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Spectral analysis of drought index (Palmer) for India

K. N. RAO, C. J. GEORGE, P. E. MORAY and N. K. MEHTA

Meteorological Office, Poona

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ABSTRACT. Values of (Palmer's) drought index computed for 23 sub-divisions of India for a period of about 60 years have been examined by power spectrum method. Very few of the peaks are significant at 95 per cent level though persistence in the case of some sub-divisions is indicated. No definite periodicity could be recognised in these series and even the oscillatory character noted in some of them need further critical examination.

1. Introduction

The present study is an attempt to examine periodicity of droughts in India by 'Power Spectrum Analysis'. For assessment of droughts and to facilitate their scientific study, Palmer (1965) used the water budgeting concept for developing a numerical index of drought, taking into account, rainfall, soil moisture and evapotranspiration. The formula for computing Palmer Index is

$$X_i = 0{\cdot}897\, X_{i-1} + Z_i/3$$

where X_i is the Palmer index value of month i and Z_i is the weighted rainfall anomaly of the month obtained from the hydrologic accounting procedure.

Employing this technique, monthly values of Palmer index for different sub-divisions of India have been computed from 1901 onwards. The accumulated values of the index for the monsoon season (June to September) have been worked out for the following 23 sub-divisions of India.

(1) Gangetic West Bengal, (2) Orissa, (3) Bihar Plateau, (4) Bihar Plains, (5) Uttar Pradesh East, (6) Uttar Pradesh West, (7) Punjab & Haryana, (8) Rajasthan West, (9) Rajasthan East, (10) Madhya Pradesh West, (11) Madhya Pradesh East, (12) Gujarat, (13) Saurashtra & Kutch, (14) Madhya Maharashtra, (15) Marathwada, (16) Vidarbha, (17) Coastal Andhra Pradesh, (18) Telangana, (19) Rayalaseema, (20) Tamil Nadu, (21) Interior Mysore North, (22) Interior Mysore South, (23) Kerala.

The remaining sub-divisions (Assam, Jammu & Kashmir, Konkan, Coastal Mysore), Palmer drought index could not be calculated satisfactorily due to very high runoff over these areas.

2. Method of analysis

The method followed is that due to Blackman and Tukey as given in the *W.M.O. Technical Note* No. 79 (1966). The analysis is carried out in three steps on a given time series of N equally spaced values of the variate X :

(i) Computation of auto-correlation coefficient of series for Lags zero to m ($m < N$), is given by the formula—

$$\gamma_L = \frac{N}{N-L}\;\frac{\sum_{i=1}^{N-L}(X_i-\bar{X})\,(X_{i+L}-\bar{X})}{\sum_{i=1}^{N}(X_i-\bar{X})^2}$$

where,

γ_L — Auto-correlation coefficient,

N — Number of years of data,

X_i — Variate in i^{th} term,

L — Period of Lag = 0, 1, 2, 3, ... , m

(ii) The cosine transforms of these $m+1$ auto-correlation coefficients, are computed to obtain $m+1$ raw estimates of power spectrum.

$$B_0 = \frac{1}{2m}\,(\gamma_0+\gamma_m) + \frac{1}{m}\sum_{L=1}^{m-1}\gamma_L$$

$$B_i = \frac{\gamma_0}{m} + \frac{2}{m}\sum_{L=1}^{m-1}\left(\gamma_L \cos\frac{\pi_i\,\gamma_L}{m}\right) + $$

$$+\ \frac{\gamma_m}{m}\,(-1)^i$$

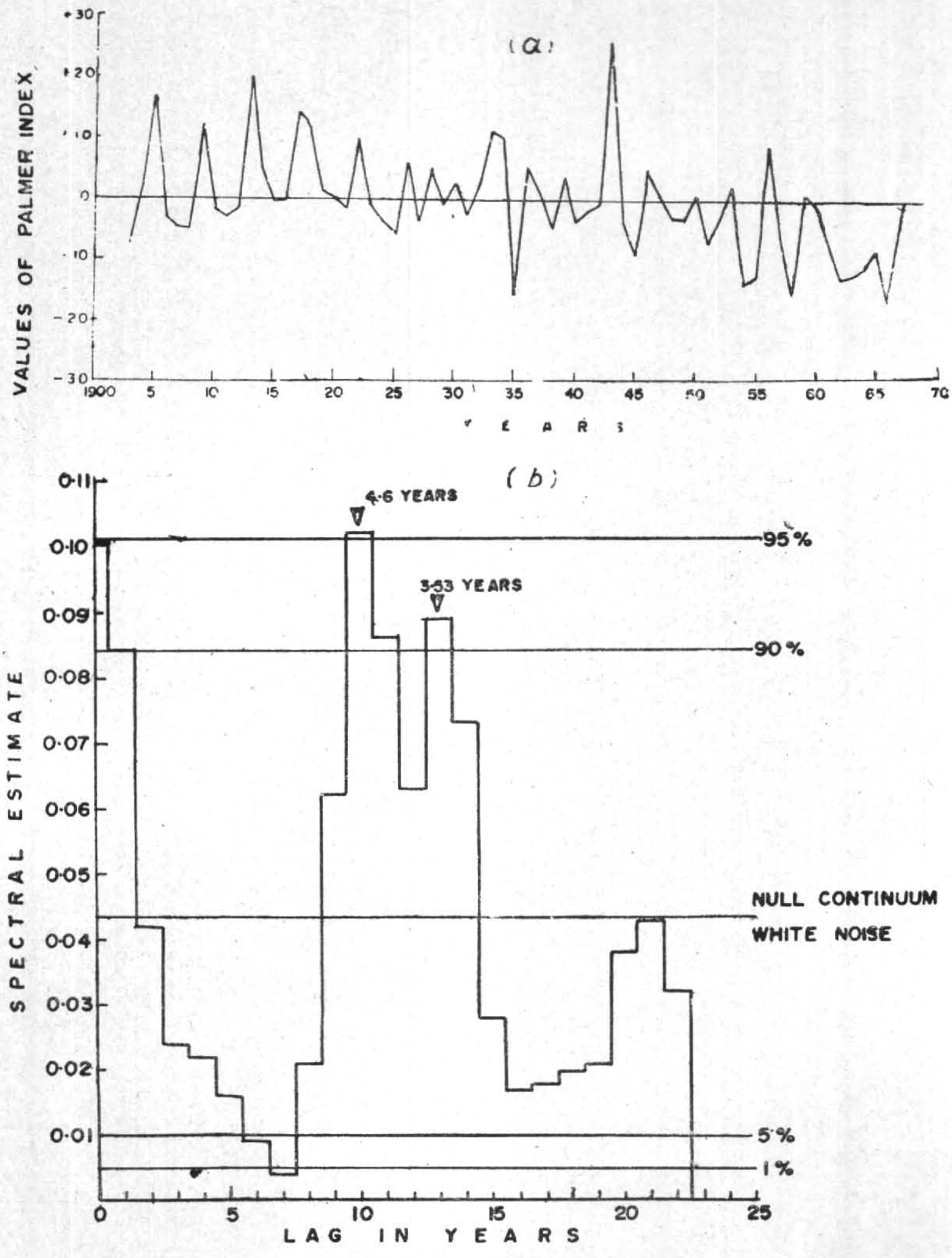


Fig. 1. Gangetic West Bengal, Jun to Sep (1903-1967)

