

Wave Prediction for the east coast of India under storm conditions in the Bay of Bengal

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ABSTRACT. Cyclonic storms are frequent in the Bay of Bengal particularly during the NE monsoon period. Some of these storms are severe and generate high waves which cause havoc in the coastal regions. This paper presents an analysis of the cyclonic storm which hit coastal Andhra Pradesh on 7 November 1969.

Wave prediction under storm conditions involves an analysis of moving fetches and variable wind speeds. Wilson's graphical method incorporating the latest available wave prediction relations was used for wave prediction. The predicted waves which are deep water waves, were modified to account for refraction, shoaling and bottom friction effects as they enter shallower waters. The predicted waves were compared with the waves observed by the Visakhapatnam outer harbour authorities.

The results of the analysis suggest that (i) Wilson's graphical method can be applied for wave prediction for Indian coasts under storm conditions, (ii) the recommended value of bottom friction factor appears to be low and (iii) waves of considerable height are experienced even in areas not in the direct path of the cyclone.

1. Introduction

The planning, design, construction and maintenance of a coastal structure require information about waves in the area. The wave data must be such that the structure is secure and operative at all times. One of the ways of obtaining wave data is through wave recorders and the analysis of wave records. But, on Indian coasts we have few such recorders, and alternatives have to be sought to get the necessary wave data. The only alternative appears to be prediction of waves from known or predicted meteorological conditions. This problem of wave prediction is being actively investigated in the Coastal Engineering Section of the Karnataka Regional Engineering College, Surathkal. The present paper discusses one aspect of the problem in detail—the problem of storm surge (the sea level rise above the normal tide level as a result of low barometric pressure and the wind set up) is not included in the present paper.

2. Wave prediction techniques

The object of wave prediction is to obtain wave characteristics from known wind characteristics. As the physical processes involved in the generation of waves by wind cannot be regarded as completely

known, the relation between wind and wave characteristics can only be empirical. The variables involved are the significant wave height (H), significant wave period (T), wind velocity (U), wind duration (t) and length of fetch (F). Such relations are incorporated in wave forecasting techniques such as, the Pierson-Neumann-James method (Pierson *et al.* 1955) and the Sverdrup-Munk-Bretschneider method (Bretschneider 1952, 1958 and 1970). Reference may also be made to *Shore Protection, Planning and Design* (Army Corps of Engineers, U.S.A., *Tech. Rep.* 4, 1966) for details of these methods. Dattatri and Renukaradhya (1970, 1971) have concluded that these forecasting methods predict reasonably well waves for the west coast of India.

Basically these forecasting methods are applicable when the generating areas (the area of water surface over which the wind blows in essentially constant direction, generating the waves) are fairly stationary and the wind reasonably steady.

In the Bay of Bengal depressions, storms and cyclonic storms are common occurrences and these results in a continuously changing wind field. The general wave prediction methods discussed earlier can be used with modification for wave prediction

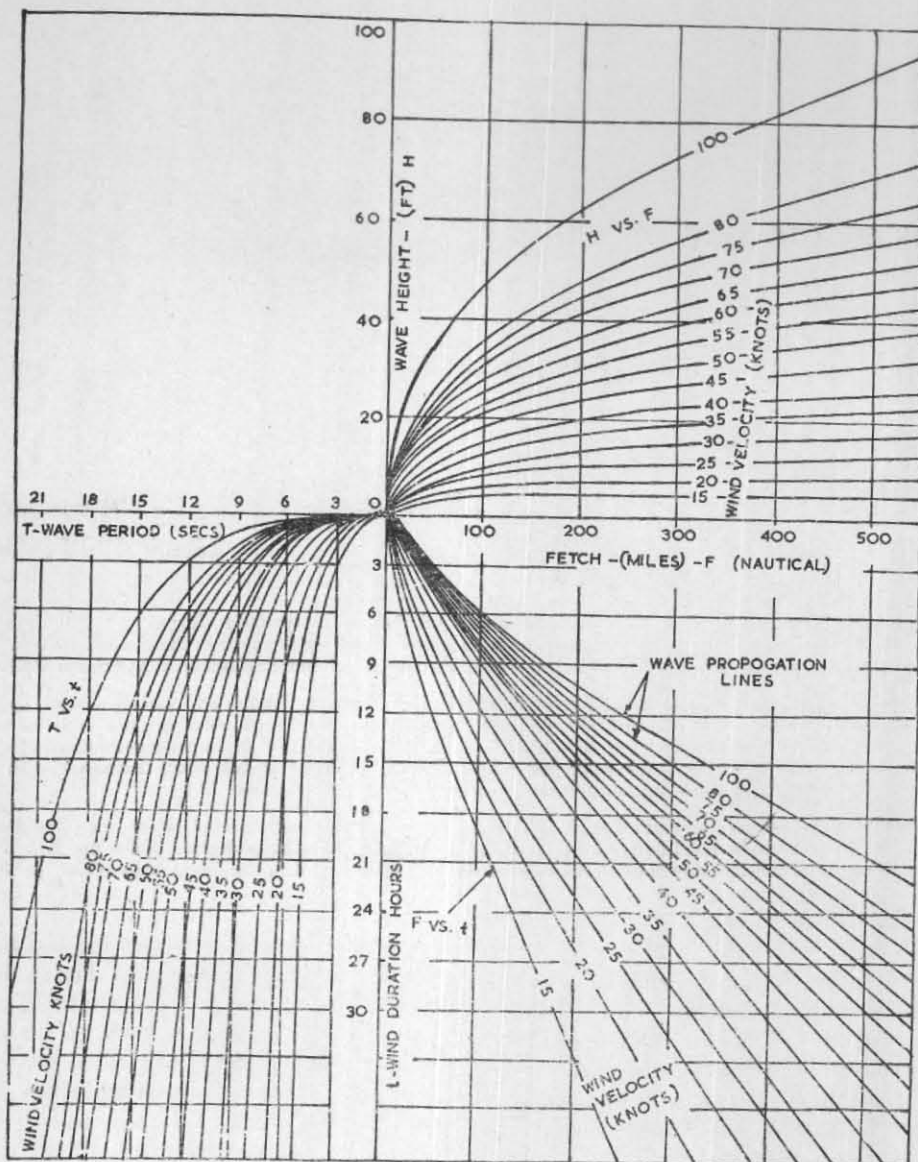


Fig. 1

Ht-FT Diagram for graphically forecasting wind generated waves in deep water

in the case of moving fetches with variable winds. Of the few methods available, the graphical approach of Wilson (1955) is perhaps the best and is widely used with a fair degree of success.

