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A scheme of initialization at every latitude

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ABSTRACT. A new scheme of initialization is suggested which can be utilized for regions which extend from middle to low latitudes. In this scheme the two fields of wind and geopotential are analysed independently and then forced to be balanced by means of a balance equation, in which each of the two fields enters with a weight, that is a continuous function of the latitude, chosen in dependence of the reliability and the relative significance of the two observed fields.

The results referring to a SW monsoon situation are finally examined.

1. Introduction

Most experiments carried out about the initialization of forecasting primitive-equations numerical models show the use of the following three schemes :

(*a*) the observed fields of geopotential and wind are analysed independently (Nitta 1968; Nitta and Hovermale 1969);

(*b*) at the starting point the wind is inferred from the geopotential field (*vice-versa* for low latitudes) (Nitta 1968; Charney 1955; Shuman 1957, Houghton and Washington 1969);

(*c*) the observed geopotential and wind fields are forced to be consistent through the equations of the model by means of forward and backward integrations around the initial time (an essentially mathematical filtering) (Miyakoda and Moyer 1968; Nitta and Hovermale 1969).

These experiments have shown that the best solution for the initialization of present primitive equation models is, for high and middle latitudes, to analyse the geopotential field and infer the wind field by means of a balance equation; the *vice-versa* (to analyse the wind field and infer the balanced geopotential field) holds for low latitudes.

Problems arise, however, when the working area stretches over latitudes ranging from middle to low. An attempt could be made to divide the area into two belts (separate or partially overlapping), and to apply to each of them suitable initialization approach. Then the fields must be connected across the boundary or within the overlapping belt. Besides the criticism for the arbitrarity of the choice of the boundary, this approach is likely to lead to serious problems over the boundary of the two belts, where spurious inbalances between wind and geopotential fields can arise (especially if the connection is purely mathematical).

For this reason Finizio and Bucchi (1972) have revised the problem of the initialization and proposed a scheme which can be utilized over areas ranging from middle to low latitudes.

In this scheme the two fields of wind and geopotential are analysed independently and then forced to be balanced by means of a balance equation, in which each of the two fields enters with a certain weight.

The choice of the weight should take into account two separate effects :

(*a*) the relative significance of the two fields according to the scale of motion and the latitude (through the Rossby number);

(*b*) the observational errors which at low latitudes prevent the possibility of inferring reliable winds from the observed geopotential gradients.

In the present application (at synoptic scale) the weight has been chosen both in dependence of the reliability and the relative significance of the two observed fields.

2. The initialization scheme

The entire procedure is automatized, and consists of three main phases :

(*a*) independent analyses of the geopotential z and the u and v components of the wind (*b*) determination of the stream function Ψ related to the rotational component of the observed wind field;

(*c*) balancement between the geopotential and the stream function.

2.1. *Analysis*

The objective analysis of the fields has been performed by means of a program (Job STAMPEDE) already catalogued in the IBM 360/40 library. This program has been already utilized by Bizzarri and Finizio (1972), with reliable results.

2.2. *Determination of the stream function*

The stream function has been determined by means of the expression :

$$\nabla^2\Psi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad (1)$$

with the boundary condition given by :

$$\frac{\partial \Psi}{\partial s} = v_n - \overline{v_n}$$

where s is the boundary of the working area (drawn anti-clockwise), v_n is the wind component normal to the boundary (towards the interior of the area), and $\overline{v_n}$ is the average of v_n along the boundary. The finite differences equations are formulated on an offset grid lattice; the geopotential and the stream function are defined at the grid points, while the wind components are defined at the centres of the grid squares. Using a well known notation (*see* for instance Shuman and Hovermale 1968), the finite differences form used (1) is :

$$\overline{\Psi}^{yy}_{xx} + \overline{\Psi}^{xx}_{yy} = \overline{V}^{y}_{x} - \overline{U}^{x}_{y}$$

where $U = \frac{u}{m}$ and $V = \frac{v}{m}$; m is the map factor.

The Ψ function is computed by means of a sequential relaxation scheme. An over-relaxation coefficient $\alpha = 1.68$ is used (this value has been chosen considering the size of the used matrices).

