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A preliminary study of the adiabatic generation and dissipation of kinetic energy by meridional and zonal winds over India and neighbourhood during the winter season

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ABSTRACT. The generation and dissipation of kinetic energy by the mean meridional and zonal winds in the free atmosphere have been estimated for the Indian region during winter. It is found that kinetic energy is generated by meridional motion and consumed by zonal motion. The magnitudes of these are largest in the upper troposphere. The results are in general agreement with those found by Kung for the American continent.

1. Introduction

The maintenance of the general circulation of the atmosphere demands a continual replenishment of the kinetic energy which is lost by frictional dissipation. The average rate at which the kinetic energy of the earth's atmosphere is being dissipated was estimated long ago by Brunt (1934) to be of the order of 5 watts in a vertical column of 1 m^2 cross-section. He also estimated that more than half of the dissipation occurs in the first 1 km above the surface. It is remarkable that these estimates remain, by and large, to be of the right order in the light of recent studies based on observational data which were not available when Brunt made his estimates. The frictional loss of kinetic energy has to be ultimately made up by the conversion of heat energy into kinetic energy by atmospheric processes.

2. Energetics of the atmosphere

2.1. The three kinds of energy with which we are concerned in atmospheric energetics are, internal energy (I), potential energy (P) and kinetic energy (K). Per unit mass of the atmosphere the values of these quantities are given by :

$$I_1 = c_v T \quad (1)$$

$$P_1 = g z \quad (2)$$

$$K_1 = \tfrac{1}{2} (u^2 + v^2 + w^2) \quad (3)$$

Making use of the First Law of Thermodynamics, the Equation of Continuity and the Equations of Motion, the time rate of change of these quantities can be readily shown (Wiin-Nielsen 1968) to be

$$\frac{dI_1}{dt} = H - \frac{p}{\rho} \nabla . \mathbf{V} \quad (4)$$

$$\frac{dP_1}{dt} = g w \quad (5)$$

$$\frac{dK_1}{dt} = - \frac{1}{\rho} \mathbf{V} . \nabla p - gw + \frac{1}{\rho} \mathbf{V} . \mathbf{F} \quad (6)$$

In Eqns. (1) to (6) the subscript 1 refers to unit mass and the remaining symbols have the following meanings :

p, ρ, T : Pressure, density and absolute temperature of the air

c_v : Specific heat of air at constant volume

$\mathbf{V}$: Three dimensional velocity vector whose zonal, meridional and vertical components are u, v and w

H : Heat absorbed per unit time

$\mathbf{F}$: Frictional force per unit volume

g : Acceleration due to gravity.

2.2. The rate of change of the three kinds of energy for the entire atmosphere is obtained by integrating over the total mass (M) or total volume (V) of the atmosphere. Thus :

$$\frac{dI}{dt} = \int_M \frac{dI_1}{dt} dm = \int_V \frac{dI_1}{dt}\rho\, dV$$

$$= \int_V H\rho dV - \int_V p\, \nabla . \mathbf{V}\, dV \qquad (7)$$

$$\frac{dP}{dt} = \int_V g\,\rho\, w\, dV \qquad (8)$$

$$\frac{dK}{dt} = -\int_V \mathbf{V}.\nabla p\, dV - \int_V g\,\rho\, w\, dV + \int_V \mathbf{V}.\mathbf{F}\, dV \qquad (9)$$

When integrated over the entire atmosphere it follows that,

$$-\int_V \mathbf{V}.\nabla p\, dV = -\int_V [\nabla.(p\mathbf{V}) - p\nabla.\mathbf{V}]\, dV$$

$$= \int_V p\, \nabla.\mathbf{V}\, dV$$

Hence,

$$\frac{dK}{dt} = \int_V p\nabla.\mathbf{V}\, dV - \int_V g\,\rho\, w\, dV + \int_V \mathbf{V}.\mathbf{F}\, dV \qquad (10)$$

2.3. The common terms with opposite signs in Eqns. (7) to (10) bring out the link between the three kinds of energy. The kinetic and potential energies are directly linked, the decrease in kinetic energy being associated with a corresponding increase in potential energy and *vice versa*. The kinetic energy obtains its replenishment from internal energy. The loss in internal energy is replenished by generation through heat absorbed.

2.4. The destruction of kinetic energy occurs by work done against frictional forces and also against gravity. The former is dissipated as heat while the latter goes to increase the potential energy. The generation of kinetic energy represented by the term $-\mathbf{V}.\nabla p$ results from the work done against pressure gradient force or by cross-isobaric flow. Ageostrophic motion is thus the direct mechanism for the production of kinetic energy in the atmosphere to compensate for its frictional dissipation.

2.5. The term $-\int_V p\nabla.\mathbf{V}\, dV$ in Eq. (7) for the rate of change of I gives the net contribution from regions of convergence and divergence in the atmosphere. Since divergence is generally associated with areas of high pressure and convergence with regions of low pressure, it follows that the net value of the integral would be negative. This decrease of internal energy goes to replenish the kinetic energy.

