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Size distribution of Raindrops — Part VI

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ABSTRACT. A. C. Best has found an empirical formula for the size distribution of raindrops. If $F(x)$ is the fraction of liquid water in the air comprised by drops of diameter less than x, $1 - F = \exp[-(x/a)^n]$, where a and n are constants for any particular rainfall. Contrary to the general supposition that the above formula applies to the average size distribution of a number of samples of rain, it has been found that the formula holds for *individual* samples, in 90 out of 104 cases of general monsoon rains and in 134 out of 169 cases of thunderstorm rains recorded at Poona. The formula fails where the liquid water distribution is multimodal (generally bimodal) or is J-shaped.

The parameters a and n are found to be independent for general rains but are connected by the average relation $a\,n = 7{\cdot}1$ mm for thunderstorm rains. There is also a correlation between the intensity of precipitation I and the parameter n. For general rains the average relation found is $I = 6{\cdot}9 \exp[-0{\cdot}14\,(n-5{\cdot}5)^2]$ and that for thunderstorm rains is $I = 975 \exp(-n)$.

1. Introduction and Results

Best (1950) has shown that most of the available data on the drop-size distribution is in accordance with the formulae —

$$1-F = \exp[-(x/a)^n] \text{ and } a = AI^p$$

where F is the fraction of liquid water in the air comprised by drops of diameter less than x; a and n are constants for a particular rainfall. From the first of these equations it follows that —

$$\log\log_{10}[1/(1-F)] = -0.36 + n\,(\log_{10}x - \log_{10}a)$$

and that $\log\log_{10}[1/(1-F)]$ plotted against $\log_{10}x$ should be a straight line with slope n. The value of a can be calculated from the intercept.

Following the usual practice, the whole of the data on thunderstorm rains consisting of 169 samples was divided into 17 groups and average values obtained. On using these for plotting $\log\log_{10}[1/(1-F)]$ against $\log_{10}x$, it was found that in 9 cases a straight line was obtained, in 7 cases two straight lines were obtained and in one case three straight lines were obtained. Suspecting that this non-agreement was due to averaging of such heterogeneous material, Best's lines were plotted for two individual samples of rain with identical value of the intensity of precipitation, *viz.*, 3·3 mm/hr, having very different diameter spectra of liquid water. The average values for these two samples were calculated and a similar graph was plotted. It was found that the graph was a straight line for each of the individual samples with its own values of a and n as shown in Figs. 1(a) and 1(b). For the average values, however, the plot showed a sharp kink so that two straight lines with different slopes were obtained as shown in Fig. 1(c). Realising that the individual samples were more suitable and that there was a danger in calculating averages of such heterogeneous material (very different diameter spectra), Best's lines were plotted for all the 169 individual samples. It was found that a straight line was obtained for 134 samples, showing that Best's formula holds for individual samples. It was further found that the 35 samples for which the formula does not apply, could be divided into three categories as follows —

(a) Two straight lines with the second line having a higher slope were obtained in 21 cases. An exactly similar result was obtained by Best (1950) for the East-Hill data which he has attributed to the small amount of data corresponding to that line.

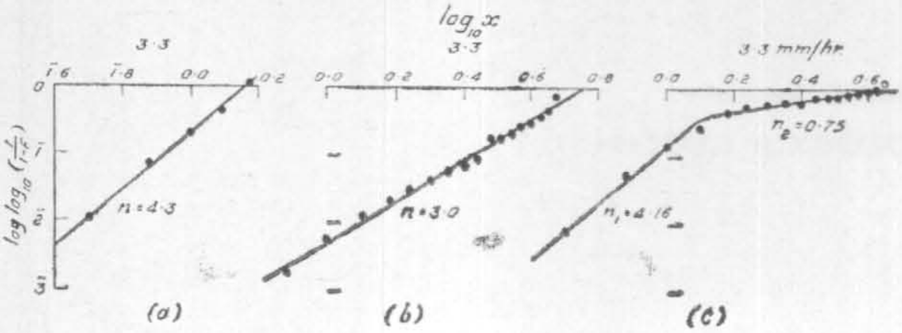

Fig. 1. Best's formula log $\log_{10}[1/(1-F)]$ plotted against $\log_{10}x$ for two identical intensities and their average