2.3. *Balancement*

The Ψ and Z fields so far obtained are fully independent. At this stage the linear balance equation is applied in the following form :

$$\nabla^2\Psi = \frac{g}{f}\,\nabla^2 Z - \frac{1}{f}\vec{\nabla}\Psi \cdot \vec{\nabla} f \qquad (2)$$

f being the Coriolis parameter. None of the two fields is assumed to be fixed, but both are mutually approaching in the following way. Let $R^{(n)}(I,J)$ be the residual in the point (I,J) at the n-th iteration :

$$R^{(n)}(I,J) = \overline{\Psi^{(n)}}^{yy}_{xx} + \overline{\Psi^{(n)}}^{xx}_{yy} + \frac{1}{\overline{f}^{xy}}\left(\overline{\Psi^{(n)}}^{yxy}_{x}\,\overline{f}^{y}_{x} + \overline{\Psi^{(n)}}^{xyx}_{y}\,\overline{f}^{x}_{y}\right) - \frac{g}{\overline{f}^{xy}}\left(\overline{Z^{(n)}}^{yy}_{xx} + \overline{Z^{(n)}}^{xx}_{yy}\right) \qquad (3)$$

The $(n+1)$th order approximation for Ψ and Z is computed by means of the two expressions :

$$\Psi^{(n+1)}(I,J) = \Psi^{(n)}(I,J) + \frac{K}{4}\,\alpha\,R^{(n)}(I,J)\,2\Delta x^2 \qquad (4)$$

$$Z^{(n+1)}(I,J) = Z^{(n)}(I,J) - \frac{1-K}{4}\,\alpha R^{(n)}(I,J)\,2\Delta x^2\,\frac{\overline{f}^{xy}}{g} \qquad (5)$$

Thus, the residual is ascribed to both fields according to the weight K, which takes into account the relative importance of the two fields.

2.4. *Determination of the weight K*

In middle and high latitudes, in the free atmosphere, the direction and spacing of the isoipses are closely related to the wind. On the other hand, in low latitudes the observed patterns of the geopotential and the stream lines are often independent. Very interesting research in this field has been carried out by Worthley (1959), who has computed the geostrophic winds at various latitudes using geopotential measurements taken each 300 n.m. along well-known flight paths of the U. S. Air Forces. These geostrophic winds have been compared with the wind components normal to the flight path, measured by means of Doppler-effect methods in the centres between the points where the geopotential was measured. These results have been elaborated by Finizio and Bucchi (1972), assuming that the weight to be given to the geopotential must decrease according to the percentage deviation between geostrophic and actual wind. Finally, the weight K is found varying along the latitude φ according to the expression :

$$K(\varphi) = \frac{2.8}{\pi}\,\varphi - \frac{3.2}{\pi^2}\,\varphi^2 \qquad (6)$$

3. Application

In order to test this scheme in tropical areas. reference has been made to a work of Ramanathan, *et al.* (1971) concerning a situation (00 GMT of June 16, 1966) of monsoonic depression in the northern Bay of Bengal. For that situation the availability of information was larger than ordinarily.

In that work the researches of the Poona Institute of Tropical Meteorology deduce the

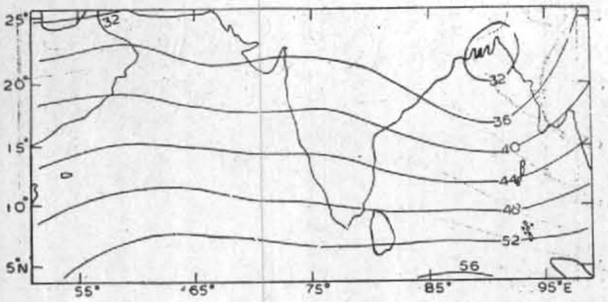

Fig. 1(a). Observed stream function for 850 mb ($10^5 m^2/s$) on 16 Juue 1966 (00GMT)

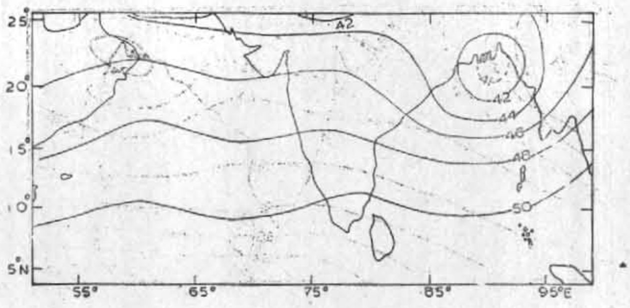

Fig. 1(b). Observed geopotential (in dam *minus* 100) fo 850 mb on 16 June 1966 (00 GMT)