3. Graphical method and wind field diagram

(a) *Wilson's graphical method*

The basic wave prediction relations used by Wilson are those of the Sverdrup-Munk-Bretschneider method (SMB method). This method was first developed by Sverdrup and Munk (1947) in connection with the landing of the army on the French coasts during World War II. The method was based on theoretical development and the relations so obtained showed a fair measure of

agreement with actual field observations. Bretschneider (1952, 1958 and 1970) has slightly modified these relationships in the light of additional field data and these wave prediction relationships are known today as the Sverdrup-Munk Bretschneider method of wave forecasting. The latest revision of these relations was in 1970 by Bretschneider and these are used in the present analysis. These relations are available in the form of design curves from which the wave characteristics can be determined if the wind characteristics are known.

To facilitate graphical solution, the SMB relations between the wave and the wind characteristics were put in a three quadrant diagram by Wilson and was referred to by Wilson as the *Ht-FT*

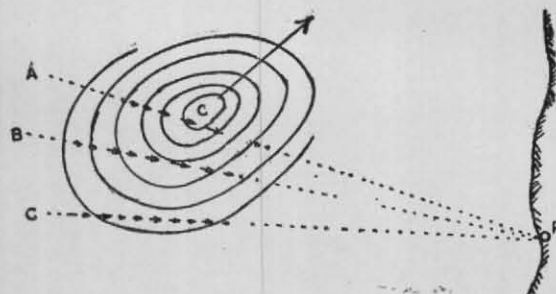


Fig. 2 (a)

Stationary storm in relation to a coastal station

diagram. For the present work the latest revision of the SMB method (Bretschneider 1970) was utilised to draw the Ht - FT diagram as shown in Fig. 1. From this diagram, one can read directly H and T from known values of U , F and t .

The continuously changing nature of the wind is represented by Wilson in the form of a space-time wind-field diagram. The Ht - FT diagram is superposed on the space-time wind-field diagram for the evaluation of the characteristics of the waves generated at any specific point in space and time within the wind field. The diagrams can be relatively shifted when a new point in the wind-field requires investigation. The graphical solution eliminates the elaborate integrations which are otherwise necessary. The exact procedures for performing the graphical integration are given by Wilson (1955) and the same procedures were followed in the present analysis. The mathematical justification for this method has been provided by Wilson (1963) in another paper.

(b) *Preparation of the space-time wind-field diagram*

In any specific wave prediction problem involving moving fetches, the most important step would be obtaining the space-time wind-field diagram. Fig. 2 (a) shows a depression or storm considered for the moment to be stationary. Assuming the area to be in the northern hemisphere, the wind circulation will be counter clockwise at an angle of 15-20 degrees across the isobars. As the winds are not steady in direction or magnitude, the waves generated by these winds will certainly be short crested due to interference effects of multidirectional influences. At increasing distances from the storm centre the multidirectional influences reduce and waves assume longer crests. In such cases it is reasonable to assume that the long crested waves that finally emerge from the wind area in a particular direction are those that have been consistently acted upon by the wind components directed along that line of action. This means that along any

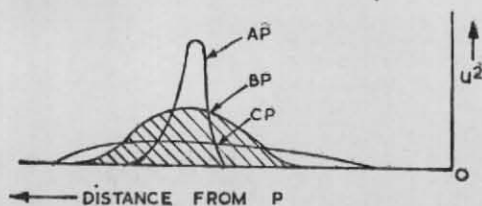


Fig. 2 (b)

Determination of optimum fetch line for generating highest waves

particular radial line such as AP (Fig. 2 a) the resolved horizontal component of the surface wind velocity will alone be responsible for the characteristics of the waves propagating along AP towards the coast. It is reasonable to further assume that the highest waves of all will originate only from one optimum radial direction, along which the wind velocity component has maximum wave generating capacity. As this depends on the square of the wind velocity, the particular radial direction for which the integral $\int U^2 dx$ is a maximum (x is the distance from the coastal station), will generate the highest waves. Fig. 2 (b) shows the typical distribution of U^2 values with distance x and the particular curve with maximum area beneath it will then determine the optimum fetch line (such as BP in Fig. 2 b) for generating the highest waves.

If the storm were to move forward in the direction shown in Fig. 2 (a) then along any particular line such as AP there will be continuously changing distribution of radial wind velocity component with time. This changing pattern is depicted in the space-time wind-field diagram shown in Fig. 2(c). Corresponding to times t_1, t_2, t_3, \dots , for which the storm positions are known, the wind velocity components along the particular radial direction can be computed as a function of the distance from the observing station P, and plotted as shown in Fig. 2(c). Contours of wind velocities can now be interpolated on the diagram. This diagram is the space-time wind-field diagram. This diagram can now be used with the Ht - FT diagram to yield the wave characteristics along the particular radial direction (optimum fetch line).

To investigate adequately the nature of waves converging on the coastal station P from different directions, it will be necessary to select a number of representative directions such as A_1P, A_2P, A_3P, \dots , (Fig. 2 d) and to submit the wind fields appropriate to each direction to the graphical procedure.

