n$ are independent random variables and probability density functions, $p_1(x_1)$, $p_2(x_2)\ldots\ldots\ldots\ldots\ldots,p_n(x_n)$ exist, then as stated by Gnedenko (1963)

$$p(x_1 x_2,\ldots\ldots,x_k)=p_1(x_1)p_2(x_2)\ldots p_k(x_k)\ldots \quad (1)$$

for every positive integer $k \leqslant n$.

Let $\xi_1, \xi_2,\ldots\ldots\ldots\ldots\ldots, \xi_n$ denote rainfall in n specific periods during monsoon, d-day periods, all these periods being non-overlapping. Then, monsoon rainfall ξ may be considered as n-dimensional variable. Let the range on each of the n dimensions be sub-divided into m equal intervals. Then the n-dimensional sample space is subdivided into m^n cells of n-dimensional contingency table. If N is the total number of observations of the n-dimensional variable, the number of observations expected in each of the m cells can be computed on the assumption of independence by utilising equation (1). The actual number of observations falling in each of these m^n cells of the n-dimensional contingency table can be counted from the rainfall data. The next step is to apply χ^2 test to the n-dimensional contingency table to test the null hypothesis H_0 of independence against the alternative hypothesis H_1 of association or non-independence, adopting level of significance $\alpha = \cdot 05$. Number of observations expected in each of the m^n cells on the hypothesis of independence is N/m^n. χ^2 test is to be generally applied in cases where theoretical cell frequency is not less than 5. Hence for applicability of χ^2 test, $N/m^n \leqslant 5$. In the present study $N=60$. Hence it has not been possible to investigate beyond tripletwise independence.

3. Rainfall data

11 rainguage stations over India were selected. These are : Ahmadabad, Allahabad, Bombay (Colaba), Calcutta (Alipore), For Cochin, Jaipur Madr s, Nagpur, Port Blair, Simla and Vishakhapatnam. These stations are shown in Fig. 1. From the daily rainfall for the period 1901-1960 rainfall for each of the standard pentads during monsoon season was obtained for each of the stations. Rainfaill data for Port Blair are not available for 1931-32 and 1943 45. Rainfall for the pentads numbered 31 to 55 was obtained for all stations. In addition, rainfall was obtained for the pentads 56 to 67 for the southern stations Visakhapatnam, Port Blair, Madras and Fort Cochin which get good amount of rainfall during the northeast monsoon season. Pentad 31 corresponds to the period 31 May—4 June and 67, to 27 November—1 December. Rainfall totals for 10, 15, 20 and 25-day periods were also computed. Next, terciles and medians of d-day period rainfall distribution were obtained. In the present study, two-dimensional sample space was divided into 3^2, *i.e.*, 9 cells by means of terciles of the two distributions and the three-dimensional sample space was divided into 2^3, *i.e.*, 8 cells by means of medians of the three distributions. As terciles have been used to partition the sample space, it has been decided to consider only those d-day (where d is 5, 10, 15, 20, 25) periods for which rainfall distributions have non-zero lower terciles. With this criterion, the total period considered for each of the stations and for each of the time scales of rainfall is given in Table 1. The first d-day period commences on the first date given under the appropriate column of this table and other periods follow successively without any overlap, the last period ending on the next date under the same column.

As consecutive periods may be more dependent than non-consecutive periods, it was decided to investigate separately the independence of consecutive pairs and non-consecutive pairs as well as that of triplets with all consecutive periods, triplets with only two consecutive periods and triplets with no consecutive periods for d-day periods with d upto and including 15. For 20 and 25-day periods this distinction becomes meaningless since the number of pairs or triplets

TABLE 1

Total period considered

Station	Rainfall for 5-day	10-day	15-day	20-day	25-day
Ahmadabad	15 Jun to 7 Sep	10 June to 17 Sep	31 May to 27 Sep	31 May to 27 Sep	31 May to 2 Oct
Allahabad	20 Jun to 27 Sep	10 Jun to 27 Sep	,,	,,	,,
Bombay (Colaba)	5 Jun to 2 Oct	31 May to 27 Sep	,,	,,	,,
Calcutta (Alipore)	31 May to 2 Oct	31 May to 27 Sep	,,	,,	,,
Fort Cochin	31 May to 21 Nov	31 May to 26 Nov	31 May to 26 Nov	31 May to 26 Nov	31 May to 21 Nov
Jaipur	30 Jun to 7 Sep	10 Jun to 17 Sep	31 May to 27 Sep	31 May to 27 Sep	31 May to 2 Oct
Madras	10 Jun to 1 Dec	10 Jun to 26 Nov	31 May to 26 Nov	31 May to 26 Nov	31 May to 21 Nov
Nagpur	5 Jun to 2 Oct	31 May to 27 Sep	31 May to 27 Sep	31 May to 27 Sep	31 May to 2 Oct
Port Blair	31 May to 1 Dec	31 May to 26 Nov	31 May to 26 Nov	31 May to 26 Nov	31 May to 21 Nov
Simla	31 May to 22 Sep	31 May to 27 Sep	31 May to 27 Sep	31 May to 27 Sep	31 May to 2 Oct
Visakhapatnam	31 May to 17 Oct	31 May to 6 Nov	31 May to 11 Nov	31 May to 26 Nov	31 May to 21 Nov