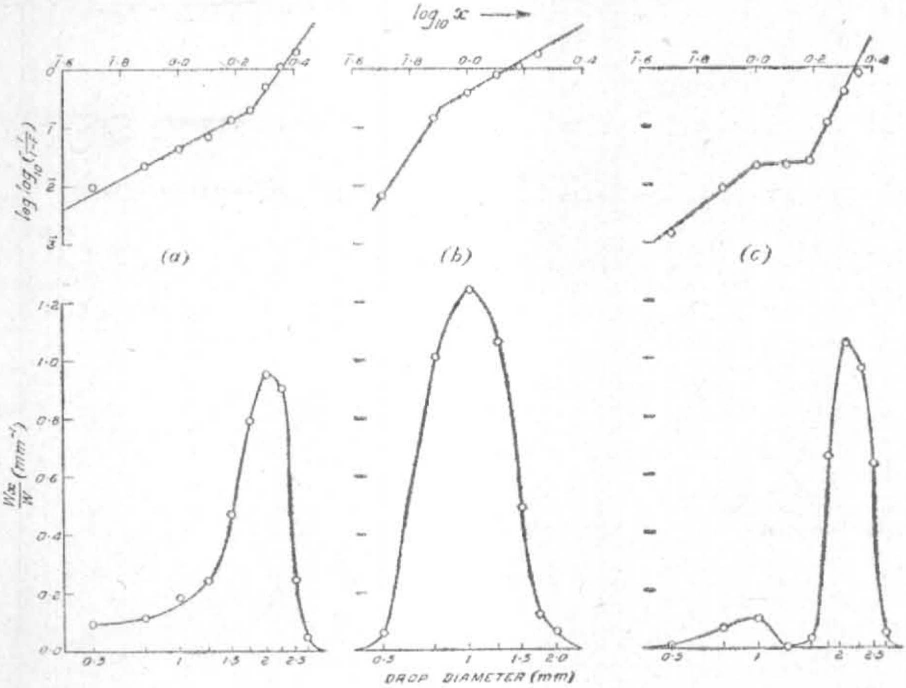

Fig. 2. Best's formula log $\log_{10}$ $[1/(1-F)]$ plotted against $\log_{10}x$ for drop size distributions which do not satisfy Best's formula

(b) Two straight lines with the first line having a higher slope were obtained in 7 cases.

(c) Three straight lines were obtained in 7 cases again.

To bring out the cause of the failure of the formula, liquid water distribution curves were plotted below each line. To facilitate comparison, $\log_{10} x$ values have been plotted instead of x. If $W_x\,dx$ is the amount of liquid water in unit volume of air comprised by drops of diameter between x and $x+dx$ and W is the total amount of liquid water in a unit volume of air, W_x/W gives the fraction of liquid water in a given volume of air per unit range of diameter. The values of W_x/W are plotted as ordinates. The results are shown in Fig. 2. It is seen that when the liquid water distribution curve has an initial long tail and rises to a sharp maximum towards larger diameters (J-shaped distribution), two straight lines with the second line having a higher slope are obtained as in Fig. 2(a). When, on the other hand, the curve has an initial sharp rise and a long tail towards larger diameters, again two straight lines with first line having a higher slope are obtained as in Fig. 2(b). Here the liquid water curve looks rather symmetrical because of the distortion due to logarithmic plotting along x. When the liquid water distribution curve is bimodal, three straight lines are obtained as in Fig. 2(c). The Best's formula applies to a unimodal