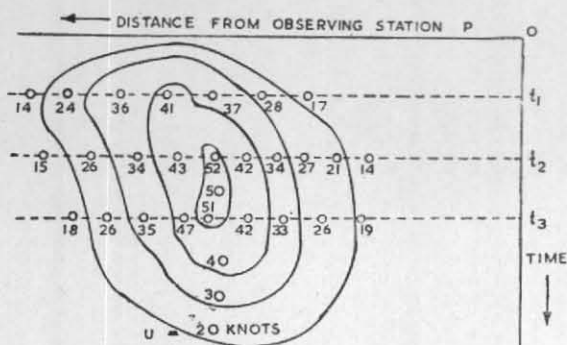


Fig. 2(c)

Space-time wind-field for a particular radial direction

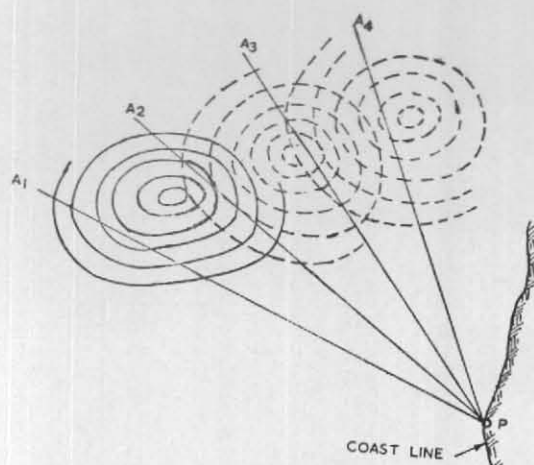


Fig. 2(d)

Moving storm in relation to a coastal station

4. Application of the Wilson's graphical approach to the November 5-7, 1969 cyclone in the Bay of Bengal

The east coast of India is subjected to the effects of depressions or storms originating in the Bay of Bengal. The largest number of cyclonic storms occur during the months of October and November (India met. Dep. 1964) causing extensive damage to life and property along the coast. The damage is mainly due to squally winds that accompany such conditions, and the large wind generated waves acting on land and structures not normally exposed to them. Often the situation could be worsened by a storm surge but this aspect has not been considered in the present paper.

The *Indian Daily Weather Reports* published by the India Meteorological Department, Poona were carefully studied and the depression which formed in the Bay of Bengal on 5 November 1969 and later intensified into a severe cyclonic storm that hit the Andhra Pradesh coast was selected for analysis. This cyclone crossed the coast between Kakinada and Masulipatnam on the afternoon of 7 November 1969.

The reason for selecting this cyclone was that wave observations were available for this period with the Visakhapatnam Outer Harbour authorities and the predicted waves could be compared with the recorded waves. Fig. 3 shows the synoptic sequence of surface pressure patterns at intervals of 12 hours that indicate the progress of this cyclone towards the coast. These maps show the general configuration of isobars around the cyclone. Perfect symmetry is absent and the pressure gradients tend to be less on the bottom left hand side. Isobars for pressures below 996 mb are absent near the cyclone centre. Probably, general lack of information in these regions is responsible for this.

(a) Space-time wind-field diagram

To enable the preparation of the space-time wind-field diagram, it is necessary to evaluate the wind velocities at a number of points on the optimum fetch line with time. Wilson (1957—Appendix A) has shown that in a cyclone which is moving with velocity V , the horizontal wind velocity U can be computed from the following relation:

$$\frac{U^2}{r} + \frac{UV}{r} \sin \theta + 2\omega U \sin \phi = \frac{1}{\rho} \frac{dp}{dr} \quad (1)$$

where U is the horizontal wind velocity, V the cyclone propagation velocity, ω the angular velocity of rotation of the earth, ϕ the latitude of the the point considered, θ the angle of bearing (from centre) of the point considered, taken positive counter clockwise from the direction of travel of the cyclone, ρ the density of air, p the pressure and r the radius from centre to the point considered.

For large values of r , or remote from the cyclone eye, the first two terms of Eq. (1) become insignificant and the solution of the equation yields the geostrophic wind velocity

$$U_g = \frac{1}{2\omega \rho \sin \phi} \frac{dp}{dr} \quad (2)$$

This expression is applicable when the isobars are relatively straight. If the isobars are curved, corrections to the geostrophic wind involving the first term of Eq. (1) would give the gradient wind U_g . Near the cyclone centre where the full Eq. (1) is involved, U may be designated as the gradient wind U_g . Determination of surface wind from U_g or U_g requires knowledge the ratio between the surface wind and geostrophic or gradient wind