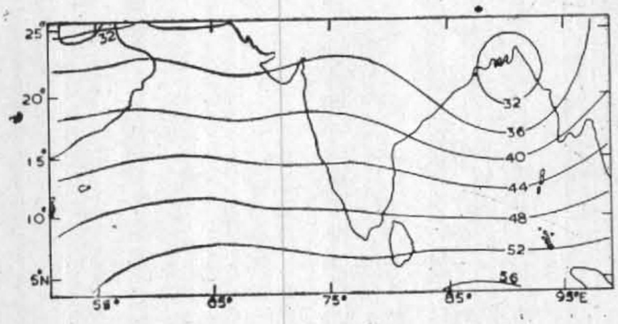

Fig. 1 (c). Balanced stream function for 850 mb (10^5 m^2 /s) on 16 June 1966 (00 GMT)

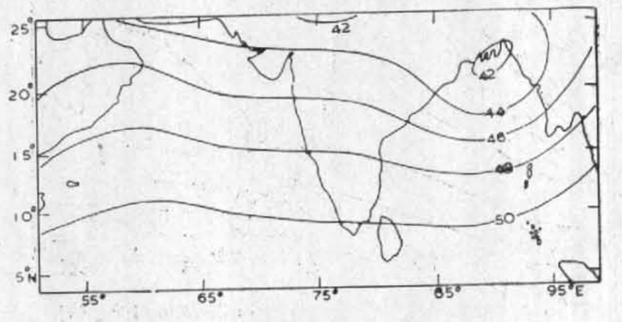

Fig. 1 (d). Balanced geopotential (in dam *minus* 100) for 850 mb on 16 June 1966 (00 GMT))

geopotential field by means of the balance equation from the stream function inferred from the observed wind.

Taking into account the particular area (from 2.5°N to 27.5°N and from 50°E to 100°E), a Mercator projection has been used (map factor $m = 1/\cos\varphi$) with a grid mesh $\triangle x = 277$ km at the equator (correcsponding to $\triangle\varphi_{eq} = \triangle\lambda_{eq} = 2.5°$). Figs. 1 and 2 show the patterns Ψ_0, Z_0, Ψ_b, Z_b (index $_0$ and b stand for objective analysis and balanced field) respectively for the topography of 850 and 300 mb.

3.1 *Comparison*

A qualitative comparison between the maps shows that, besides the general agreement, there is some different feature between the fields before and after the balancement ; for instance, the ridge of the field at 850 mb over the Indian Peninsula is lower, particularly in the southern belt, where the kinematic field is more important in the balancement; and the weak ridge in the easterly stream between the anticyclonic circulations at 300 mb appears to be displaced towards east, especially in the lower latitudes.

A quantitative comparison between balanced and observed fields has been made in the grid points. Firstly, the mean square differences $\sigma\Psi$ between Ψ_o and Ψ_b and σ_Z between Z_o and Z_b have been computed. These are reported in Table 1 together with σ^*_Z as computed by Ramanathan *et al.* (1971).

After having recognized the fair agreement between the configurations, the agreement between the gradient of Ψ_0 and Ψ_b, and Z_0 and Z_b have been evaluated by means of the following winds :

$$\vec{V}_0 = (u_0, v_0), \text{ observed wind;}$$

$$\vec{V}_1 = \vec{K} \times \vec{\nabla}\Psi_0 = (u_1, v_1);$$

$$\vec{V}_2 = \vec{K} \times \vec{\nabla}\Psi_b = (u_2, v_2)$$

$$\vec{V}_{go} = \frac{g}{f} \vec{K} \times \vec{\nabla} Z_0 = (u_{g0}, v_{g0});$$

$$\vec{V}_{gb} = \frac{g}{f} \vec{K} \times \vec{\nabla} Z_b = (u_{gb}, v_{gb})$$

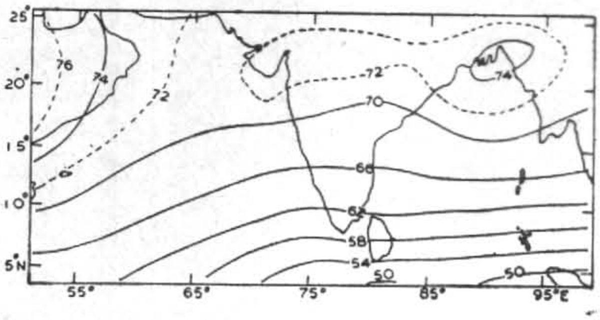

Fig. 2(a). Observed stream function for 300 mb (10^5 m^2/s) on 16 June 1966 (00GMT)