2.6. If the atmosphere is assumed to be in hydrostatic equilibrium, Equations (7), (8) and (9) become :

$$\frac{dI}{dt} = \int_V H\rho\, dV - \int_V p\nabla_2.\mathbf{U}\, dV - \int_V g\rho w\, dV \qquad (11)$$

$$\frac{dP}{dt} = \int_V g\rho w\, dV \qquad (12)$$

$$\frac{dK_H}{dt} = -\int_V \mathbf{U}.\nabla_2 p\, dV + \int_V \mathbf{U}.\mathbf{F}\, dV \qquad (13)$$

where ∇_2 is the two-dimensional del operator and $\mathbf{U}$ is the horizontal wind vector with zonal component u and meridional component v. As before, when the integration is performed over the entire atmosphere we have :

$$-\int_V \mathbf{U}.\nabla_2\, p\, dV = \int_V p\, \nabla_2.\mathbf{U}\, dV$$

Hence,

$$\frac{dK_H}{dt} = \int_V p\nabla_2.\mathbf{U}\, dV + \int_V \mathbf{U}.\mathbf{F}\, dV \qquad (14)$$

Eqns. (11) to (14) again show the link between the kinetic and internal energies. However, in the hydrostatic case the potential energy is linked with the internal energy and not with the kinetic energy as in the general non-hydrostatic case. It is well known that in an atmosphere in hydrostatic equilibrium the relation, $P/I = (R/c_v) \backsim (2/5)$ is always maintained.

3. Generation and dissipation of kinetic energy

3.1. In the hydrostatic case the rate of generation of kinetic energy per unit mass is $-(1/\rho)\mathbf{U}.\nabla p$. This can also be represented by $-\mathbf{U}.\nabla\phi$, where ϕ is the geopotential and ∇ is the horizontal del operator on an isobaric surface. Integrated over the entire atmosphere and for a sufficiently long period of time, the generation term has to be positive to compensate for the dissipation of kinetic energy by turbulent friction into heat. However, the sign of the term $-\mathbf{U}.\nabla\phi$, which depends on whether the flow is with or against the pressure gradient can vary with location and with time. It is probable that on account of geographically anchored features in the planetary flow patterns brought about by the land-sea configuration and orographical features, there exist preferred regions of kinetic energy generation in the atmosphere which undergo seasonal variations.

3.2. In a recent paper Kung (1971) has reported the results of study of the generation of kinetic energy over the area comprised of USA and Canada. The generation term $-\mathbf{U}.\nabla\phi$ consists of the zonal component $-u(\partial\phi/\partial x)$ and the meridional component $-v(\partial\phi/\partial y)$. Kung examined separately the behaviour of these two terms as a function of latitude and height for the area from 20° to 70°N and from the surface to 50-mb level. The study, based on the twice-daily radiosonde/rawin data of 144 stations for a one-year period, showed that at the lower latitudes kinetic energy is produced by meridional motion and destroyed by zonal motion while the reverse conditions obtain in middle and higher latitudes. The production and destruction have their maxima at the level of the jet stream. In the lower boundary layer of the atmosphere, both zonal and meridional motions produce kinetic energy at all latitudes.

3.3. Making use of the radiosonde/rawin data of selected stations Holopainen (1963) estimated the frictional dissipation of kinetic energy ($\mathbf{V}.\mathbf{F}$) in the atmosphere over the British Isles for a few days in January 1954 by integrating Eq. (6) over triangular areas whose corners were at the locations of the upper air stations. He arrived at a mean value of 10·4 watts/m^2 for a vertical column from the surface to 200 mb with two pronounced regions of dissipation, one below 900 mb (surface friction layer) and the other near 300 mb (just below the level of maximum wind).

3.4. It should be noted that the destruction of kinetic energy brought about by the negative value of the term $-\mathbf{U}.\nabla\phi$ is quite different from the frictional dissipation of kinetic energy. As discussed by Lorenz (1955) the available potential energy of the atmosphere [which is a fraction of the total potential energy $(P+I)$] is the source for the kinetic energy. The production/destruction of kinetic energy indicated by positive/negative values of $-\mathbf{U}.\nabla\phi$ represent exchange between the available potential energy and kinetic energy. These exchanges are adiabatic and thermo-dynamically reversible unlike the frictional dissipation of kinetic energy into heat which is irreversible.

4. Scope of the present study

4.1. Kung has remarked that the American continent is the only part of the globe with the required density of upper air network for evaluating the generation and dissipation of kinetic energy by meridional and zonal winds. Although from the standpoint of upper air observational data the eastern hemisphere is less favourably placed than the American continent, the meteorological features over these two parts of the globe differ considerably. As is well known, the monsoon circulation over South Asia is a unique feature of this part of the globe. The orographical features also differ considerably over the two parts. A study of the characteristic features of the general circulation over the eastern hemisphere is thus a necessary supplement to similar studies over the western hemisphere.

4.2. As a first step in this direction, we have attempted to investigate the behaviour of the generation/dissipation of kinetic energy by meridional and zonal winds over India and neighbourhood covering the region between latitudes 8° to 30°N and longitudes 50° to 100°E. An investigation of this type has become possible because of the replacement of the old radiosonde instruments over the Indian aerological network with new sondes of the audio-modulated type with better performance characteristics. The mean monthly zonal and meridional winds and contour heights at standard isobaric levels from 850 to 100 mb at all the rawin/radiosonde stations over India and adjoining areas have been made use of in this study for evaluating the terms $-u(\partial\phi/\partial x)$ and $-v(\partial\phi/\partial y)$ along the meridian of 77·5°E and along the parallel of 25°N. The results for January 1971 are presented in this paper.