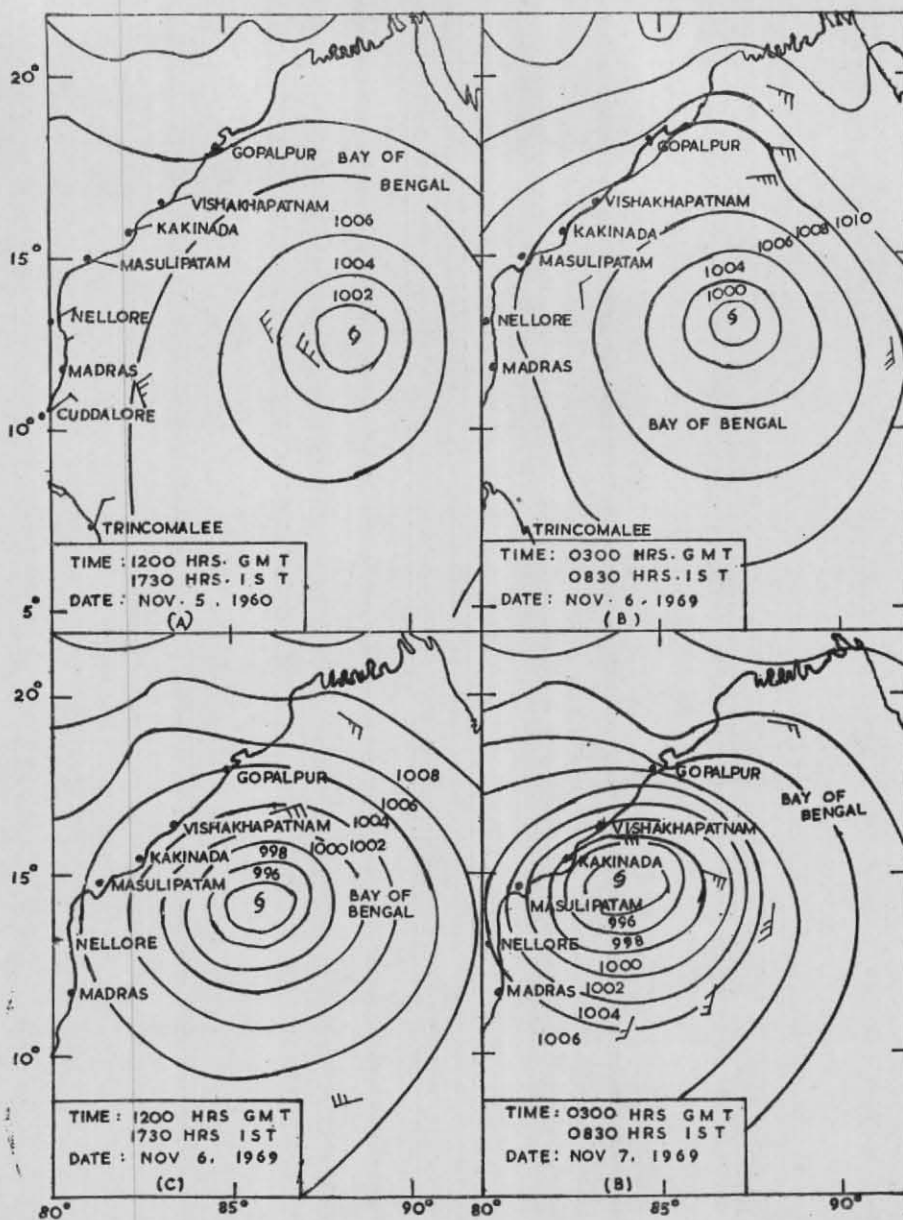


Fig. 3

12-hourly synoptic maps for the Bay of Bengal showing cyclone of 5 7 November 1969

with regard to frictional effects. In *Tech. Rep.* (US Army 1966) it is recommended that the sea-air temperature difference can be used as a measure of the frictional effects. As very little information about the sea-air temperature difference within the ambit of the Bay cyclone was available the assumption was made that it could be taken as zero. For these conditions the ratio of surface wind velocity to geostrophic or gradient wind velocity is 0.63 (US Army 1966). Surface wind velocities so computed are considered to be the mean wind speeds,

The direction of the wind was assumed at an angle of 18 degrees towards the cyclone centre from the tangent to the isobars.

The main difficulty is in determining the wind velocities in the central portions of the cyclone where the winds are strongest but no information is available about the pressure patterns. Wilson (1957) has provided a procedure for evaluating the cyclone characteristics which will aid the determination of the wind velocities near the cyclone centre.

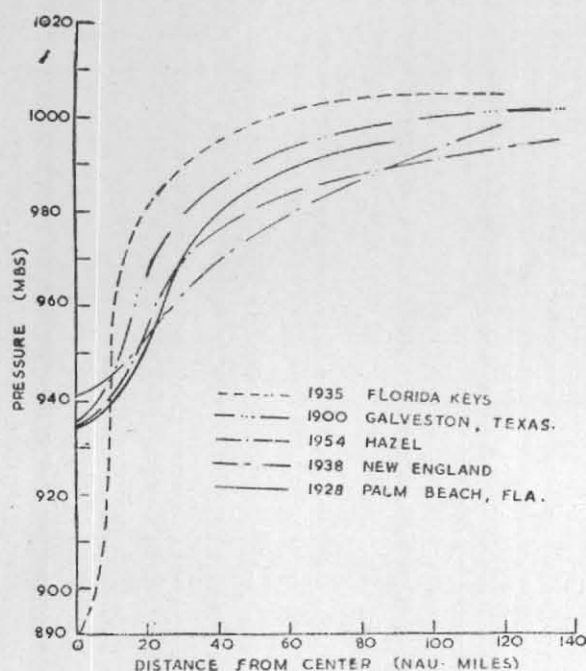


Fig. 4 (a)

Pressure profiles of damaging hurricanes
(From Gilman and Myers (1958))

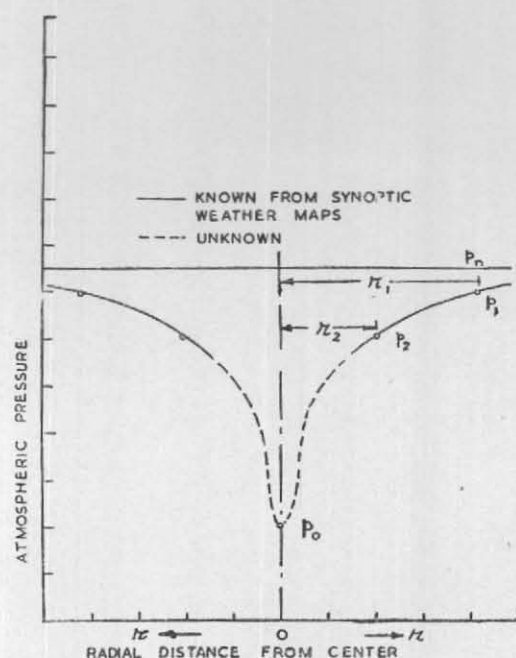


Fig. 4 (b)

Schematic diagram of pressure profiles of storm

(b) Determination of cyclone characteristics

Gilman and Myers (1958) have shown that the rate of change of pressure p with distance r from the cyclone centre is of the form

$$p = p_0 + (p_n - p_0) e^{-R/r} \quad (3)$$

in which p is the pressure at radius r , p_0 the minimum pressure at the centre of the storm, p_n the pressure 'outside' the storm (theoretically at r equal to infinity) and R is the radial distance from the storm centre at which maximum winds are encountered. Gilman and Myers (1958) have shown that Eq. (3) has been found to fit a good many hurricanes, such as those shown in Fig. 4 (a), fairly well.