is relatively quite small and hence this distinction was not maintained for periods of these lengths.

Utilising equation (1) for independence cell frequencies were computed. The empirical cell frequencies were counted. Next, χ^2 test was applied to the several contingency tables to test the null hypothesis of independence against the alternative hypothesis of non-independence. The level of significance adopted was 5 per cent. Computations were performed on CDC 3600 at TIFR Bombay and on IBM 1620 at the Institute of Tropical Meteorology, Poona. For two-dimensional contingency tables, the number of degrees of freedom was 4 and for three dimensional contingency tables it was 1.

4. Discussion of the results

The results of the χ^2 tests of goodness of fit have been tabulated. Tables 2(a), 2(b), 2(c), 2(d), and 2(e) deal with tests for pairwise independence of 5, 10, 15, 20 and 25-day rainfall respectively, while Tables 3(a), 3(b), 3(c), 3(d) and 3(e) deal with those for tripletwise independence. The tables give the total number of cases under each category and the number of cases in which χ^2 was significant at 5 per cent level for appropriate degrees of freedom together with the percentage of such cases (Tables 3a and 3b give the pairs of successive pentads and ten-day periods which are non-independent).

4.1. *Pairwise independence*

Table 2(a) suggests in general pairwise independence of successive pentad rainfall at Calcutta, Nagpur, Allahabad, Port Blair, Simla and Madras and non-independence at other places. However, the details of the pairs of successive pentads for which rainfall is not independent are given in the

TABLE 2

Pairwise independence of rainfall

Station	No. of pairs with successive periods: Total	No. of pairs with successive periods: χ^2 significant at 5% level	No. of pairs with non-successive periods: Total	No. of pairs with non-successive periods: χ^2 significant at 5% level	Successive period pairs for which χ^2 significant at 5% level. First pentad of the pair
		(a) *Pairwise independence of pentad rainfall*			
Ahmadabad	16	9 (56)	120	16 (13)	35,37,39,43,44,45,46,48,49
Allahabad	19	3 (16)	171	3 (2)	36,44,51
Bombay (Colaba)	23	9 (39)	253	15 (6)	34,38,40,42,44,45,46,48,54
Calcutta (Alipore)	24	0	276	9 (3)	Nil
Fort Cochin	34	7 (21)	561	27 (5)	35,45,49,50,54,55,58
Jaipur	13	4 (31)	78	5 (6)	41,44,46,48
Madras	34	5 (15)	561	31 (6)	40,61,62,63,65
Nagpur	23	2 (9)	253	7 (3)	42,54
Port Blair	36	5 (14)	630	28 (4)	42,44,50,57,66
Simla	22	3 (14)	231	13 (6)	34,35,38
Visakhapatnam	27	5 (19)	351	17 (5)	33,34,37,47,56
		(b) *Pairwise independence of 10-day rainfall*			
Ahmadabad	9	1 (11)	36	1 (3)	3
Allahabad	10	1 (10)	45	1 (2)	7
Bombay (Colaba)	11	0	55	3 (5)	Nil
Calcutta (Alipore)	11	1 (9)	55	5 (9)	6
Fort Cochin	17	3 (18)	136	6 (4)	6,10,12
Jaipur	9	1 (11)	36	2 (6)	7
Madras	17	0	136	8 (6)	Nil
Nagpur	11	1 (9)	55	1 (2)	1
Port Blair	17	2 (12)	136	9 (7)	15,17
Simla	11	3 (27)	55	1 (2)	7,8,11
Visakhapatnam	15	2 (13)	105	2 (2)	3,12
		(c) *Pairwise independence of 15-day rainfall*			
Ahmadabad	7	1 (14)	21	1 (5)	
Allahabad	7	0	21	0	
Bombay (Colaba)	7	0	21	1 (5)	
Calcutta (Alipore)	7	0	21	0	
Fort Cochin	11	0	55	4 (7)	
Jaipur	7	0	21	2 (10)	
Madras	11	0	55	0	
Nagpur	7	0	21	1 (5)	
Port Blair	11	0	55	4 (7)	
Simla	7	0	21	0	
Visakhapatnam	10	0	45	3 (7)	