5. Method of analysis and computation

5.1. The monthly mean contour height fields as well as the fields of u and v at standard isobaric levels were separately analysed by plotting the data on charts. Isopleths of u, v and ϕ were drawn at suitable intervals. Typical charts for 700 and 200-mb levels are shown in Figs. 1, 2 and 3. From the analysed charts the values of u and v were picked out at 2·5-degree intervals along 77·5° E and 25°N. Values of ϕ at intervals of 2·5°E /W and N/S around these grid points were picked out from the contour charts to evaluate the gradients $\partial\phi/\partial x$ and $\partial\phi/\partial y$. The values of

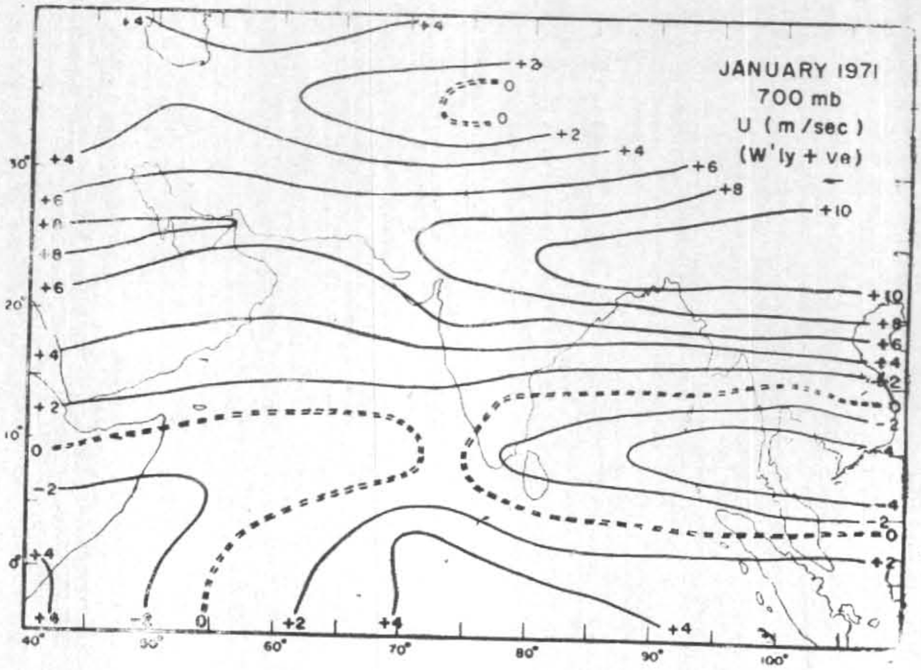

JANUARY 1971
700 mb
U (m/sec)
(W'ly + ve)

Fig. 1 (a)

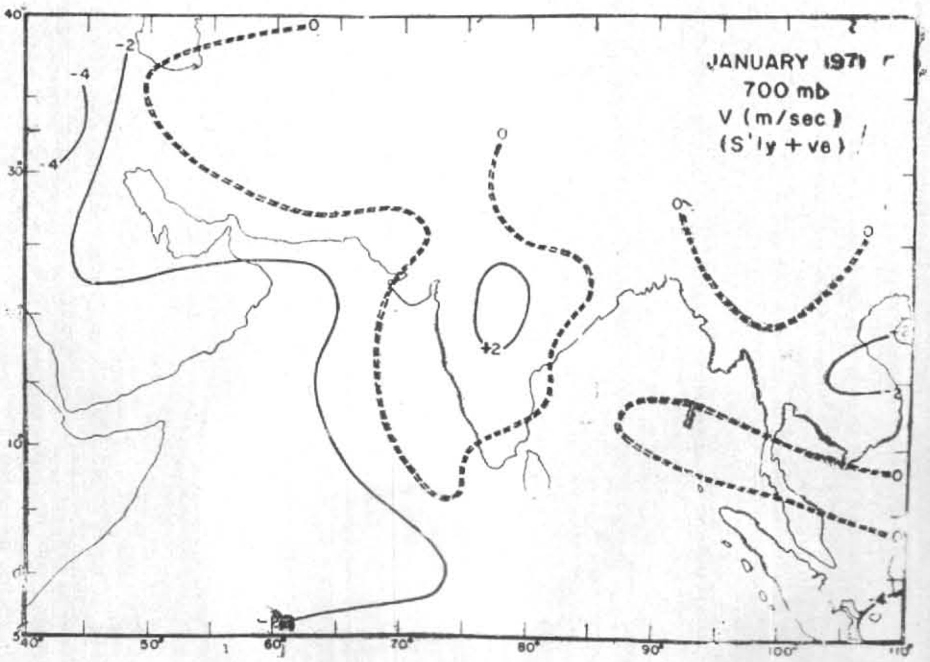

JANUARY 1971
700 mb
V (m/sec)
(S'ly + ve)

Fig. 1 (b)