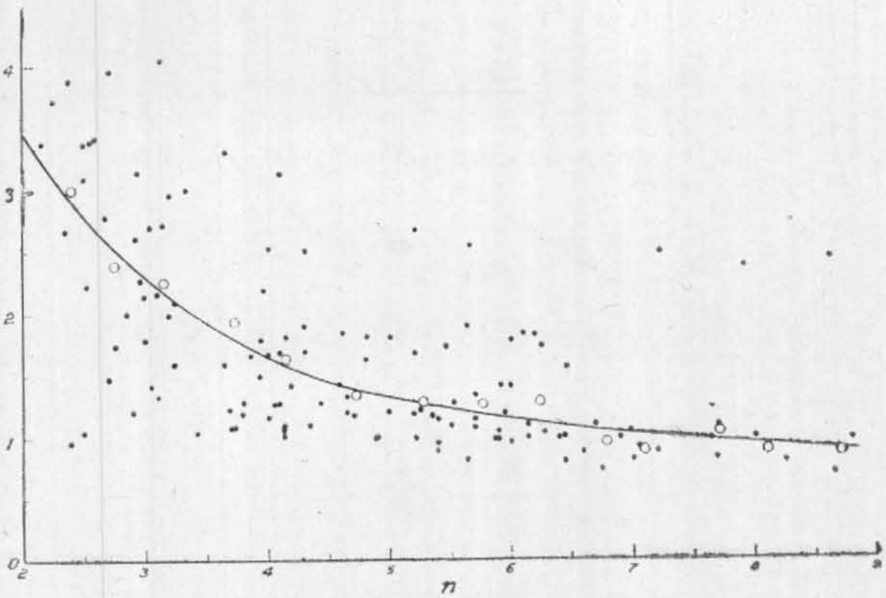

Fig. 3(a). Variation of the parameter a with parameter n for thunderstorm rains

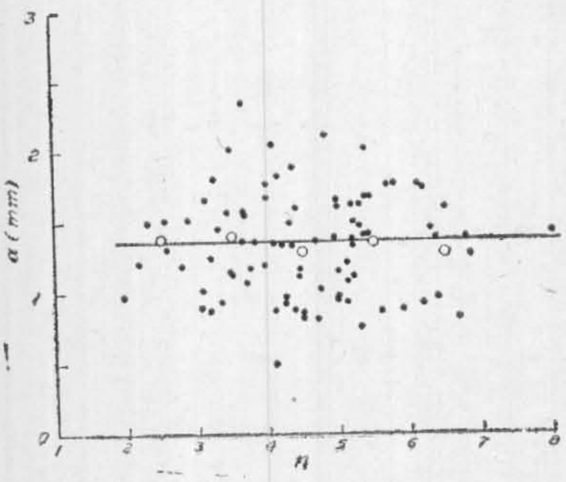

Fig. 3(b). Variation of parameter a with parameter n for general rains

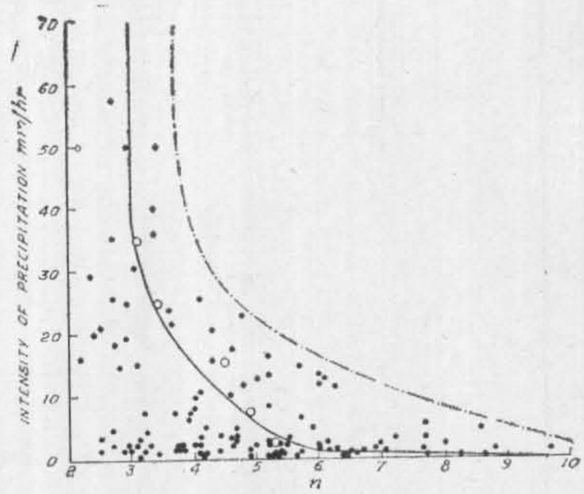

Fig. 4(a). Variation of the intensity of precipitation with the parameter n for thunderstorm rains

distribution which is very nearly symmetrical about the mode diameter. Best's formula seems to give the best smoothened out version of the observed distribution of liquid water even for individual samples.