(*a*) Values of Palmer Index, (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 22$ yr)

$$B_m = \frac{1}{2m}[\gamma_0 + (-1)^m \gamma_m] + + \frac{1}{m}\sum_{L=1}^{m+1} (-1)^L \gamma_L$$

where B_0 is spectral estimate for zero lag, and B_m for th e maximum lag of m time units.

(*iii*) The final spectral estimates are obtained by smoothing the raw estimates by a 3 term weighted moving averages with weights equal to 0.25, 0.50 and 0.25 respectively as under :

$$S_1 = 0{\cdot}50\ (B_0 + B_1)$$

$$S_i = 0{\cdot}25\ (B_{i-1} + 2B_i + B_{i+1})$$

$$S_m = 0{\cdot}50\ (B_{m-1} + B_m)$$

If a pure sine wave is present in the time series, spectrum will have comparatively narrow peak at the appropriate wave length. If the series contains oscillations of more complex type, the spectrum will have two or more peaks, one at the fundamental wave length and other at wave lengths corresponding to some of its higher harmonics. Finally, if the series contains Markov type persistence, the spectrum will be distorted across all wave lengths. In particular, the amplitude of the spectrum will tend to decrease from the longer to the shorter wave lengths. This characteristic where one finds ıelatively high spectral mass concentrated at the very lowest frequency is called "red noise".

3. Significance tests for Power Spectrum

If we have a time series of length N, whose frequency distribution approximates to the Gausian (Normal) distribution, the statistical significance of γ_L for the null hypothesis, of randomness is made by computing the value of

$$(\gamma_1)_t = \frac{-1 + t_g\sqrt{N-2}}{N-1}$$

where , t_g is the value of the standard deviate in the Gausian (Normal) distribution corresponding to the desired significant level of γ_1. For 95 per cent probability point, the value of t_g is 1.645 (for infinite degrees of freedom) which gives the equation

$$(\gamma_1)_t = \frac{-1 + 1{\cdot}645\ \sqrt{N-2}}{N-1}$$

If the value of γ_1 is less than $(\gamma_i)_t$ one may conclude that the series is random. But if it is equal to or more than $(\gamma_i)_t$ it means the series is significantly non-random, and we proceed further to locate the non-random variations present in the series. For this purpose, we first complete the null continuum from the equation

$$R_i = \frac{1}{m+1} \cdot \frac{1-\gamma_i^2}{1+\gamma_i^2 - 2\gamma_i \cos(\pi_i/m)}$$

A smooth curve drawn through the plot of R_i (Harmonic number $i = 0, 1, 2, .., m$) gives the null continuum for the computed spectrum and should be assumed for that of Markov's 'red noise'.

In cases where, γ_1 is less than $(\gamma_i)_t$, *i.e.* when a spectrum is of 'white noise', the null continuum is given by a horizontal straight line, whose value everywhere is equal to $1/(m+1)$.

According to Tukey, the ratio of magnitude of any null continuum is distributed as chi-square divided by the number of degrees of freedom, *i.e.*, (χ^2/ν).

Degree of freedom ν is given by

$$\nu = \frac{2N - m/2}{m}$$

This is utilised to compute the critical confidence limits of the null continuum (usually 95 % and 5 %) by multiplying the local values of the null continuum with fixed multiples representing the 95 per cent and 5 per cent probability points of distribution for the appropriate number of degrees of freedom.

Table of χ^2/ν given in *W. M. O. Technical Note* No. 79 (p.74) has been used for the purpose. In the same way, continuum for other confidence limits have been determined and superposed on the computed spectrum along with the null continuum, one can see at a glance, if any of the spectral estimates lie outside this limits. If none does, it means that the computed spectrum belongs to a population, the spectrum of which approximates to the null continuum. If however, any spectral estimate is found to exceed the upper confidence limit, it indicates the presence of significant oscillations in the series. Similarly, if any spectral estimate falls below the lower confidence limit, repetitions of phenomena with these periods may be considered to be very rare. Spectral estimates exceeding the upper confidence limit are called spectral peaks and those falling below the lower confidence limit are called spectral troughs or gaps. The period corresponding to any spectral estimate is given by the relation, $P = 2\ (m+1)/L$, where