There are two unknowns p_0 and R to be determined in the above equation and these can be evaluated by solving two simultaneous equations obtained by substituting two sets of known values for p and r as shown in Fig. 4 (b). But Wilson (1957) reports that the p_0 values so evaluated were too high to be valid.

This was overcome by Wilson (1957—Appendix A) by using polar or spiral diagrams, the basis for which is still Eq. (3). These spiral diagrams give isolines of pressure p , applicable to different values of R at a specific pressure p_0 , with normal pressure p_n taken as 1020 mb. Vijayakumar (1971) has re-

drawn these spiral diagrams to the same scale as the Indian Daily Weather maps, so that they can be directly used on them.

The idea of superposition with coincidence of centres was to rotate any particular spiral diagram (prepared as a transparency) until best fit could be obtained between the isoline distributions along some radial line of the spiral diagram and the underlying synoptic chart. The criterion for judging which particular spiral diagram was most applicable to any synoptic weather situation was the degree of straightness of the lines of concurrent pressures.

Fig. 5 shows the superposition of the spiral diagram of 900 mb central pressure over the isobaric pattern of the cyclone under investigation. The actual line of agreement of concurrent pressure is not straight but this is the best approximation that could be obtained using the several spiral diagrams. This gives $p_0=900$ mb, and $R=22$ n. miles. The central pressure of 900 mb for the cyclone indicates that the cyclone is very severe. (The *Indian Daily Weathers Report* had described it as a severe cyclonic storm).

It should be noted that these are only estimates based on the pressure patterns and the accuracy depends on the accuracy of the pressure observations. It is unfortunate that very few observations about the cyclone characteristics in the Bay of Bengal are available for the purpose of comparison. Under

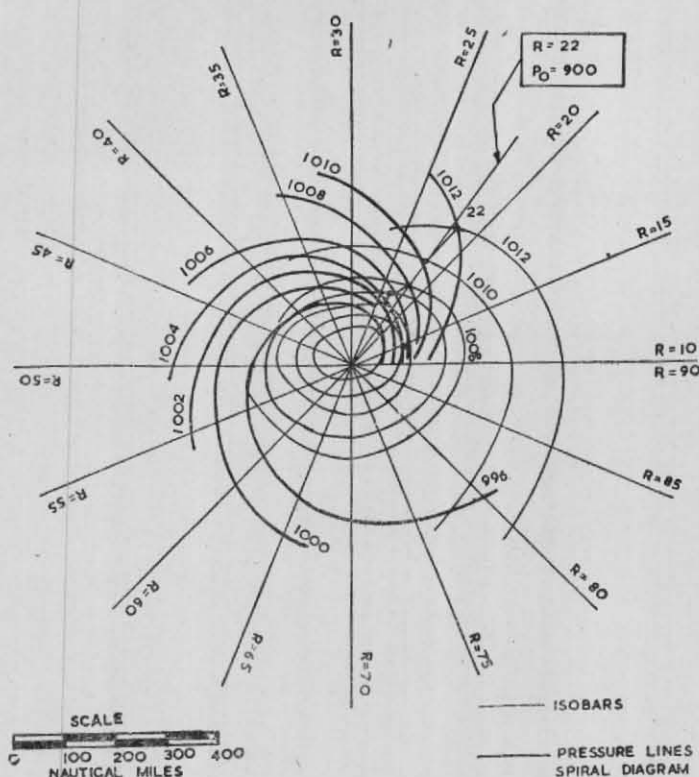


Fig. 5. Best-fit determination of cyclone characteristics

Spiral diagram gives isolines of pressures (mb) for cyclone with central pressure of 900 mb. Each radial line, defining a value of radial distance "R" (n.m) to maximum winds gives distribution of pressure from storm centre
 [Spiral diagram for $P_c=900$ mb gives nearest approach to straight (radial) line concurrence of pressure distribution at $R=22$ n.m.]

these circumstances it is assumed that the spiral diagrams which were actually developed on the basis of observed data of the Gulf of Mexico hurricanes is also applicable to the Bay of Bengal.

The cyclone speed was evaluated at 10 kt. For computing the wind velocities near the centre of the cyclone the p_0 and R values determined above were used. With these values of p_0 and R , dp/dr for any particular value of r was determined from Eq. (3) and this value of dp/dr was used in Eq. (1) to determine the gradient wind U_g . Wilson (1957) has provided graphical aids for these computations and these were used in the present analysis. Near the eye of the cyclone the surface wind velocities so evaluated were around 100 kt which is not unusual for a cyclone with a central pressure of 900 mb. At distances remote from the cyclone centre, where the isobar spacings could be scaled from the synoptic maps, the geostrophic wind equation was used for velocity computations.

In the determination of surface wind velocities pressure patterns were exclusively used and observational data if any from shore stations as recorded in the *Indian Daily Weather Reports*, was used merely as a check on the results. Table 1

TABLE 1

Comparison between computed and observed wind velocities

	Wind velocities (kt)							
	Computed	7.9	22.8	37.6	14.8	37.6	18	37.4
Observed	15	20	34	13	30	19	38	30

gives a comparison between the computed and observed wind velocities for the few cases, where observed data was available from the shore stations.