For general rains it is found that Best's formula gives straight lines for 90 out of 104 samples recorded. Out of the 14 cases where the formula does not apply, one belongs to category (a), seven belong to category (b) and six belong to category (c).

It seems at least as a possibility, that for a sample of rain to which Best's formula does not apply, we get simultaneously rain of two different physical origins, *e.g.*, rain from two different clouds at different levels. Nature might produce such a hybridisation, giving rise to very unsymmetrical liquid water spectrum, as observed on the ground level.

2. Parameters a and n

To examine whether there is any correlation between a and n, values of a are plotted against

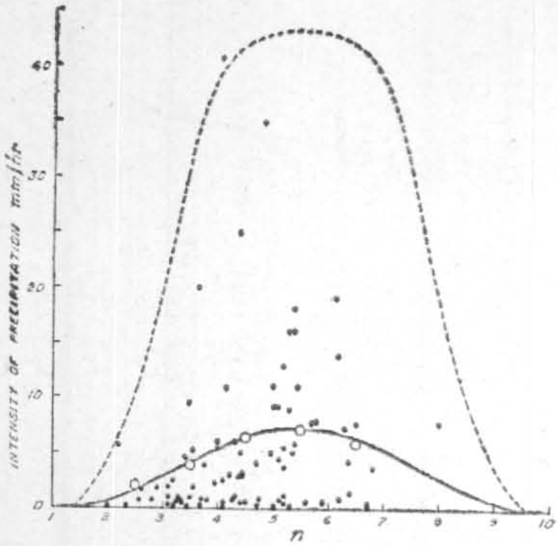

Fig. 4 (b). Variation of the intensity of precipitation with the parameter n for general rains

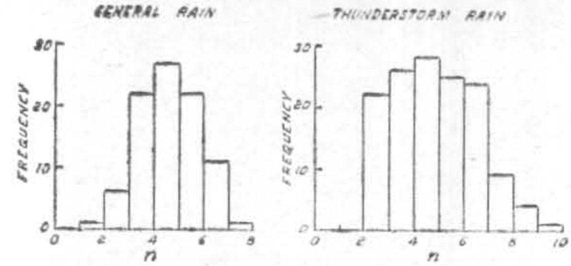

Fig. 5. Frequency polygons for the parameter n for thunderstorm rains and for general rains

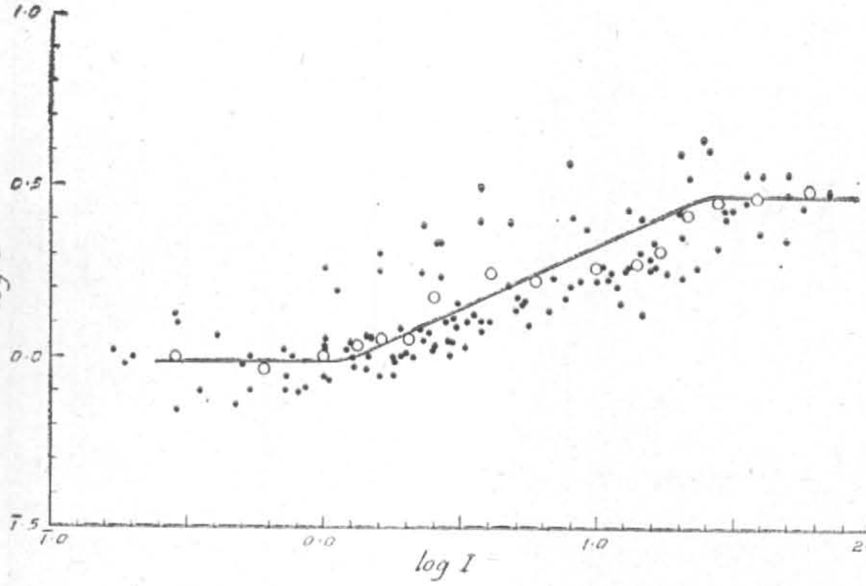

Fig. 6. Variation of the parameter a with I for thunderstorm rains

the corresponding values of n and the results are shown in Fig. 3(a) for thunderstorm rains and in Fig. 3(b) for general rains. It is seen that the the average value of a decreases as n increases and the average relation is found to be $a\,n = 7 \cdot 1$ mm. For general rains, a remains practically constant for all values of n, showing that a is independent of n.