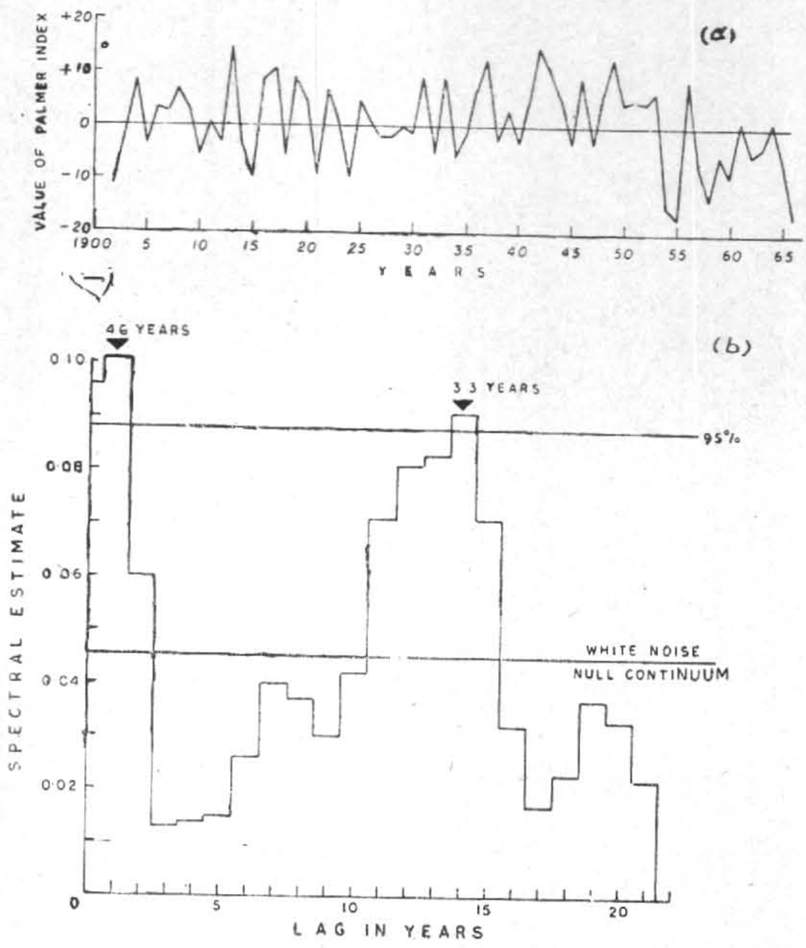


Fig. 2. Bihar Plateau, Jun-Sep (1903-1966)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 21$ yr)

m is the maximum lag and L denotes the lag period of the spectral estimate in question.

4. Analysis

The time series of Palmer index values of the monsoon season for twentythree sub-divisions listed in the introduction have been analysed*. Some of these are shown in Figs. 1(a) to 10(a) and their power spectrum in Figs. 1(b) to 10(b).

(1) *Gangetic West Bengal* — This is a white spectrum. The peak at 10th lag corresponds to a period of 4.6 years and is significant at 95 per cent level; the other spectral peak significant at 90 per cent level is observed at 13th lag with corresponding period of 3.53 years (Fig. 1).

(2) *Orissa* — The spectrum is that of white noise. Spectral peak significant at 90 per cent level is observed at 12th lag corresponding to a period of 3.83 years; the peak is, however, not significant at 95 per cent level.

(3) *Bihar Plateau* — The spectrum is that of white noise. One spectral peak at 14th lag corresponding to a period of 3.14 years is significant at 95 per cent level. A long period (44 years) corresponding to 1st lag is also seen at same significance level (Fig. 2).

(4) *Bihar Plains* — The spectrum is that of red noise. The spectrum peak at 20th lag is significant at 90 per cent level corresponding to a frequency of 2.3 years.

