Indian J. Met. Geophys. (1973), **24**, **4**, 345-352

551·553·21 (540)

Dynamical parameters derived from analytical functions representing Indian monsoon flow

S. T. AWADE and G. C. ASNANI

Indian Institute of Tropical Meteorology, Poona

(*Received 1 March 1973*)

ABSTRACT. Normals of July upper wind (zonal component only) along Long. $77\frac{1}{2}$° E were considered between Lat. 5°N and 25°N, at isobaric levels from 850 to 150 mb at interval of 50 mb. Parabola of the type $u=a+b\log_{10}p+c(\log_{10}p)^2$ was fitted by the least square method to the zonal wind at each latitude at an interval of $2\frac{1}{2}$-degree latitude. To the values of a, b and c at each latitude, a parabola of the type $a=a_0+a_1\phi+a_2\phi^2$ was fitted. Here u is zonal wind in kt, p is pressure in mb and ϕ is latitude in degrees. The fitted values of u agree well with observed values.

Assuming geostrophic approximation through thermal wind relationship, meridional profile of temperature at each isobaric level has been determined in analytical form and compared with observed temperatures. Assumption of geostrophic balance in respect of climatological zonal winds appears to be justified. The method therefore gives detailed structure of temperature field and other parameters in a quantitative form which is otherwise difficult to build through ordinary analysis of temperature values at individual stations.

Static stability, vertical shear and absolute vorticity on isobaric and on isentropic surfaces are calculated with the help of these values. It is found that the zonal current is inertially stable and also barotropically stable along $77\frac{1}{2}$°E between 5° and 25° N.

The analysis of maximum wind suggests that the easterly jet maximum is perhaps close to Lat. 9°N.

1. Introduction

Wind shears in the vertical and in the horizontal, temperature gradient in meridional direction, static stability, Richardson number and absolute vorticity are some of the parameters essential for understanding of the dynamics of zonal current (Charney 1947, Fjortoft 1950, Kuo 1956 etc). It is possible to calculate these parameters at individual stations straightway from observations, but to get smooth values representing large scale features, it is better to have recourse to smoothing process. We feel that to get smooth profile, it is better to fit an analytical function to one basic parameter and to derive all other parameters from this function. We choose zonal wind as a basic parameter and derive all other quantities from the distribution of this zonal wind.

July is a representative monsoon month for the Indian region. The region of India and neighbourhood from equator to 25°N is fairly free from high mountains and is also well in the grip of summer monsoon. We consider this month suitable for the study of the zonal current. Longitude $77\frac{1}{2}$°E was taken as representative of the Indian region.

We have taken the layer from 850 to 150 mb for the purpose of fitting a curve in the vertical. Easterly jet stream maximum occurs between 150 and 100 mb. We thought that inclusion of 150 to 100 mb layer along with that from 850 to 150-mb layer will necessitate the fitting of a very complicated curve. Aiming at simplicity of a curve, we confined ourselves to the layer from 850 to 150 mb.

From the analysis of the flow pattern, we found that the latitudinal belt from equator to 5°N could not be easily taken along with the region north of 5°N. While we fitted analytical function north of 5°N, we used only graphical interpolation from 5°N to equator.

However, in fixing the value of zonal wind from 5°N to equator, we were somewhat guided by the consideration that at the levels of weak meridional flow, the relative vorticity of zonal wind may be kept as zero at the equator and absolute vorticity may not become negative north of the equator.

Thus, we have fitted analytical functions from 5°N to 25°N and from 850 mb to 150 mb, along the longitude $77\frac{1}{2}$°E during the month of July. We adopted the graphical method of smoothing from 5°N to equator,

TABLE 1

Observed and fitted zonal wind component (kt)

Level (mb)	0°N Obs	0°N Fit	2·5°N Obs	2·5°N Fit	5°N Obs	5°N Fit	7·5°N Obs	7·5°N Fit	10°N Obs	10°N Fit
850	7·0	7·0	13·6	14·4	19·0	21·5	21·8	23·2	22·3	23·9
800	8·0	8·0	16·0	15·4	20·8	20·2	22·0	22·0	22·0	22·7
700	13·0	13·8	17·8	14·9	18·2	17·1	21·0	18·8	20·5	19·5
600	8·0	9·5	10·0	10·6	12·0	12·7	13·0	14·4	13·0	15·0
500	6·5	3·8	6·6	4·6	7·0	6·5	7·2	7·9	7·2	8·6
400	—2·6	—4·5	—2·4	—3·9	—2·5	—2·6	—2·2	—1·4	—0·5	—0·8
300	—17·0	—17·4	—16·6	—17·4	—16·2	—16·9	—15·5	—16·4	—13·0	—15·9
200	—39·0	—40·7	—41·0	—41·2	—44·0	—41·9	—48·0	—42·5	—43·0	—42·3
150	—50·0	—50·0	—55·6	—57·2	—61·0	—63·1	—63·0	—64·7	—63·8	—64·7