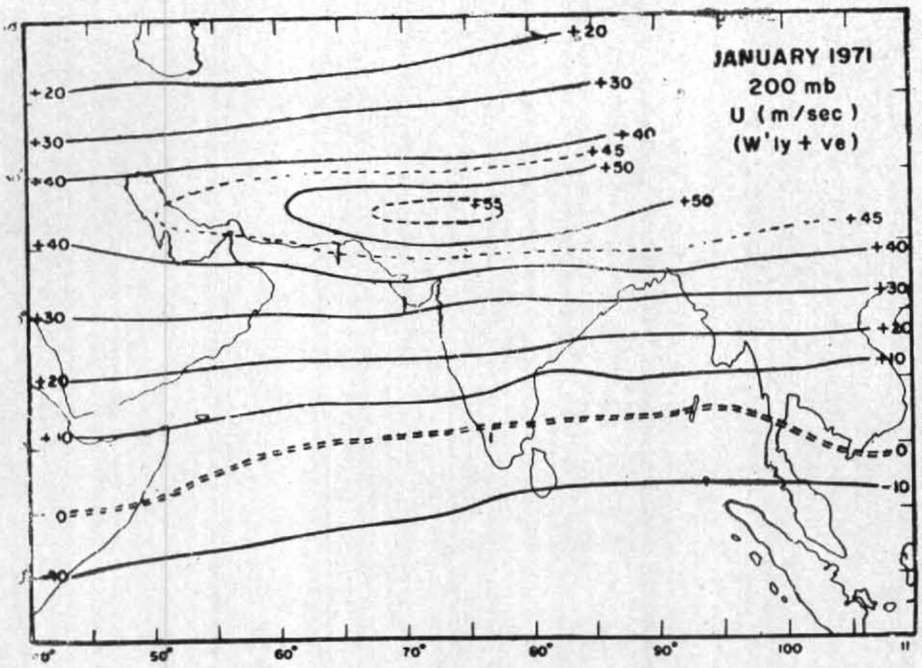

JANUARY 1971
200 mb
U (m/sec)
(W'ly + ve)
+20
+30
+40
+45
+50
+55
+10
0
-10
50°
60°
70°
80°
90°
100

Fig. 2 (a)

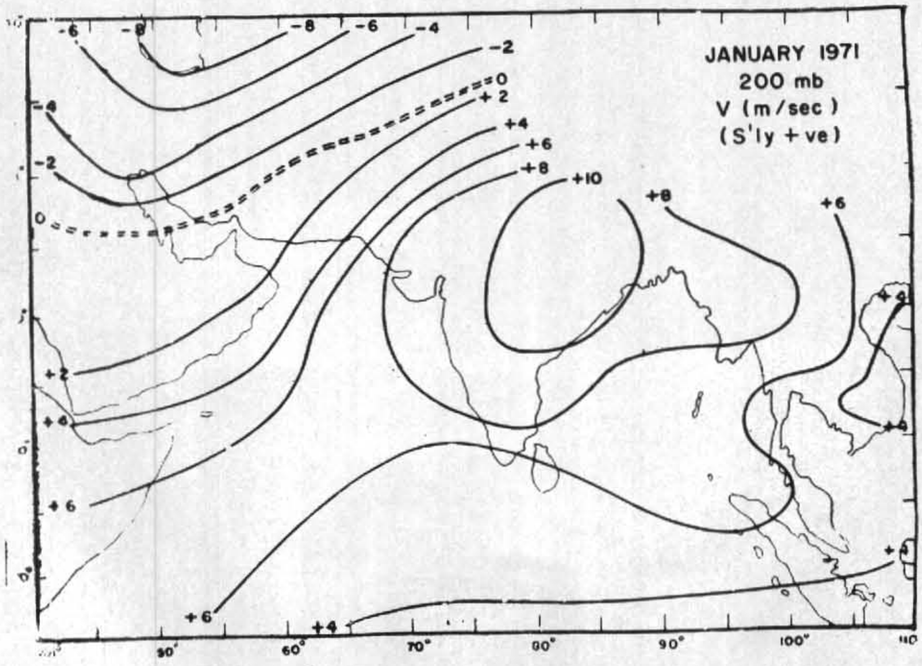

JANUARY 1971
200 mb
V (m/sec)
(S'ly + ve)
-8
-6
-4
-2
0
+2
+4
+6
+8
+10
70°
90°
100°

Fig. 2 (b)