*Full analysis may be seen in India met. Dep. pre-published Sci. Rep. No. 169

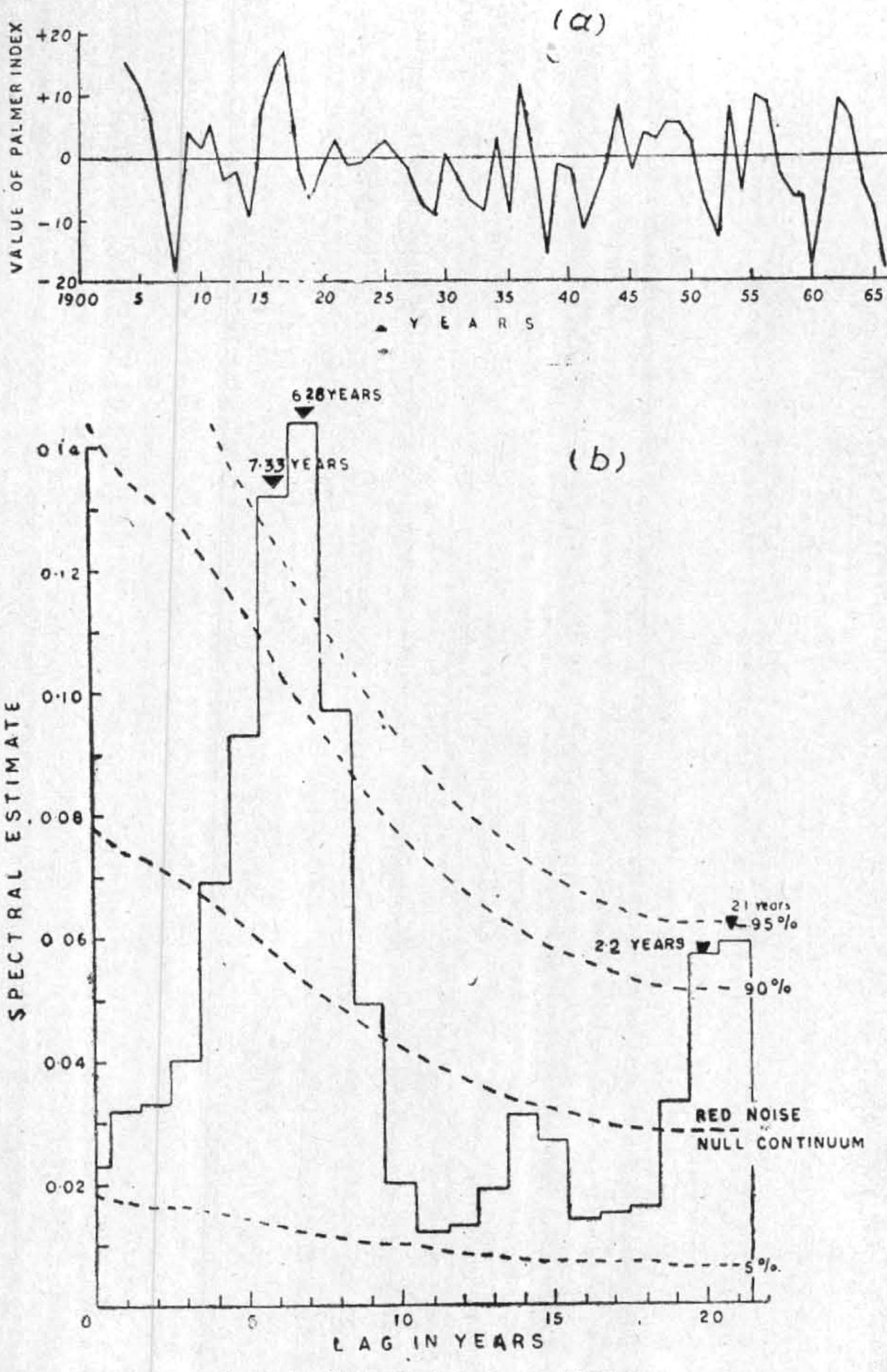


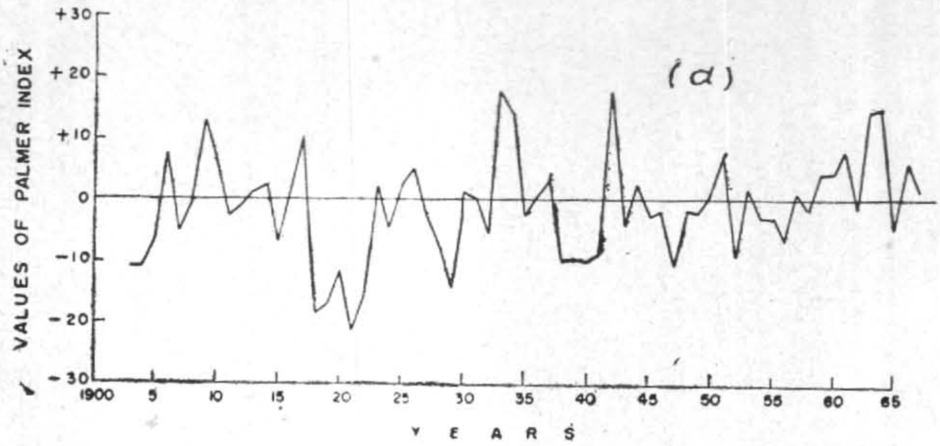
Fig. 3. East Uttar Pradesh, Jun-Sep (1904-1966)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag m=21 yr)

(5) *East Uttar Pradesh* — The spectrum is that of red noise. The spectral peaks for 6th and 7th lags correspond to frequencies of 7.33 and 6.28 years respectively and are significant at 95 per cent level. Other peaks are significant at 90 per cent level with corresponding periods of 2.2 and 2·1 years at 20th and 21st lags respectively (Fig. 3).

(6) *West Uttar Pradesh* — The spectrum is indicative of persistence. The spectral peak corresponding to 15th lag with a period of 2·9 years and



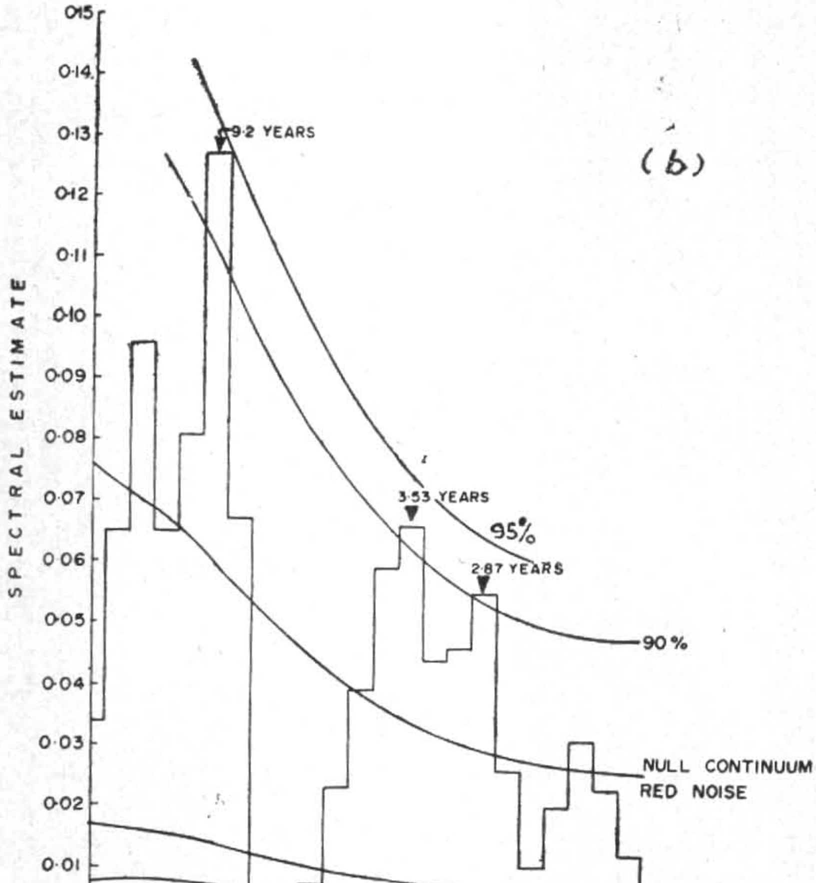


Fig. 4. Punjab & Haryana, Jun-Sep (1903-1967)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 22$ yr)