TABLE 1 *(contd)*

Level (mb)	12·5°N Obs	12·5°N Fit	15°N Obs	15°N Fit	17·5°N Obs	17·5°N Fit	20°N Obs	20°N Fit	22·5°N Obs	22·5°N Fit	25°N Obs	25°N Fit
850	23·0	23·4	25·2	21·8	24·6	19·1	18·0	15·3	11·0	10·4	6·0	4·4
800	21·0	22·2	19·4	20·7	18·2	18·1	14·0	14·3	9·0	9·5	4·0	3·6
700	19·0	19·1	18,0	17·7	16·8	15·3	13·0	11·8	6·5	7·2	1·0	1·6
600	12·7	14·8	12·0	13·6	10·5	11·4	7·5	8·3	2·0	4·2	—1·0	—0·7
500	6·5	8·5	6·0	7·6	3·5	5·9	1·0	3·4	—2·0	0·1	—2·5	—3·9
400	1·2	—0·7	0·5	—1·1	—2·2	—2·1	—5·5	—3·6	—7·8	—5·5	—6·0	—8·1
300	—9·5	—15·4	—10·4	—15·0	—13·0	—14·7	—15·0	—14·4	—14·0	—14·2	—11·8	—14·0
200	—40·0	—41·2	—38·0	—39·3	—37·0	—36·6	—35·0	—33·1	—31·0	—28·7	—20·0	—23·5
150	—63·0	—63·2	—61·0	—60·0	—57·0	—55·2	—51·0	—48·8	—43·0	—40·7	—31·0	—31·1

Obs—Observed Fit—Fitted

2. Data and computational procedure

For the present study, *Normals (1965) of Rawin winds based on afternoon data,* issued by India Meteorological Department were the main source of data.

2.1. *Fitting a function to the zonal wind*

The zonal component u of the normal wind was plotted at each isobaric level for the Indian region. Smooth isopleths were drawn and values read along $77\frac{1}{2}°$ E at $2\frac{1}{2}$-degree latitude interval from 5° to 25°N. At each latitude point, a smooth vertical profile was drawn for u *versus* p. From this profile, the values of u were picked up at 50-mb interval, from 850 to 150 mb. Thus at each latitude point, we had 15 values along the vertical. We fitted, by the least square method, the parabolic curve,

$$u = a + b \log_{10} p + c (\log_{10} p)^2 \tag{1}$$

where, u is in kt and p is in mb. At each isobaric level from 5° to 25°N, we had nine values of a, b and c. To these nine values of a, b and c, we fitted by the least square method, the curve

$$a = a_0 + a_1 \phi + a_2 \phi^2 \qquad 2(a)$$

$$b = b_0 + b_1 \phi + b_2 \phi^2 \qquad 2(b)$$

$$c = c_0 + c_1 \phi + c_2 \phi^2 \qquad 2(c)$$

where, ϕ represents the latitude in degrees. Thus we had the fitted function :

$$u = (a_0 + a_1 \phi + a_2\phi^2) + (b_0 + b_1 \phi + b_2 \phi^2) (\log_{10} p) + (c_0 + c_1\phi + c_2\phi^2) (\log_{10} p)^2 \tag{3}$$

The consequence of second degree fit for u at an isobaric level is that over the distance for which the fit has been attempted, relative vorticity ζ varies linearly with y. This has analogy with the usual β-plane approximation in which coriolis parameter f is assumed to vary linearly with y. Combination of the two assumptions gives linear variation of absolute vorticity with y between 5° and 25°N.

2.2. *Goodness of fit*

An analysis of variance for the parabolic curve u against $\log_{10}p$ shows that the linear and second degree terms are significant for all latitudes even at 1 per cent level.