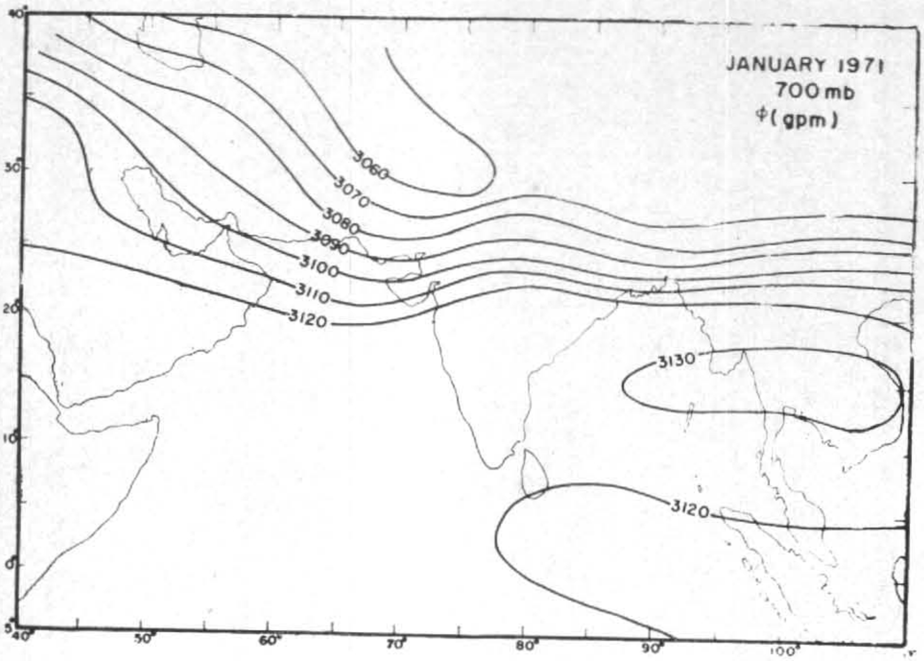

JANUARY 1971
700 mb
Φ (gpm)
3060
3070
3080
3090
3100
3110
3120
3130
3120
40°
50°
60°
70°
80°
90°
100°
30°
20°
10°
0°

Fig. 3 (a)

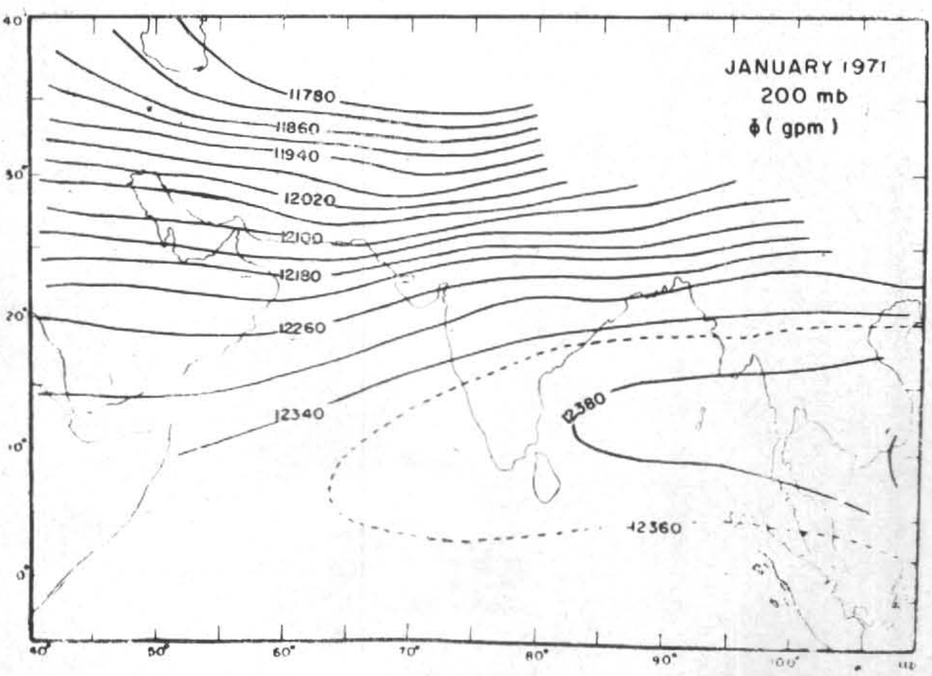
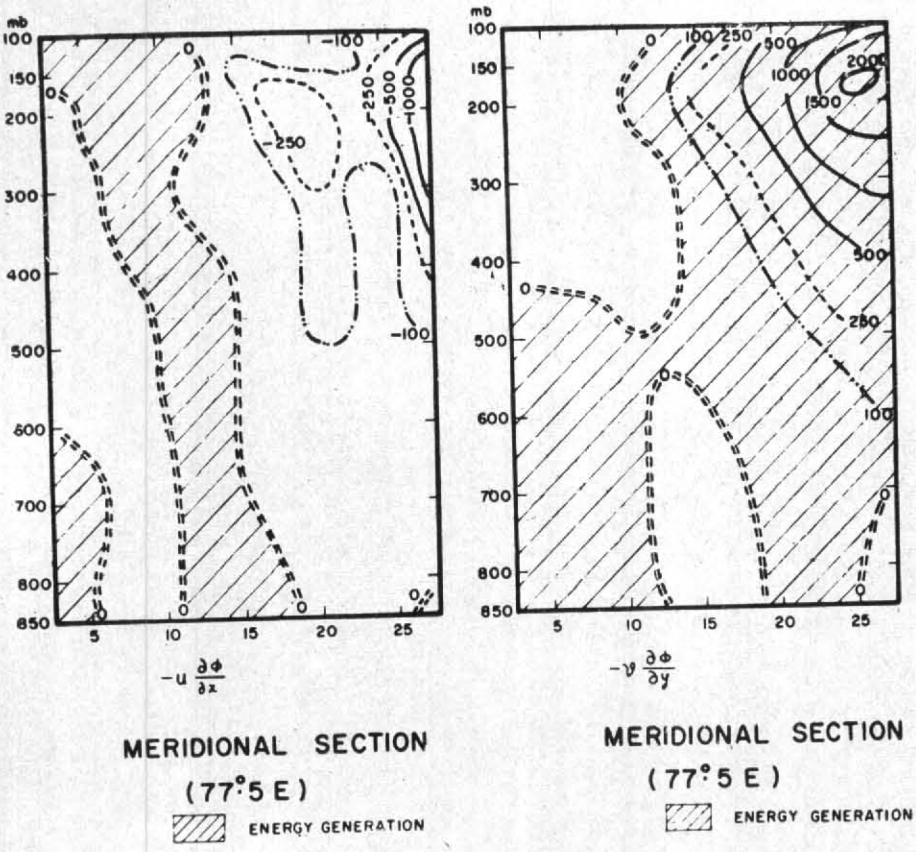