To examine whether there is any correlation between the intensity of precipitation I and the parameter n, the values of I are plotted against the corresponding values of n, shown in Fig. 4(a) for thunderstorm rains and in Fig. 4 (b) for general rains. For thunderstorm rains the average relation between I and n is found to be —

$$I = 975 \exp\ (-n).$$

For general rains the corresponding relation is—

$$I = 6 \cdot 9 \exp\ [-0 \cdot 14 (n - 5 \cdot 5)^2].$$

It is also of interest to examine the frequency of occurrence of particular values of n. Fig. 5 gives the frequency polygons for thunderstorm rains and for general rains.

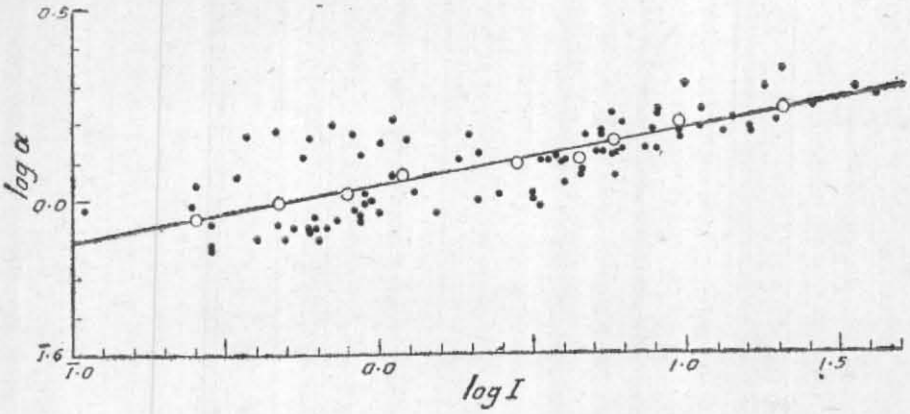

Fig. 7. Variation of the parameter a with I for general rains

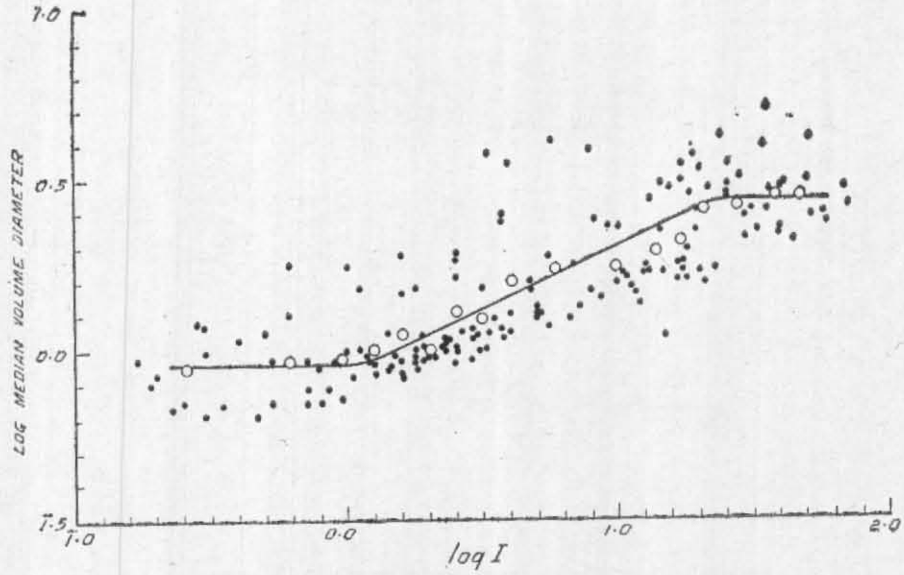

Fig. 8. Variation of the median volume diameter with I for thunderstorm rains

The parameter a is related to the intensity of precipitation I. On plotting a against I to a double log scale, it is found that a attains constant limiting values towards low and high intensities and increases linearly with I for intermediate values as shown in Fig. 6 for thunderstorm rains. For general rains a logarithmic plot of a against I gives a straight line throughout the whole range of I as in Fig. 7.