Similar analysis of variance for parabolic curve fitted to coefficients a, b and c against latitude also shows that first degree and second degree terms are significant for all the coefficients at

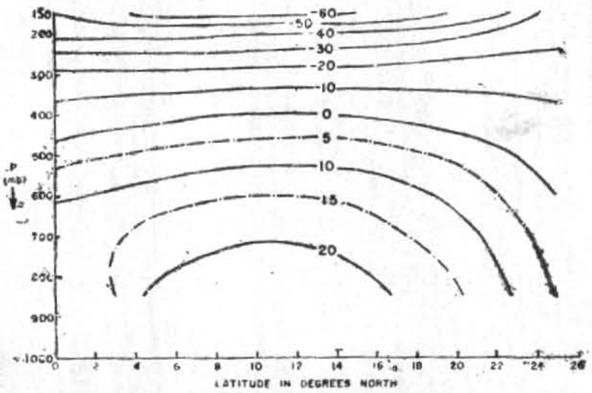

Fig. 1

Vertical cross-section of zonal wind (fitted values) in kt. Positive values indicate westerlies

1 per cent level except for the coefficient c which is significant at 2·5 per cent level.

Table 1 gives observed and fitted zonal winds. It will be seen that the difference between the observed and fitted values rarely exceeds two knots and only at four places it exceeds four knots. Thus, the fitted values agree very well with the observed values. As explained in Section 2·3 below, the fitting was done by graphical interpolation south of 5°N. All subsequent calculations referred to in this paper for latitude belt 5° to 25°N were done on the basis of Eq. (3). The vertical cross-section of fitted u is shown in Fig. 1. Positive u's indicate westerlies and negative u's indicate easterlies.

The values of coefficients are given below—

$$\left.\begin{array}{lll} a_0 = -673{\cdot}20, & a_1 = -56{\cdot}94, & a_2 = 2{\cdot}93 \\ b_0 = 426{\cdot}43, & b_1 = 39{\cdot}76, & b_2 = -2{\cdot}03 \\ c_0 = -65{\cdot}41, & c_1 = -6{\cdot}73, & c_2 = 0{\cdot}34 \end{array}\right\} \tag{4}$$

2.3. *Calculation of u south of 5°N*

From the calculations of Section 2·2 above, we had at 5°N, the values of u and $\partial u/\partial y$ at 50-mb interval. We also had the observed normal wind values at equator (Gan Island, Lat. 0·7°S, Long. 73·1°E). We did the graphical interpolation for u and $\partial u/\partial y$ between 5°N and the equator, keeping $\partial u/\partial y$ nearly zero at the equator at levels where meridional component of normal wind is small. Using these interpolated values of u between equator and 5°N at various isobaric levels, we calculated other parameters using geostrophic relationship through shear wind wherever necessary.