	(d) *Pairwise independence of 20-day rainfall*		(e) *Pairwise independence of 25-day rainfall*	
	No. of pairs		No. of pairs	
	Total	χ^2 significant at 5% level	Total	χ^2 significarnt at 5% level
Ahmadabad	15	1 (7)	10	0
Allahabad	15	0	10	0
Bombay	15	0	10	0
Calcutta	15	0	10	0
Fort Coehin	36	0	21	1 (6)
Jaipur	15	0	10	0
Madras	36	2 (6)	21	2 (10)
Nagpur	15	0	10	0
Port Blair	36	2 (6)	21	1 (5)
Simla	15	2 (13)	10	1 (10)
Vishakhapatnam	36	2 (6)	21	1 (5)

Note—(*i*) Ten-day period number 1 is 31 May to 9 June and number 18 is 17 to 26 November

(*ii*) The number of degress of freedom is 4 for all χ^2 tests reported under Tables 2(a) to 2(e). Bracketted figures are percentages.

TABLE 3

Tripletwise independence of rainfall

Station	No. of triplets with three consecutive periods		No. of triplets with two consecutive periods		No. of triplets with non-consecutive periods	
	Total	χ^2 significant at 5% level	Total	χ^2 significant at 5% level	Total	χ^2 significant at 5% level
		(a) *Tripletwise independence of pentad rainfall*				
Ahmadabad	15	15 (100)	210	173 (82)	455	206 (45)
Allahabad	18	13 (72)	306	166 (54)	816	338 (41)
Bomboy (Colaba)	22	21 (95)	462	340 (74)	1540	785 (51)
Calcutta (Alipore)	23	11 (48)	506	262 (52)	1771	1013 (57)
Fort Cochin	33	24 (73)	1056	673 (64)	5456	2417 (44)
Jaipur	12	11 (92)	132	97 (74)	220	127 (58)
Madras	33	14 (42)	1056	516 (49)	5456	2402 (44)
Nagpur	22	13 (59)	462	210 (45)	1540	711 (46)
Port Blair	35	21 (60)	1190	723 (61)	6545	2929 (45)
Simla	21	12 (57)	420	220 (52)	1330	557 (42)
Visakhapatnam	26	16 (62)	650	311 (48)	2600	1034 (40)
		(b) *Tripletwise independence of 10-day rainfall*				
Ahmadabad	8	6 (75)	56	33 (59)	56	32 (57)
Allahabad	9	3 (33)	72	33 (46)	84	46 (55)
Bombay	10	6 (60)	90	54 (60)	120	59 (49)
Calcutta	10	3 (30)	90	43 (48)	120	49 (41)
Fort Cochin	16	12 (75)	240	136 (57)	560	273 (49)
Jaipur	8	7 (88)	56	36 (64)	56	33 (59)
Madras	16	9 (56)	240	96 (40)	560	219 (39)
Nagpur	10	7 (70)	90	50 (55)	120	67 (56)
Port Blair	16	10 (62)	240	132 (55)	560	279 (50)
Simla	10	10 (100)	90	52 (58)	120	28 (23)
Visakhapatnam	14	8 (57)	182	85 (47)	364	168 (46)
		(c) *Tripletwise independence of 15-day rainfall*				
Ahmadabad	6	3 (50)	30	13 (45)	20	10 (50)
Allahabad	6	3 (50)	30	14 (47)	20	9 (45)
Bombay	6	4 (67)	30	15 (50)	20	14 (70)
Calcutta	6	2 (33)	30	19 (63)	20	13 (65)
Fort Cochin	10	5 (50)	90	46 (51)	120	75 (62)
Jaipur	6	4 (67)	30	13 (43)	20	6 (30)
Madras	10	5 (50)	90	39 (43)	120	34 (28)
Nagpur	6	3 (50)	30	19 (63)	20	12 (60)
Port Blair	10	3 (30)	90	45 (50)	120	49 (41)
Simla	6	3 (50)	30	16 (53)	20	8 (40)
Visakhapatnam	9	2 (22)	72	28 (39)	84	44 (52)