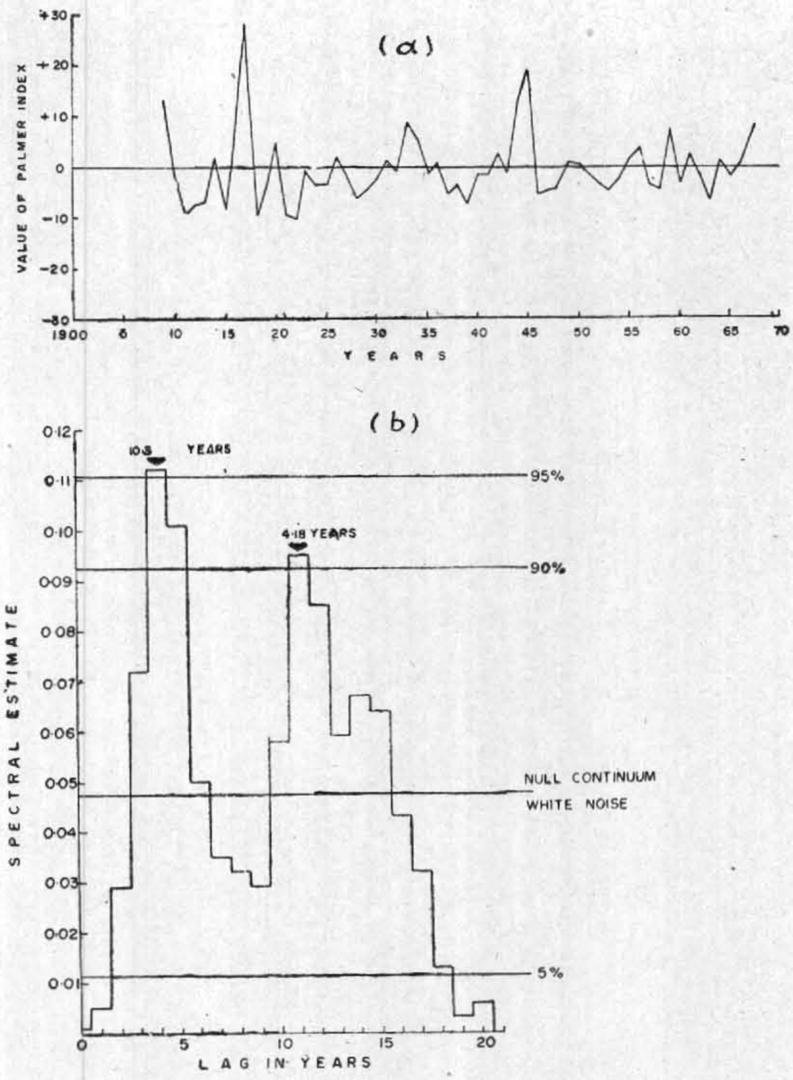


Fig. 5. West Rajasthan, Jun-Sep (1909-1967)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag m = 20 yr)

the other at the 5th lag with a period of 8·8 years are significant at 90 per cent level.

(7) *Punjab & Haryana* — The spectrum is that of red noise. All the three peaks are significant at 90 per cent level, with periods of 9·2, 3·53 and 2·87 years corresponding to 5th, 13th and 16th lags (Fig. 4).

(8) *West Rajasthan* — The spectrum has no persistence. The spectral peak at 4th lag corresponds to a period of 10·5 years and is significant at 95 per cent level. The other spectral peak significant at 90 per cent level observed at 11th lag corresponds to a period of 4·18 years (Fig. 5).

(9) *East Rajasthan* — The spectrum is that of red noise and does not indicate any periodicity (Fig. 6).

(10) *West Madhya Pradesh* — The spectrum is indicative of persistence. The spectral peaks for the 3rd and 4th lags corresponding to frequencies of 14·66 years and 11 years respectively are both observed at 90 per cent significance level (Fig. 7).

(11) *East Madhya Pradesh* — The spectrum is indicative of persistence. It does not indicate any periodicity at even 90 per cent level as it lies within the confidence limits of the population. The spectral estimate at 20th lag appears to be

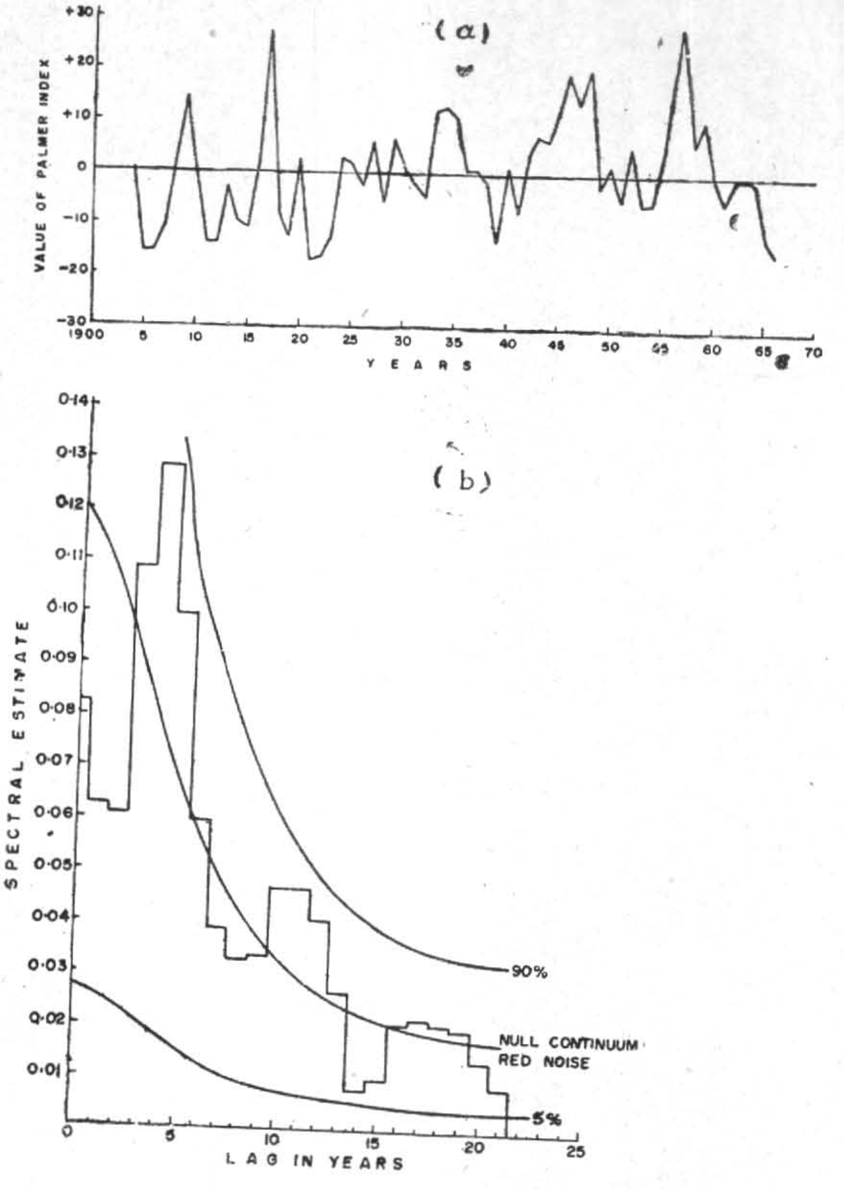


Fig. 6. East Rajasthan, Jun-Sep (1904-1966)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 21$ yr)

very near the 90 per cent significance level with corresponding periodicity of 2·2 years.

(12) *Gujarat* — The spectrum has no persistence. The spectral peak at 9th lag corresponding to frequency of 4·66 years is significant at 90 per cent level.

(13) *Saurashtra & Kutch* — The spectrum is indicative of persistence. The spectral peaks (*i*) at 13th lag corresponds to a period of 3·38 years and (*ii*) at 12th lag with 3·66 years and are significant at 90 per cent level.

(14) *Madhya Maharashtra* — The spectrum is indicative of persistence. Spectral peak at 9th lag corresponding to a period of 4·9 years is significant at 90 per cent level.