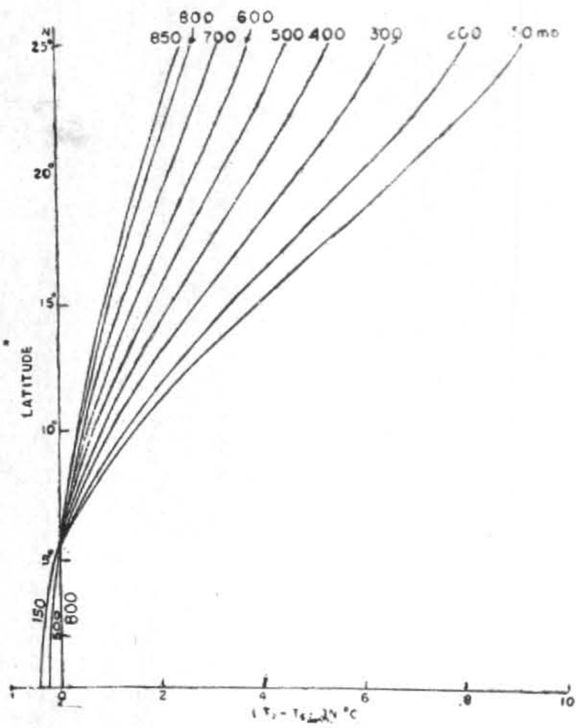

Fig. 2

Latitudinal variation of temperature at different isobaric levels

TABLE 2

Vertical wind shear in kt per 100 mb

Lat. (°N)	p (mb) 850	800	700	600	500	400	300	200	150
0		—4.0	—1.0	4·9	6·9	10·4	17·1	22·7	22·1
5	2·3	2·7	3·7	5·2	7·5	11·2	18·2	34·3	52·3
7·5	2·2	2·7	3·7	5·3	7·7	11·6	19·0	35·9	54·9
10	2·2	2·6	3·7	5·3	7·7	11·7	19·2	36·4	55·6
12·5	2·2	2·6	3·6	5·2	7·5	11·5	18·8	35·5	54·3
15	2·1	2·5	3·5	4·9	7·2	10·8	17·7	33·4	51·0
17·5	2·0	2·4	3·3	4·6	6·6	9·9	16·0	30·1	45·8
20	1·9	2·2	3·0	4·1	5·8	8·6	13·7	25·5	38·6
22·5	1·8	2·0	2·6	3·5	4·8	6·9	10·8	19·6	29·4
25	1·6	1·8	2·2	2·7	3·6	4·9	7·3	12·6	18·2

2·4. *Calculation of vertical wind shear*

From Eq. (3), it follows that —

$$\frac{\partial u}{\partial p} = \frac{1}{p}[(b_0+b_1\phi+b_2\phi^2) + 2\,(c_0+c_1\phi+ + c_2\phi^2)\log_{10}p]\log_{10}\mathrm{e} \qquad (5)$$

We employed Eq. (5) to calculate $\partial u/\partial p$ between Lat. 5° and 25°N. Vertical wind shears south of 5°N were calculated graphically. The vertical wind shear expressed in kt/100 mb is shown in Table 2.

2·5. *Calculation of temperature*

Here we introduced geostrophic approximation through thermal wind relationship :

$$\frac{\partial u}{\partial p} = \frac{R}{fp}\;\frac{\partial T}{\partial y} \qquad (6)$$

Hence,

$$\frac{\partial T}{\partial y} = \frac{f}{R}[(b_0+b_1\phi + b_2\phi^2) + 2(c_0+c_1\phi + +c_2\phi^2)\log_{10}p]\log_{10}\mathrm{e} \qquad (7)$$

Integrating Eq. (7) with respect to ϕ, we get :

$$T_{\psi,p} - T_{\psi_0,p} = \log_{10}\mathrm{e}.\ r.\ 2\Omega\Big[-\ b_0\cos\psi + +b_1(\sin\psi - \psi\cos\psi)+b_2\{2\,\psi\sin\psi - -(\psi^2-2)\cos\psi\} +2\log_{10}p\,\{-c_0\cos\psi + c_1(\sin\psi - \psi\cos\psi) + c_2\{2\psi\sin\psi - -\overline{\psi^2-2)}\cos\psi\}\Big]_{\psi_0}^{\psi} \qquad (8)$$

where ψ is the latitude in radians, ϕ is latitude in degrees, Ω is the angular velocity of earth and r is radius of the earth. We took ψ_0 corresponding to latitude 5°N. The R.H.S. of Eq. (8) then gives the latitudinal variation of temperature from 5°N to any latitude for which Eq. (8) is valid. These profiles for various isobaric levels are shown in Fig. 2. In this diagram, 5°N has been taken as the reference latitude.

If we know the absolute value of temperature at 5°N at a pressure level, we can get through Eq. (8) or through Fig. 2 the value of temperature at any latitude at the same pressure level. The values of temperature at 5°N were obtained from analysis of radiosonde observations in that region. From this, we obtained temperature values at various latitudes and at different pressure levels. These were compared with radiosonde temperature values and were found to agree very well. This gives an indirect confirmation that thermal wind relationship which is based on geostrophic relationship, holds very well over the Indian region 5° to 25°N for climatological zonal winds in the monsoon season.

Eq. (6) was used to evaluate temperature at isobaric levels south of 5°N. For use of this equation, values of $\partial u/\partial p$ as determined graphically in Section 2·4 were used. The temperature profiles thus derived for latitude south of 5°N are also shown in Fig. 2. Due to extreme closeness of the lines, the graphs are shown only for pressure levels at 800, 500 and 150 mb.

2.6. *Calculation of static stability*

There are various measures of static stability. Here, we present only one measure of static stability given by —

$$S = -\frac{\alpha}{\theta}\frac{\partial\theta}{\partial p} = \frac{R}{p^2}\left(KT - \frac{\partial T}{\partial \log_e p}\right) \quad (9)$$

in the usual notation.