JANUARY 1971
200 mb
Φ (gpm)
11780
11860
11940
12020
12100
12180
12260
12340
12380
12360
40°
50°
60°
70°
80°
90°
100°
30°
20°
10°
0°

Fig. 3 (b)



(a) (b)

Fig. 4

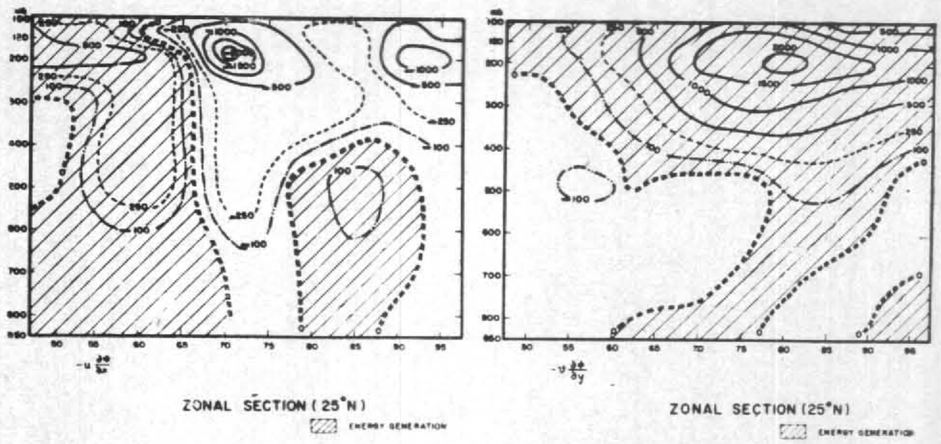
u and v in metres per second at the grid points were multiplied by the contour height differences in gpm in the zonal and meridional directions for five-degree intervals centred round the grid points. These products are proportional to $-u(\partial\phi/\partial x)$ and $-v(\partial\phi/\partial y)$ in arbitrary units.

5.2. For example, if $v = 10$ m/sec at a grid point and $\triangle\phi = 100$ gpm for five-degree separation in latitude centred at the grid point, then $v\triangle\phi = 1000$ units. Over the area of the study 5-degree $\approx 5\cdot5 \times 10^5$ m. Hence $v(\partial\phi/\partial y) = vg(\partial z/\partial y) = (g/5\ 5)$ ergs/gm/sec. Mass in grams in a column of 1 m^2 cross-section and 50 mb thickness is $(5 \times 10^8)/g$ grams. Hence kinetic energy for such a column is $(5 \times 10^8)/5.5$ ergs/sec $\approx 9\cdot1$ watts. Thus 1000 units (arbitrary) corresponds to kinetic energy generation/dissipation of 9·1 watts for a column of 1 m^2 cross-section and 50-mb thickness. This gives the conversion factor.

5.3. The calculated values in arbitrary units were plotted graphically as a function of latitude/longitude and height and isopleths were drawn at appropriate intervals. The results for the meridional and zonal components of the energy generation term are shown separately in Figs. 4(a), 4(b) and 5(a), 5(b) along longitude 77·5°E and latitude 25°N respectively. The total kinetic energy generation obtained by adding the zonal and meridional contributions is depicted in Figs. 6(a) & 6(b) for the zonal and meridional sections.