(c) Space-time wind-field diagram

The primary aim of the present investigation was to study the effect of the cyclone on the wave characteristics in its direct path as well as in areas adjacent to the path. It was decided to study the wave characteristics at four coastal stations: Kakinada, Visakhapatnam, Kalingapatnam and Gopalpur. No station to the south of Kakinada was considered because the cyclone crossed the coast about 15 n. miles to the south of Kakinada and only in northern portion of the cyclone, winds were directed towards the coast,

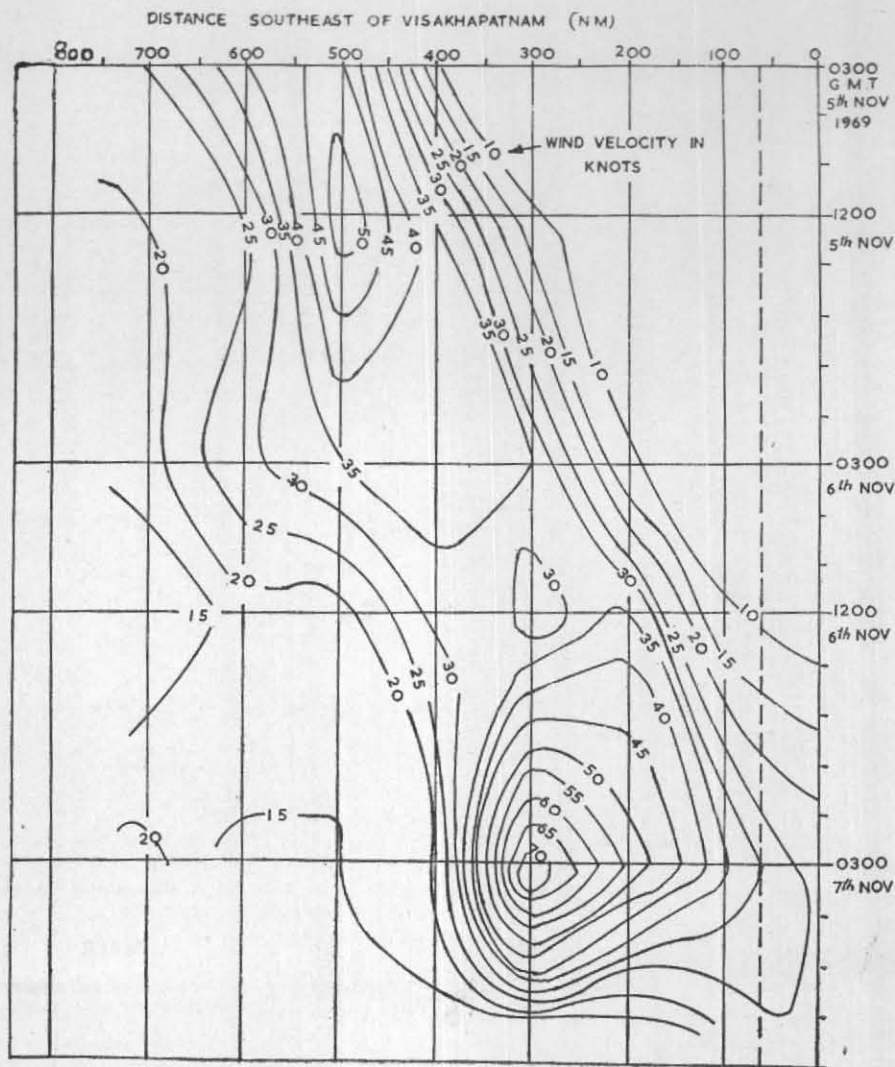


Fig. 6

Space-time wind-field for the 5-7 November 1969 cyclone along an approach direction southeast of Visakhapatnam

The determination of the optimum fetch line was done by procedures earlier explained, and wind velocities at regular intervals over the optimum fetch line were evaluated. Knowing the angle between the direction of the wind and the optimum fetch line the wind velocity components for the four coastal stations were evaluated. These were plotted on a space-time grid to yield the space-time wind-field diagram. Contours were interpolated at 5 kt intervals. The space-time wind-field diagram for Visakhapatnam forecast point is shown in Fig. 6.

(d) *Graphical prediction of wave characteristics and arrival times*

The space-time wind-field diagrams described above were prepared as transparencies and were

superposed on the *Ht-FT* diagram (Fig. 1) which was specially prepared for the present work based on the latest wave prediction relations given by Bretschneider (1970). Starting points in the wind fields for the graphical integration were chosen so as to give the largest possible end results of significant wave height and period. The graphical integration followed the procedures given by Wilson (1955 and 1957). The complete wave history with time was predicted for the Visakhapatnam forecast point where observed data were available for comparison. For other forecast points only the maximum significant wave heights caused by the cyclone were predicted.

Fig. 7 represents the graphical integration performed for the Visakhapatnam forecast point. Seven starting points for the graphical integration