This parameter occurs in thermodynamic energy equation and is frequently used in dynamical meteorology. The values of T were available from Section 2·5. The plot of T against $\log_e p$ at all latitudes showed a linear relationship between 850 and 150-mb levels of the form :

$$S \approx \frac{R}{p^2}\left(KT - \frac{T_{850} - T_{150}}{\log_e 850 - \log_e 150}\right) \quad (10)$$

The values of S so obtained are given in Table 3. It will be seen that S increases with latitude as well as height. However, its variation with latitude is not large but its variation with height is substantial; it increases by one order of magnitude from 850 to 150-mb level.

2·7. *Calculation of* $-\partial u/\partial y$ *on isobaric surfaces*

From Eq. (3), we immediately obtained $-(\partial u/\partial y)$ on the isobaric surfaces. This is the contribution of zonal current to relative vorticity. Figs. 3(a) and 3(b) give vertical variation of $-(\partial u/\partial y)$ and $f-(\partial u/\partial y)$ respectively. It is interesting to find from Fig. 3(a) that $-(\partial u/\partial y)$ is nearly zero at all levels around latitude 10°N. Further, at levels below 300 mb this quantity is negative (anticyclonic vorticity) south of 10°N and positive (cyclonic vorticity) to the north. The opposite holds above 300-mb level.

TABLE 3

Static stability $S=-(\alpha/\theta)(\partial\theta/\partial p)$

(Unit : 10^{-4}m sec^{-2} mb^{-2})

p (mb)	Lat. (°N) 5	10	15	20	25
850	139	142	147	153	157
700	191	196	204	213	219
500	324	334	350	370	383
300	667	698	749	808	849
200	1044	1121	1244	1387	1487
150	1411	1554	1785	2052	2236

2·8. *Calculation of* $-(\partial u/\partial y)$ *on isentropic surfaces*

For inertial stability of zonal current, we have the criterion $f-(\partial u/\partial y)_\theta \geqslant 0$, where subscript θ denotes differentiation along an isentropic surface (Kuo 1956). This criterion of Kuo is based on linearised theory of infinitesimal perturbation on a zonal current of finite relative vorticity. The authors are not aware of theoretical extension of this criterion, but it appears plausible to infer from Kuo's work that in his criterion, vorticity arising out of both zonal and meridional motion can perhaps replace the vorticity of the zonal current. In other words, a fluid layer is inertially stable if $f+[\partial v/\partial x-\partial u/\partial y]_\theta \geqslant 0$.

Analysis of normal winds shows that between the equator and latitude 5°N there is appreciable meridional component also along with the zonal component. Hence $\partial v/\partial x$ would also contribute significantly to relative vorticity in this region. Elsewhere, *i.e.*, north of 5°N, the contribution $\partial v/\partial x$ appears to be small compared to that of $-\partial u/\partial y$. Hence it appears reasonable to conclude from Fig. 4 that absolute vorticity on an isentropic surface north of 5°N is positive and hence to conclude that normal flow in the monsoon season is inertially stable, at least north of Lat. 5°N.

2·9. *Calculation of jet maximum*

On inspection of wind between equator and 25°N, it was found that the easterly wind maximum lay north of 5°N. Hence we could use Eq. (3) for the analysis of wind maximum. From the condition $\partial u/\partial \phi = 0$, one gets the latitude of maximum wind at constant pressure