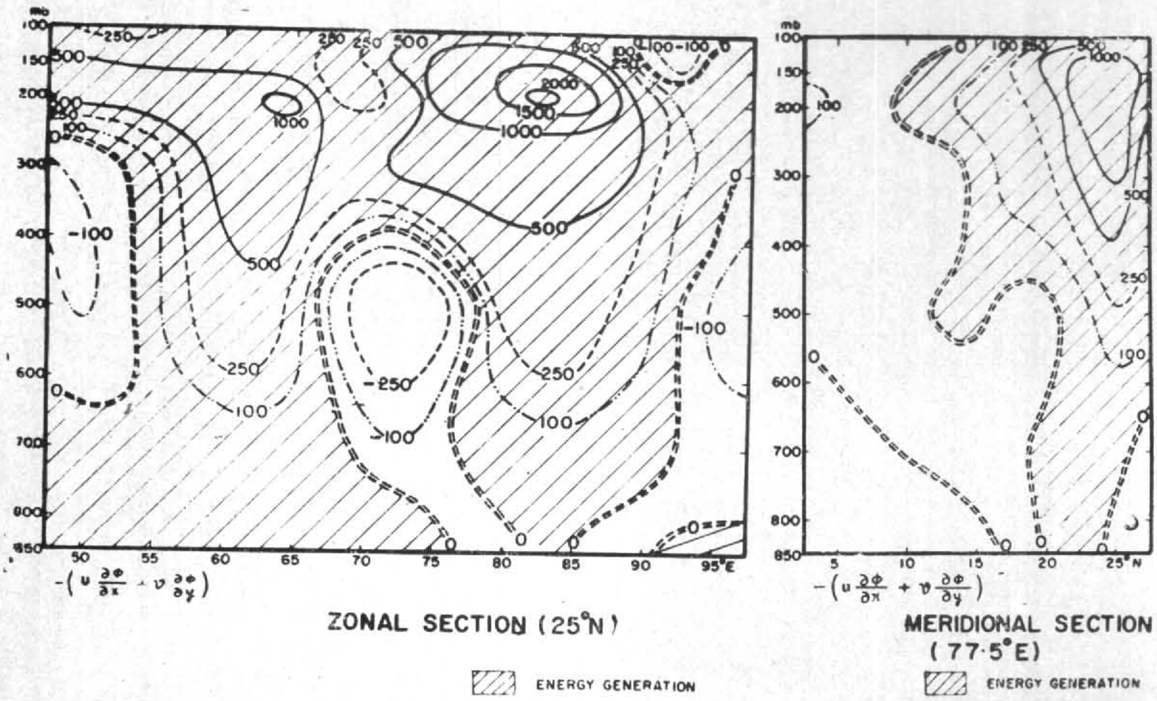
6. Results

6.1. *Meridional Section* (77·5°E) — Figs. 4(a), (b) and 6 (b) show that north of 15°N the meridional component of the mean motion generates kinetic energy while the zonal component dissipates kinetic energy, the production exceeding the dissipation. The largest production and dissipation occur near the level of the jet stream at about 200 mb. The maximum rate of production by the v-component of the motion is about 18 watts/m^2 in a column of 50 mb thickness; the maximum dissipation is about half this value. Thus, the net maximum rate of production of kinetic energy by the mean motion near the level



(a) (b)

Fig. 5



(a) (b)

Fig. 6

of the sub-tropical westerly jetstream around latitude 25°N is about 9 watts/m² in a column of 50 mb thickness.

6.2. *Zonal Section (25°N)* — The section is close to the mean latitude of the sub-tropical westerly jet stream over India and neighbourhood in the winter months. Figs. 5(a) and 5(b) show that practically over the entire upper troposphere from 50° to 100° E the meridional motion generates kinetic energy; the zonal motion consumes kinetic energy over most of this region except to the west of about 65°E where zonal motion also contributes to the generation of kinetic energy. This feature is due to the occurrence of a quasi-stationary trough in the circulation with trough