The median volume diameter d_{50} plotted against I to a double log scale is shown in Fig. 8. This shows features similar to the a–I relation, for thunderstorm rains.

According to Best's formula the median volume diameter $d_{50} = 0{\cdot}69^{1/n}\, a$. To find the amount of divergence from the Best's formula shown by individual samples, the ratio d_{50}/a is plotted against n in Fig. 9 (a) for thunderstorm rains and in Fig. 9(b) for general rains. The calculated curve is shown by full lines and the individual points by dots. Taking into account the fact that the values of d_{50} and a are estimated to an accuracy of about 5 per cent the remaining dispersion must be attributed to a real divergence from Best's formula.

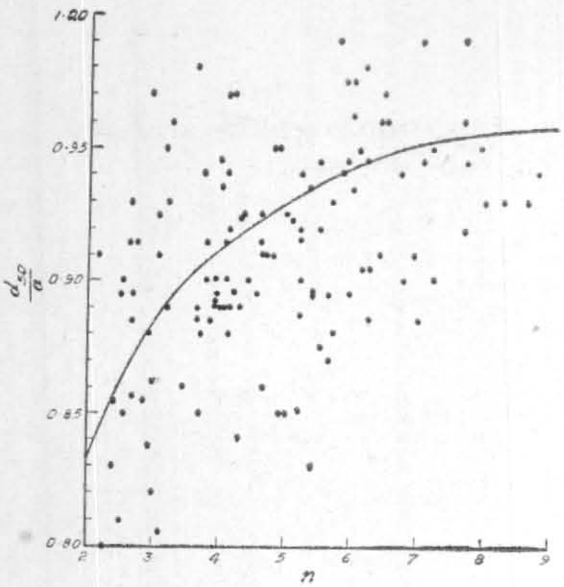

Fig. 9(a). The ratio d_{50}/a as a function of the parameter n for thunderstorm rains

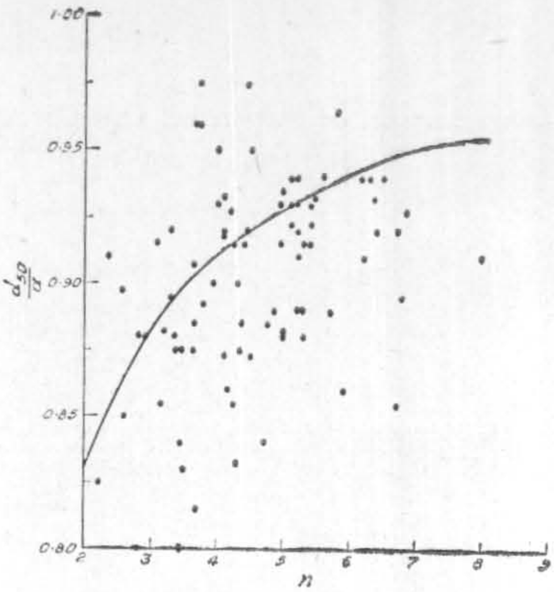

Fig. 9(b). The ratio d_{50}/a as a function of the parameter n for general rains

3. Conclusion

In conclusion it may be said that Best's theory in the present form is inadequate. The present investigation only shows that most of the diameter spectra can be represented by an equation empirically suggested by Best. The large amount of scatter obtained in Figs. 3,4,5 etc excludes it from being accepted as a physical theory.

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