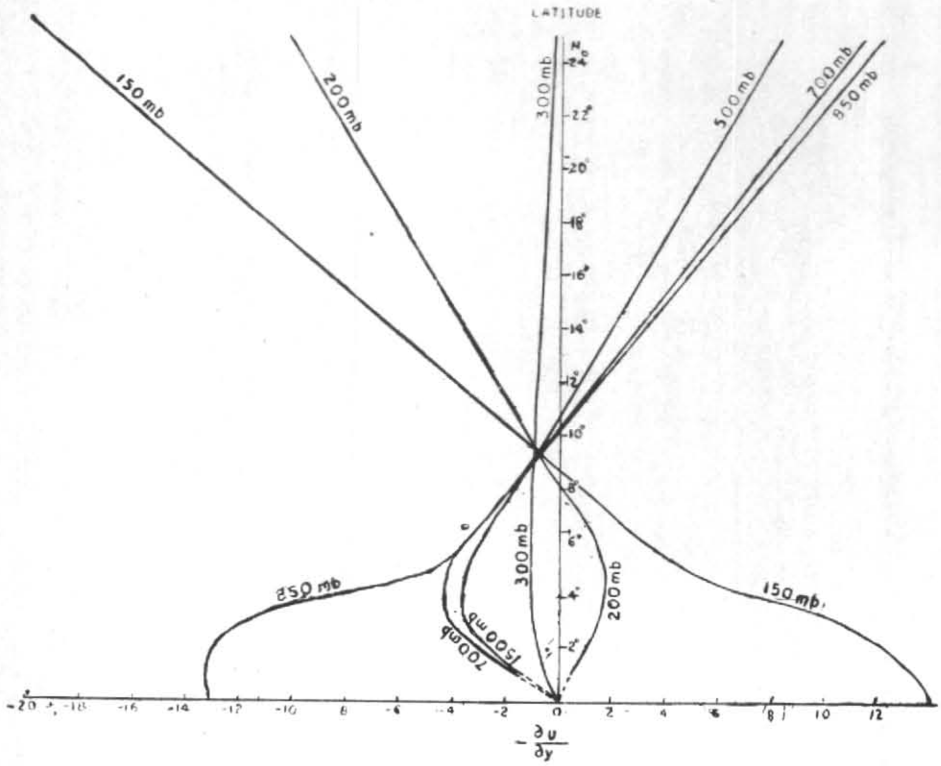

Fig. 3(a)

Values of $-(\partial u/\partial y)$ on isobaric surfaces
(units : 10^{-6} sec^{-1})

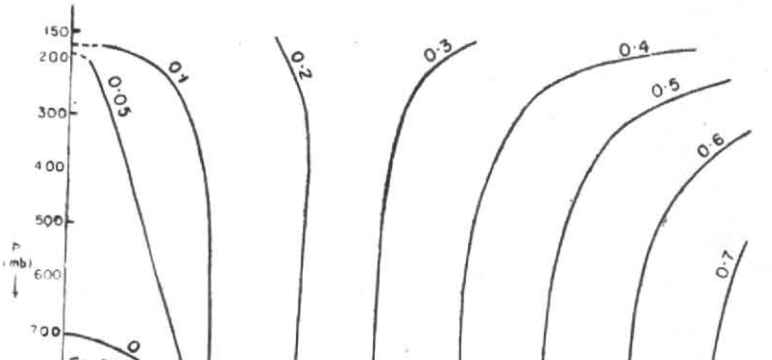

Fig. 3(b)

Vertical cross-section of $(f-\partial u/\partial y)$ on isobaric surfaces
(units : 10^{-4} sec^{-1})

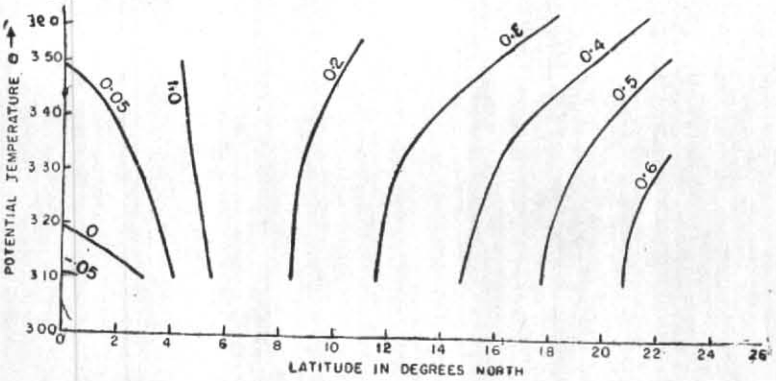

Fig. 4

Vertical cross-section of $(f - \partial u/\partial y)$ on isentropic surfaces (units : 10^{-4} sec^{-1})

surface as :

$$\phi = -\frac{a_1 + b_1 \log_{10} p + c_1 (\log_{10} p)^2}{2\{a_2 + b_2 \log_{10} p + c_2 (\log_{10} p)^2\}} \tag{11}$$

Values of ϕ and the corresponding values of maximum wind are given below :

p (mb)	Lat. of max. wind (°N)	Values of easterly max. wind (kt)
150	8·80	64·9
200	8.04	42·5
250	5·53	27.5

3. Discussion and Conclusions

3.1. As already stated, there is close agreement between the observed and the fitted zonal winds as well as the temperatures. This suggests two things :

(*i*) Method of smoothing adopted in this paper is reasonably sound.

(*ii*) Assumption of geostrophic balance of climatological zonal winds in the region of Indian monsoon appears to be justified.