(15) *Marathwada* — The spectrum is that of red noise. The spectral peaks at lags 4th and 6th are significant at 95 per cent level with periods

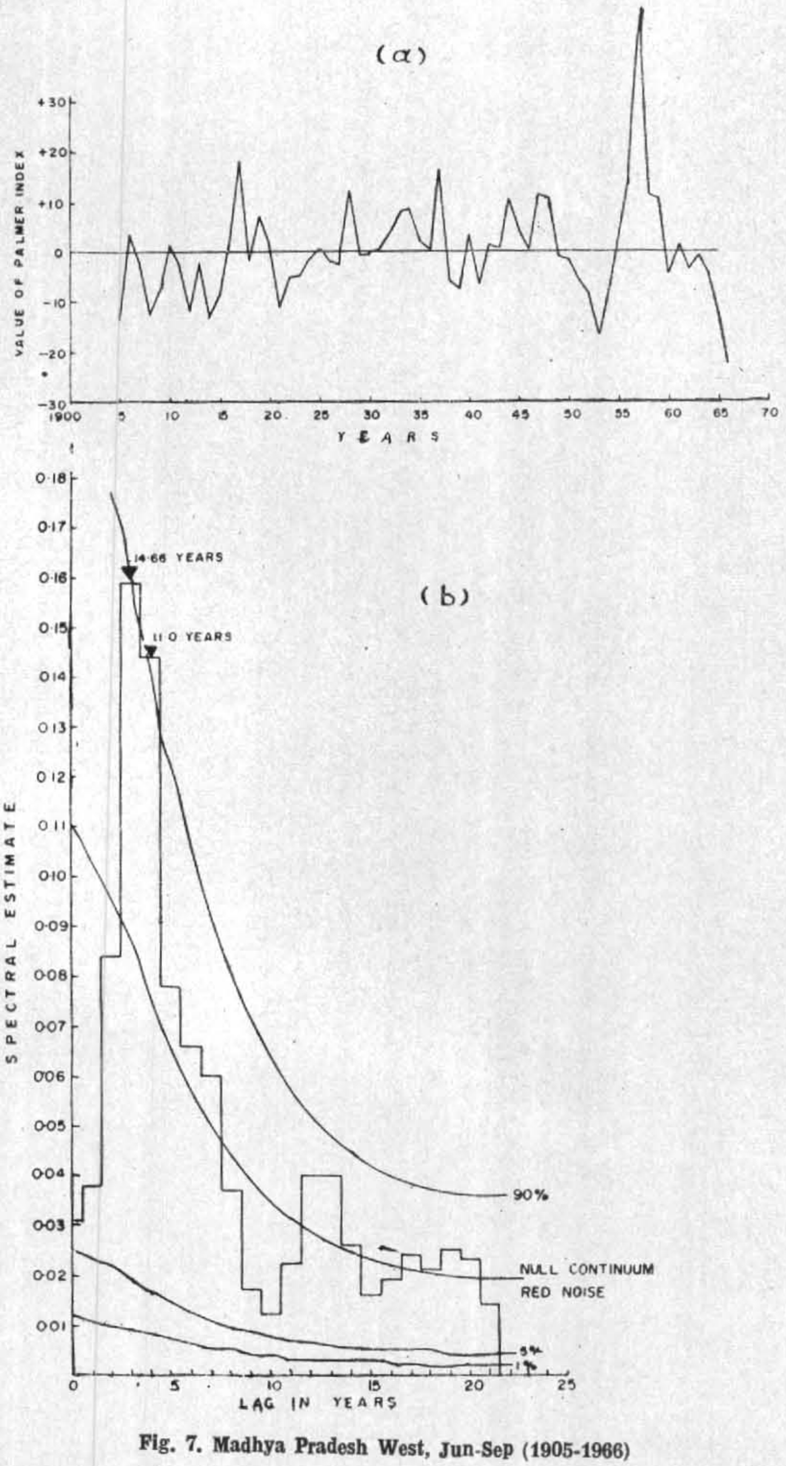


Fig. 7. Madhya Pradesh West, Jun-Sep (1905-1966)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 21$ yr)

of 8·4 years and 6 years respectively. The spectral peak at 8th lag is significant at 90 per cent level (and nearly so at 95 per cent) corresponding to a period of 5·4 years.

(16) *Vidarbha* — The spectrum is that of red noise and it does not indicate any periodicity.

(17) *Coastal Andhra Pradesh* — The spectrum

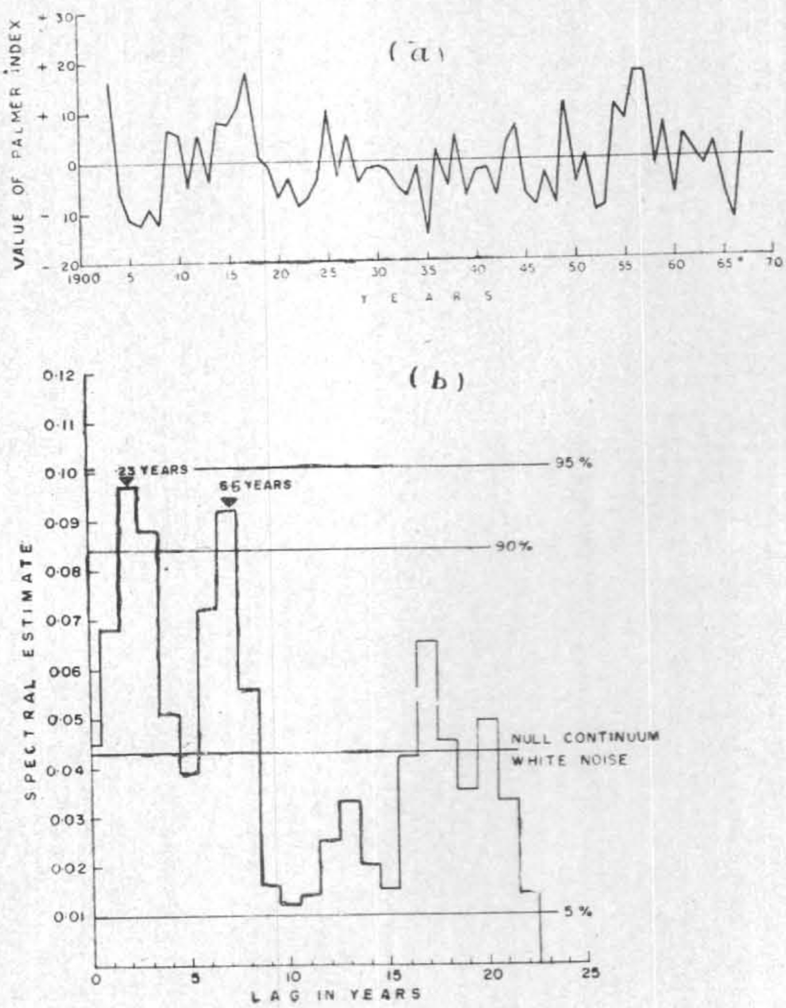


Fig. 8. Coastal Andhra Pradesh, Jun-Sep (1903-1967)

(*a*) Values of Palmer Index

(*b*) Power Spectrum of Palmer Index

(Max. lag $m = 22$ yr)

is that of white noise. The spectral peaks at 2nd lag corresponding to the period of 23 years are significant at 90 per cent level. Another peak at same significance level is at the 7th lag corresponding to a period of 6·7 years (Fig. 8).