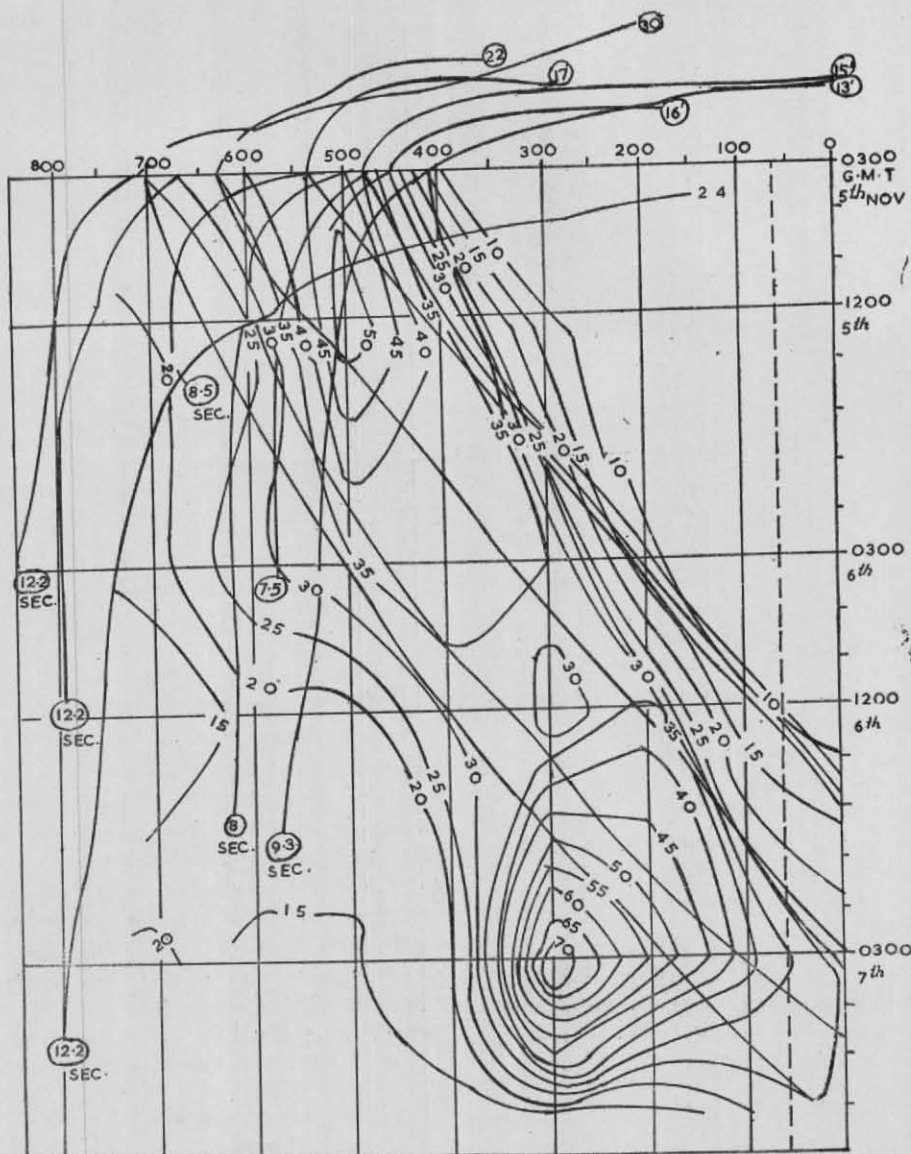


Fig. 7

Deviation of wave characteristics from several different points of origin within the cyclone

were considered. The curves rising above the points of origin record the wave height growth, and that to the left of the origin going down record the wave period growth. The figures in circles indicate the deep water significant wave height and the period. The wave arrival times can be directly read as the time ordinates.

The wave heights and periods at various intersections are plotted for the corresponding arrival times. Envelope lines are drawn to give the history of wave heights and periods over the duration of the cyclone. Fig. 8 gives the deep water wave characteristics at Visakhapatnam forecast point as obtained from the above graphical integration.

(c) *Predicted wave characteristics in shallow waters*

The wave characteristics predicted are for deep water (depth of water more than half the wave length). The observed wave characteristics were at a place where the depth was 45 ft. As the deep water waves enter shallow waters, they undergo changes due to refraction, shoaling and bottom friction. Generally these lead to a reduction in the wave height. With the wave orthogonals being fairly parallel, the refraction coefficient works out to be nearly unity. The shoaling and bottom friction effects were evaluated as per procedures laid down in *Tech. Rep. (US Army 1966)*.

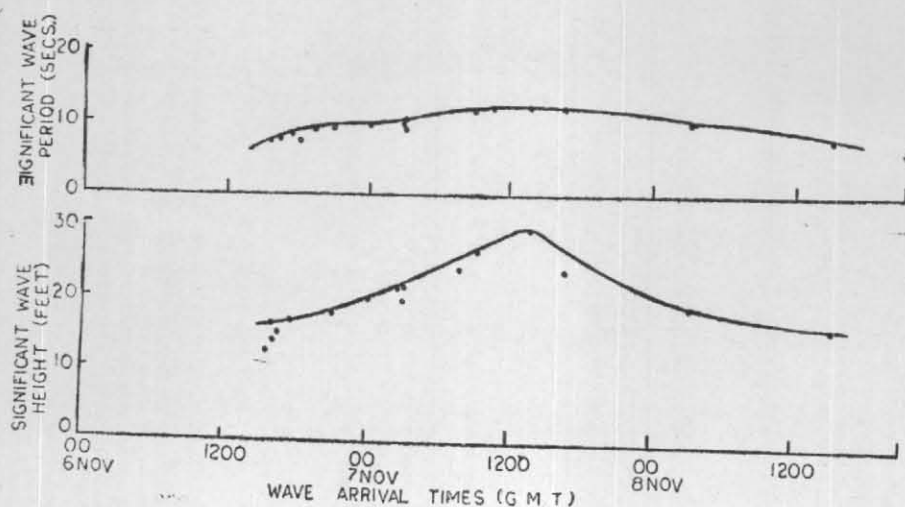


Fig. 8

Deep water wave characteristics at Visakhapatnam forecast point

The shoaling coefficient accounts for the changes in the wave characteristics as waves enter shallower waters. The wave celerity is affected by the depth of water and this affects the energy transmitted. Assuming that there is no loss of energy due to other causes, the changes in the wave height due to changes in the depth of water is calculated in terms of the shoaling coefficient.