Station	(d) *Tripletwise independence of 20-day rainfall* No. of triplets		(e) *Tripletwise independence of 25-day rainfall* No. of triplets	
	Total	χ^2 significant at 5% level	Total	χ^2 significant st 5% level
Ahmadabad	20	12 (60)	10	8 (80)
Allahabad	20	15 (75)	10	8 (80)
Bombay	20	6 (30)	10	7 (70)
Calcutta	20	11 (55)	10	3 (30)
Fort Cochin	84	42 (50)	35	13 (37)
Jaipur	20	13 (65)	10	2 (20)
Madras	84	49 (58)	35	9 (26)
Nagpur	20	0	10	3 (30)
Port Blair	84	25 (30)	35	23 (66)
Simla	20	15 (75)	10	9 (90)
Visakhapatnam	84	34 (10)	35	13 (37)

Note—The number of degrees of freedom is 1 for all χ^2 tests reported under Tables 3(a) to 3(e). Bracketted figures are percentages.

last column of Table 2(a). It is seen from this table that rainfall for successive pentads for Ahmadabad in July and August, for Bombay in August, for Cochin in September and October, for Jaipur in August, for Madras in November and for Simla and Visakhapatnam from mid-June to mid-July are non-independent. For the remaining stations there are a few pairs of non-independent successive pentads and these are not concentrated in any particular portion of the monsoon season, indicating that these may be treated as chance occurrences. Table 3(a) also brings out that rainfall in successive pentads numbered 44 and 45 is not independent at 5 of the stations, *viz.*, Ahmadabad, Allahabad, Bombay Jaipur and Port Blair.

When non-successive pentads are considered, Table 2(a) permits the drawing of the general inference that non-successive pentad rainfall is pairwise independent.

Table 2(b) hints at pairwise independence of successive 10-day rainfall at all stations with the possible exception of Simla and Cochin. The last column of this table gives for all stations the details of successive 10-day pairs which show non-independence.

Tables 2(b) clearly brings out pairwise independence of non-successive 10-day period rainfall.

Tables 2(c), 2(d) and 2(e) show that 15-day, 20-day and 25-day period rainfall is pairwise independent.

4.2. *Tripletwise independence*

Tables 4(a), 4(b), 4(c) clearly show that 5, 10, 15-day period rainfall is not tripletwise independent, irrespective of whether we consider triplets with successive periods or triplets with non-successive periods. Tables 4(d) and 4 (e) bring out the tripletwise non-independence of 20 and 25-day period rainfall also.

5. Conclusions

(*i*) Rainfall over successive 5-day periods is not pairwise independent at Ahmadabad in July and August, at Bombay in August, at Cochin in September and October, at Jaipur in August, at Madras in November and at Simla and Visakhapatnam from mid-June to mid July otherwise pentad rainfall is pairwise independent.

Except for pairwise non-independence of successive 10-day rainfall at Simla, Cochin and Port Blair in August, September, and October middle to November end respectively, successive 10-day rainfall appears to be pairwise independent. Non-successive 10-day rainfall is pairwise independent.

Rainfall over time scale of 15, 20 and 25-day is pairwise independent.

Thus in respect of rainfall over all the time scales considered in this study with the exception of forementioned cases of non-independence it is permissible,

(a) to combine rainfall of two periods to obtain distribution function and joint probability for the combined period from those for component periods, and

(*b*) to combine any number of periods to obtain the variance of rainfall for the combined period by adding the rainfall variances for the component periods.

(*ii*) Rainfall over time scales of 5, 10, 15, 20 and 25 days is not tripletwise independent. Hence rainfall for three or more periods of these durations cannot be combined for the purpose of getting the frequency distributions for the combined period from the frequency distributions for the component periods.

REFERENCES

Barger, G. L., Shaw, R .H. and Dale, R. F. 1959 Gamma distribution parameters from 2 and 3-week precipitation total in North Central Region of United States, Agriculture and Home Economics Experiment Station, Iowa State University, Ames., Iowa.

Gnedenko, B. V. 1963 Translation of second Russian Edition of 'Kurs Teorii Veroyatnostei' Translation by B.D. Seckler, Chelsea Publishing Co., New York, p. 165.

Kendall, M. G. and Stuart, A. 1963 *The Advanced Theory of Statistics.*, Vol. 1, Second Ed., p. 259, Charles Griffin Co. Ltd., London, pp. 259

Mooley, D. A. and Appa Kao, G. 1968 Statistical Distribution of pentad rainfall over India during monsoon season. India met. Dep. Sci. Rep., No. 70.

1970 *Indian J. Met. Geophys.*, 21, 2, pp. 219-230