(18) *Telangana* — The spectrum is that of white noise, *i.e.*, it has no persistence. Spectral peak significant at 90 per cent level and almost significant at 95 per cent level is observed at 5th lag with a period of 8·8 years (Fig. 10).

(19) *Rayalaseema* — The spectrum is indicative of persistence. The spectral peak at 18th lag corresponding to a period of 2·4 years is significant at 90 per cent level.

(20) *Tamil Nadu* — The spectrum is that of white noise. Two spectral peaks significant at 90 per cent level are indicative of periods (*i*) 3·07 years at the 15th lag and (*ii*) 6·6 years at 7th spectral lag.

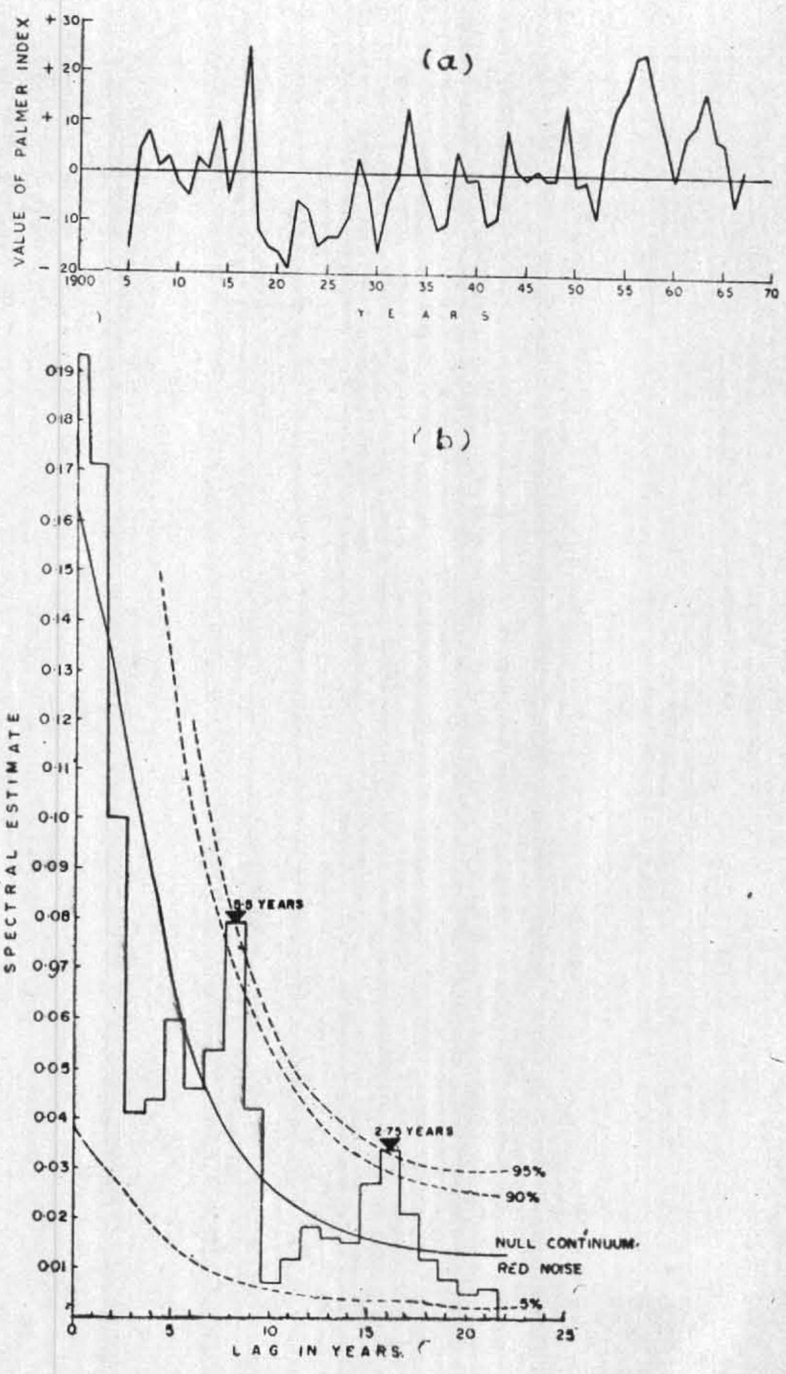


Fig. 9. Interior Mysore North, Jun-Sep (1905—1967)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 21$ yr)

(21) *Interior Mysore North* — The spectrum is indicative of persistence. It has two peaks corresponding to periods significant at 95 per cent level: (*i*) 2·75 years at the 16th lag and (*ii*) 5·5 years at 8th spectral lag (Fig. 9).

(22) *Interior Mysore South* — The spectrum is of red noise with a period of 8·8 years touching 90 per cent significant level corresponding to 6th lag.

(23) *Kerala*—The spectrum is that of white noise. It has one peak significant at 90 per cent level at 10th spectral lag with corresponding period of 4·6 years.