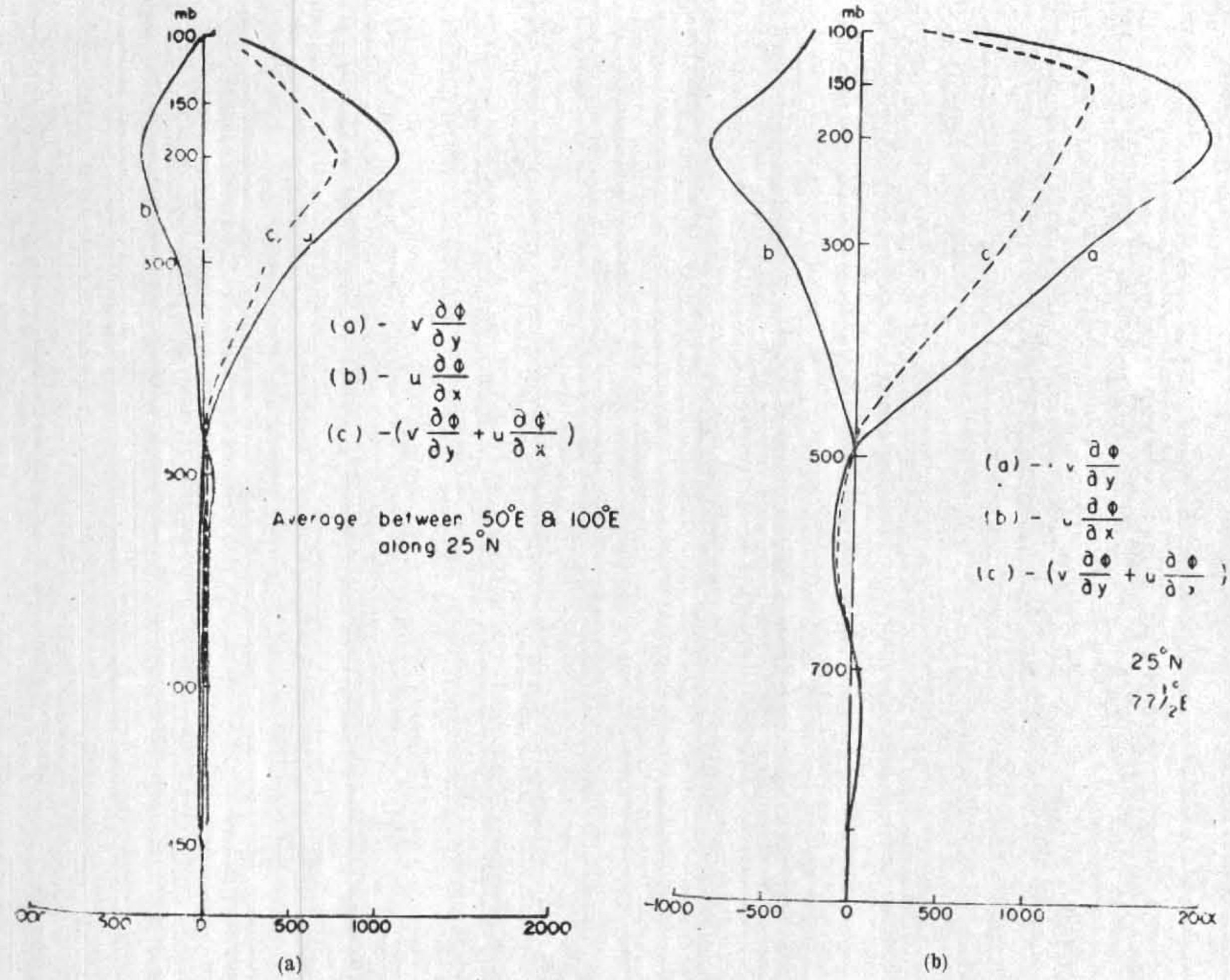


Fig. 7

axis near 65°E. Fig. 6(a) shows that the net production of kinetic energy is positive over the entire upper troposphere with a maximum of about 18 watts/m^2 in a column of 50 mb thickness around 200-mb level.

6.3. Fig. 7(a) shows the vertical profile of the generation/dissipation of kinetic energy averaged between 50° and 100°E along 25° latitude circle. The contributions by the zonal and meridional components as well as the total are shown separately. Vertical integration from 900 to 100-mb level yields the following figures —

(*i*) Generation by *v*-component : 38 watts/m^2

(*ii*) Dissipation by *u*-component : 9·5 watts/m^2

(*iii*) Net generation of kinetic energy : 28·5 watts/m^2

Fig. 7(b) shows similar profiles at 25°N and 77·5°E. The corresponding figures for (*i*), (*ii*) and (*iii*) are 81·5, 29·5 and 52 watts/m^2.

7. Discussion

7.1. The latitudinal zone covered by the present study extends further towards the equator than the American area investigated by Kung. Nevertheless, the results of the study are in general agreement with Kung's findings. The largest generation and dissipation of kinetic energy occur at and near the jet stream level in the upper troposphere. In the present investigation we have not been able to consider the generation of kinetic energy in the friction layer where there is appreciable cross-isobaric flow. The lower troposphere above the friction layer upto about 400-mb level makes very little contribution to generation of kinetic energy. According to Holopainen's study the frictional dissipation of kinetic energy is also small in this part of the atmosphere.

7.2. It is necessary to draw attention to an important difference between Kung's study based on daily values of u, v and ϕ and the present study based on monthly mean values of $\bar{u}$, $\bar{v}$ and $\bar{\phi}$. If

u', v' and ϕ' are the departures of the daily values from the monthly mean, then

$$\overline{u\frac{\partial\phi}{\partial x}} = \bar{u}\,\overline{\frac{\partial\phi}{\partial x}} + \overline{u'\frac{\partial\phi'}{\partial x}} \qquad (15)$$

$$\overline{v\frac{\partial\phi}{\partial y}} = \bar{v}\,\overline{\frac{\partial\phi}{\partial y}} + \overline{v'\frac{\partial\phi'}{\partial y}} \qquad (16)$$

Kung's estimate refer to the left hand side of Eqns. (15) and (16) while our estimates refer to the first term on the right hand side. The contribution by the transient motion terms to the generation/dissipation of kinetic energy has not been taken into account in our estimates. This is possible only if the computations are made on the basis of daily data. Nevertheless, it is possible to make an estimate of the contribution by the transient component of the meridional motion as shown below.

7.3. If u_g is the zonal component of the geostrophic wind then :

$$-\frac{\partial\phi}{\partial y} = f\,u_g \approx f u$$

Hence

$$-\overline{v\frac{\partial\phi}{\partial y}} \approx f\,\overline{u v} = f(\bar{u}\,\bar{v} + \overline{u'v'}) \qquad (17)$$

To see how far $f\bar{u}\bar{v}$ agrees with $\bar{v}(\partial\bar{\phi}/\partial y)$ we consider the mean monthly zonal and meridional winds at 200 mb over New Delhi for January 1971; $\bar{v}$=8.3 m/sec and $\bar{u}$=53.3 m/sec. The value of $f\bar{u}\bar{v}$ works out to about 18 watts/m^2 for a column of 50 mb thickness, which agrees with the computation made on the basis of mean meriodional wind and zonal pressure gradient.

7.4. Similarly, the magnitude of $f\overline{u'v'}$ may be expected to give an estimate of $\overline{v'(\partial\phi'/\partial y)}$. Rangarajan and Mokashi (1966) have computed the correlation coefficient between zonal and meridional winds at standard levels on the basis of daily data for a five-year period. From the statistical data given in their paper it can be estimated that the value of $f\overline{u'v'}$ is about 10 per cent of $f\,\bar{u}\,\bar{v}$. This shows that our estimates of generation of kinetic energy by the mean meridional motion has to be increased by about 10 per cent to take account of the contribution by the transient eddies. Although we cannot make a similar estimate of the contribution by transient eddies to the zonal generation/dissipation of kinetic energy, it is reasonable to expect that it may be of the same order.

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