In shallow waters the orbital velocities of the water particles at the bottom are finite and the sea bed acts like a hydrodynamically rough surface which offers resistance to motion. The frictional resistance can be computed from the relation

$$\tau = k \rho U_h^2 \quad (4)$$

where τ is the frictional resistance per unit area, k the bottom friction coefficient and U_h the orbital velocity of the water particle at depth h . U_h can be computed from wave theory. Using the above expression the average rate of energy dissipation by bottom friction per unit area in one wave cycle can be computed. Knowing this energy loss, the energy balance can be worked out from which the wave height after allowing for bottom friction can be evaluated. These computation can be easily carried out using the graphical aids (US Army 1966). The recommended value of the bottom friction coefficient is 0.01. But other investigators have found this value to be too low (Iwagaki and Kakimura 1967). For the Indian coasts hardly any information is available to fix this value. It was decided to assume the bottom friction coefficient at 0.01, 0.05 and 0.1 to see which of the transformed waves agrees with the observed values.

(f) Observed waves characteristics

During 1969 waves were recorded for some period off the Visakhapatnam coast with a sub-surface pressure type (OSPOS) wave recorder. Simultaneously observations from theodolite stations were also made and after comparison it was concluded by the port authorities that the theodolite observations were as reliable as the instrument records. During the period when the cyclone was crossing over, only theodolite observations were available. The authors feel that the wave height observations could be relied upon but this cannot be said about the wave period observations.

(g) Comparison between predicted waves and observed waves

Fig. 9 presents the observed wave characteristics and the predicted wave characteristics for the Visakhapatnam forecast point. The predicted wave characteristics are shown for three values of bottom friction coefficient. The wave heights, show a steep increasing tendency on 6 November 1969 at 1200 GMT, when the storm centre was about 200 nautical miles away from the coast. The heights reached a peak value of 14 ft significant wave height (30 ft deep water wave height) at 1200 GMT, on 7 November 1969. Subsequently, with the cyclone crossing the coast the wave heights decrease and reach their normal values after 8 November.

In Fig. 9, it can be seen that the predicted wave heights using a bottom friction factor of 0.05 agrees fairly well with the observed wave data. From this it will be wrong to conclude that a bottom friction factor of 0.05 should be used for the east coast of India. In the present case it happens that

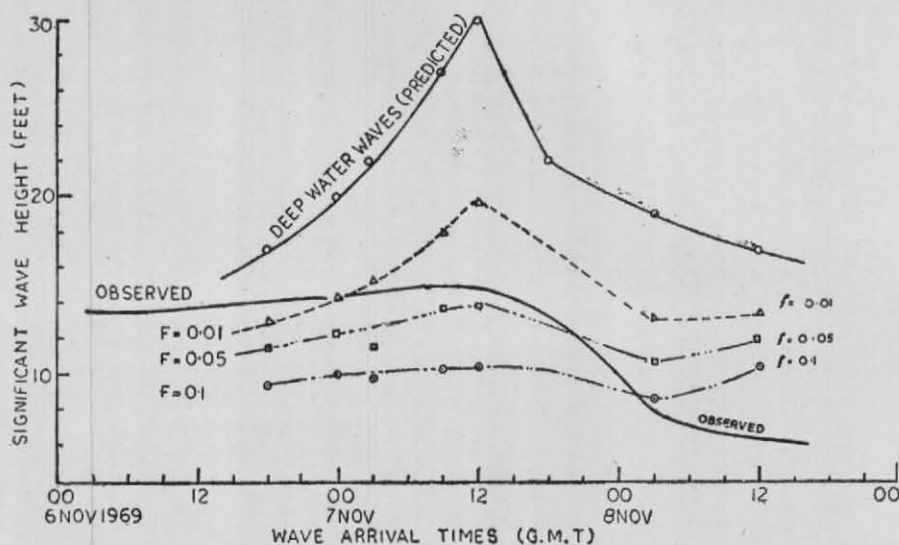


Fig. 9

Comparison between observed and predicted wave heights at Visakhapatnam

agreement is obtained for this value. It can only be stated at this stage that the recommended value of 0.01 is low.

The observed and predicted wave heights reach the maximum around the same time. The maximum significant wave height predicted was 14 ft (using a bottom friction factor of 0.05) with a significant wave period of 13 sec while the maximum significant wave height observed was 15 ft (The corresponding maximum wave heights could be 23.5 ft and 25 ft respectively). The predicted periods have not been compared with the observed periods as the observed periods were found to be not very reliable. The good agreement in Fig. 9 indicates that Wilson's graphical approach could be used to predict waves under cyclonic storm conditions for the Indian coasts.

(h) Effect of cyclones in adjacent areas

Another aspect considered in this investigation was the influence of the cyclone on the wave characteristics along the coast at large distances from the cyclone centre. The area under investigation being in the northern hemisphere only the coastal area to the north of landfall need be considered. Four coastal stations, Kakinada (the cyclone crossed the coast at a point 15 n. miles to the south of Kakinada), Visakhapatnam (about 100 n. miles from the cyclone centre), Kalingapatnam (about 175 n. miles from the cyclone centre) and Gopalpur (about 275 n. miles from the cyclone centre) were considered. Using the graphical procedure the maximum significant wave heights that could be experienced at these four coastal stations were predicted. The maximum significant wave height

at Gopalpur, as a result of this cyclone, was about 8 ft (the corresponding maximum wave height would be 13.5 ft). This indicates that over a considerable stretch of area heavy waves are experienced.

5. Conclusions

Based on this study it can be concluded that Wilson's graphical method can be used for predicting waves under cyclonic storm conditions in the Bay of Bengal. The predicted waves agreed well with the observed waves. The recommended value of 0.01 for bottom friction factor appears to be low. Good agreement was obtained with a value of 0.05 for bottom friction factor. Under the influence of the cyclone considered, waves with significant wave heights greater than 8 ft are likely to be experienced over a stretch of nearly 300 n. miles.

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