The method has yielded a detailed structure of Indian monsoon zonal current in a quantitative manner which would be hard to realise without adopting some smoothing procedure of the type adopted here. With the availability of this detailed quantitative structure, one can study numerous dynamical characteristics of the zonal current of the Indian monsoon. A few of the characteristics are presented below. Further study is in progress.

3.2. *Zonal current is inertially stable* (Sec 2·8).

3.3. As further stated in section 2·8, there is appreciable meridional component in the wind south of 5°N. Hence $\partial u/\partial y$ does not represent the whole of relative vorticity in this region. However, north of 5°N, total motion is nearly zonal and hence $-(\partial u/\partial y)$ can be taken approximately as the relative vorticity of the mean flow. If the profile of $(f - \partial u/\partial y)$ along Y-direction shows a maximum then the zonal current would be barotropically unstable (Kuo 1949). Eq. (3) assumes u as a 2nd degree function of y. In other words, $\partial^2 u/\partial \phi^2$ is approximately same at any given constant pressure level. The values of $-(\partial^2 u/\partial y^2)$ in units of 10^{-11}m^{-1}sec^{-1} at various levels from the region north of 5°N are given below :

	Level (mb)				
	850	800	700	600	500
$-\frac{\partial^2 u}{\partial y^2}$	0·76	0·75	0·71	0·65	0·54

	Level (mb)			
	400	300	200	150
$-\frac{\partial^2 u}{\partial y^2}$	0·36	0·4	—0·56	—1·10

The value of β decreases from $2{\cdot}28 \times 10^{-11}$m^{-1} sec^{-1} at 5°N to $2{\cdot}07 \times 10^{-11}$m^{-1}sec^{-1} at 25°N. From these values of β and $-(\partial^2 u/\partial y^2)$, it is clear that $(\beta - \partial^2 u/\partial y^2)$ is positive throughout the region. In other words, absolute vorticity monotonically increases with latitude and there is no maximum or minimum in its values in the region under consideration. In other words, the *mean zonal current is barotropically stable along 77 ½°E between 5°N and 25°N.*

3·4. *Latitudinal position of easterly jet maximum*

We have already shown in Section 2·9 that the latitude of the maximum easterly wind at 150-mb level is close to 9°N. From the variation in latitude of maximum wind as given by Eq. (11) in Section 2·9 and from the numerical values given in the same section, it is seen that the *surface of maximum wind slopes upwards as we go northwards.* This is in agreement with the finding of Mokashi (1971) who analysed mean monthly data of individual stations south of 20°N.

Inspection of vertical profiles of zonal wind has shown that the easterly wind minimum occurs above 150-mb level but below 100-mb level. In Section 2·9, we used Eq. (11) to locate the position of maximum wind speed at different constant pressure surfaces. Eq. (11) is strictly valid at and below 150-mb level. If we extrapolate and apply Eq. (11) upto 130-mb level, we get maximum easterly wind of 77·2 kt at Lat. 8·97°N at that level. Combining this with corresponding figures for 200 and 150 mb figures given in Section 2·9, one comes to the conclusion that *climatologically, the maximum easterly wind during July lies somewhere close to 9°N.*

According to Koteswaram (1958), the summer easterly jet has a maximum around 15°N, between 150 and 100-mb levels. At the time of his analysis, data were sparse and Koteswaram was the first to identify the existence of easterly jet over the Indian region. The present analysis suggests that the maximum is likely to be found around 9°N rather than 15°N. But it must be admitted that the *horizontal shear in this easterly jet system is very weak,* as seen from Fig. 1 itself. Hence, the location of the latitude of maximum wind in the horizontal comes to be more of academic interest at the moment.

Acknowledgements

The authors have greatly benefited from discussions with their colleagues Shri C. M. Dixit, Dr. R. N. Keshavamurty and Shri S. K. Mishra. Shrimati Leela George and Shri M. B. Gajare gave valuable assistance in computations.

REFERENCES

Charney, J. G. 1947 *J. Met.*, **4**, 5, pp. 135-162.

Fjortoft, R. 1950 *Geofys. Publ.*, **17**, 6, pp. 5-52.

Koteswaram, P. 1958 *Tellus*, **10**, 1, pp. 43-57.

Kuo, H. L. 1949 *J. Met.*, **6**, pp. 105-122.

1956 *Ibid.*, **13**, pp. 561-568.

Mokashi, R. Y. 1970 *India met. Dep. Sci. Rep.*, 156.