TABLE 1

S. No.	Sub-Division	No. of years of data used	Periodicity	Level of significance (per cent)	Lag No.	No. of degree of freedom $(2N-M/2)/M$	Type of spectrum
1	Gangetic West Bengal	67 (1902-1968)	4·6 3·53	95 90	10 13	5	White noise
2	Orissa	67 (1902-1968)	3·83	90	12	6	Do.
3	Bihar Plateau	66 (1902-1967)	44 3·14	95 95	1 14	6	Do.
4	Bihar Plains	66 (1902-1967)	2·3	90	20	5	Red noise
5	Uttar Pradesh East	65 (1902-1966)	7·33 6·28 2·2 2·1	95 95 90 90	6 7 20 21	6	Do.
6	Uttar Pradesh West	65 (1902-1966)	8·8 2·9	90 90	5 15	6	Do.
7	Punjab & Haryana	66 (1902-1967)	9·2 3·53 2·87	95 90 90	5 13 16	5	Do.
8	Rajasthan West	62 (1908-1969)	10·5 4·18	95 90	4 11	6	White noise
9	Rajasthan East	64 (1903-1966)	Nil			6	Red noise
10	Madhya Pradesh West	66 (1902-1967)	14·66 11	95 95	3 4	6	Do.
11	Madhya Pradesh East	66 (1902-1967)	2·2	90	20	6	Do.
12	Gujarat	64 (1903-1966)	4·66	90	9	6	White noise
13	Saurashtra & Kutch	63 (1904-1966)	3·66 3·38	90 90	12 13	5	Red noise
14	Madhya Maharashtra	63 (1904-1966)	4·9	90	9	5	Do.
15	Marathwada	64 (1903-1966)	8·4 6·0 5·4	95 95 90	5 7 8	6	Do.
16	Vidarbha	66 (1902-1967)	Nil		—	6	Do.
17	Coastal Andhra Pradesh	66 (1902-1967)	23 6·6	90 90	2 7	5	White noise
18	Telangana	66 (1902-1967)	8·8	90	5	6	Do.
19	Rayalaseema	66 (1902-1967)	2·4	90	18	6	Red noise
20	Tamil Nadu	67 (1902-1968)	6·6 3·07	90 90	7 15	6	White noise
21	Interior Mysore North	64 (1904-1967)	5·5 2·75	95 95	8 16	6	Red noise
22	Interior Mysore South	65 (1903-1967)	7·5	90	6	6	Do.
23	Kerala	66 (1902-1967)	4·6	90	10	5	White noise

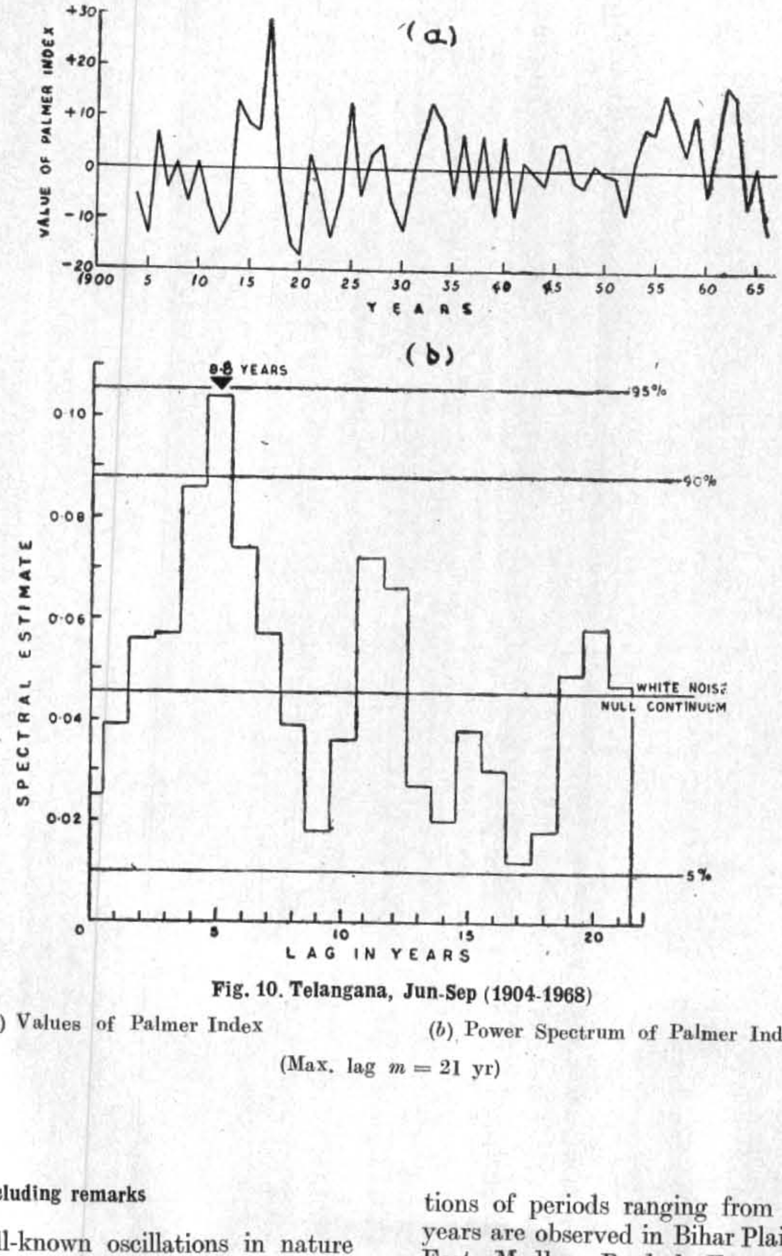


Fig. 10. Telangana, Jun-Sep (1904-1968)

(*a*) Values of Palmer Index (*b*) Power Spectrum of Palmer Index

(Max. lag $m = 21$ yr)

5. Discussion and concluding remarks

Two of the well-known oscillations in nature are: (*i*) Quasi-biennial oscillation of the equatorial zonal wind flow, and (*ii*) Sunspot cycles. The present study has been made with a view to find out whether there is any periodicity in drought incidence which can be identified with these oscillations.

Some persistence in drought occurrence is, however, seen in north India in the belt extending from Bihar Plains upto East Rajasthan. In the Peninsula, similar feature is seen in Madhya Maharashtra & Marathwada, Rayalaseema and Interior Mysore. A few spectral peaks exceed the 90 per cent level of significance in some areas and these are summarised in Table 1. Indications of periods ranging from 2·3 years to 2·5 years are observed in Bihar Plains, Uttar Pradesh East, Madhya Pradesh East and Rayalaseema. These correspond in period closely to quasi-biennial oscillation. Similarly, periods ranging from 10·5 to 11 years corresponding to sunspot cycle are indicated in Rajasthan West and Madhya Pradesh West.

The data available are for a period of about 60 years only. The above preliminary results would need to be further critically examined with reference to the observed quasi-biennial wind oscillation and sunspot cycles before conclusions can be drawn about periodicity of drought occurrence even in these limited areas. But the overall picture does not encourage one to look with confidence for periodicity in droughts.

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