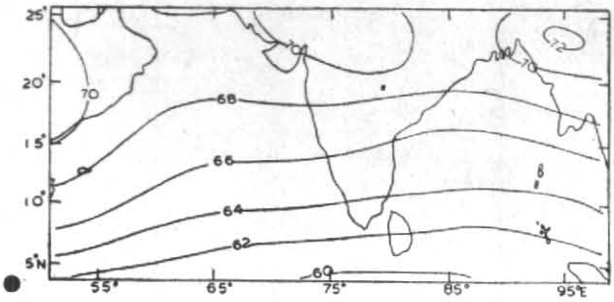

Fig. 2(b). Observed geopotential (in dam *minus* 900) for 300 mb on 16 June 1966 (00 GMT)

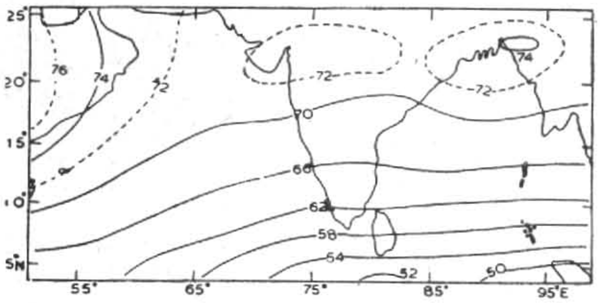

Fig. 2(c). Balanced stream function for 300 mb (10^5 m^2/s) on 16 June 1966 (00 GMT)

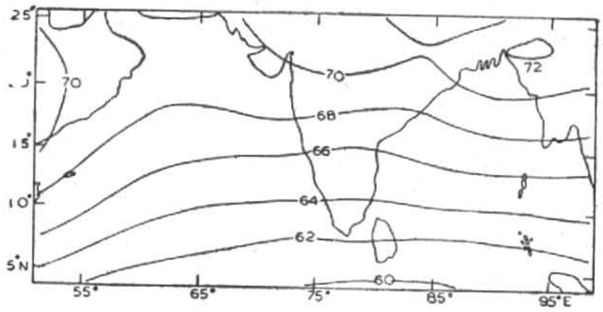

Fig. 2(d). Balanced geopotential (in dam *minus* 900) for 300 mb on 16 June 1966 (00GMT)

TABLE 1

Rootmean square differences between ψ_o and ψ_b, Z_o and Z_b

Level (mb)	σ_ψ (m)	σ_Z (m)	σ^*_Z (m)
850	2·8	3.8	6·2
300	3·4	4·7	6·7

TABLE 2

Mean square values of the winds

Level (mb)	V_0 (m/s)	V_1 (m/s)	V_{g_0} (m/s)	V_{gb} (m/s)	V_2 (m/s)
350	10·0	10·0	11·0	11·1	10·6
300	11·9	10·3	20·5	19·6	10·1

TABLE 3

Cross root mean square differences between the winds at 850 mb (in m/s)

	V_0	V_{g_0}	V_1	V_{gb}	V_2
V_o	0	5·4	2·7	4·0	2·8
V_{gs}	5·4	0	4·4	3·2	3·7
V_1	2·7	4·4	0	2·5	0·9
V_{gb}	4·0	3·2	2·5	0	2·5
V_2	2·8	3·7	0·9	2·5	0

TABLE 4

Cross root mean square differences between the winds at 300 mb (in m/s)

	V_0	V_{g_0}	V_1	V_{gb}	V_2
V_o	0	11·7	3·2	9·8	3·4
V_{go}	11·7	0	12·8	4·5	12·5
V_2	3·2	12·8	70	11·0	0·9
V_{gb}	9·8	4·5	11·0	0	11·0
V_1	3·4	12·5	0·9	11·0	0

The mean square values of these winds have been reported in Table 2.

The cross root mean square differences $\sigma_{\vec{V}}$ between the winds have been reported in Tables 3 and 4.

From Tables 3 and 4 it can be noted that $\sigma\left(\vec{V}_{gb}, \vec{V}_0\right)$ is systematically and consistently smaller than $\sigma\left(\vec{V}_{g0}, \vec{V}_0\right)$, whereas $\sigma\left(\vec{V}_2, \vec{V}_0\right)$ is systematically but non consistently smaller than $\sigma\left(\vec{V}_1, \vec{V}_0\right)$.

In fact the agreement between $\vec{V}_1$ and $\vec{V}_2$ is noticeable, according to that, at low latitudes, the balancing is mainly determined by the kinematic field.

Finally, it is evident the large difference between the observed and the geostrophic winds at 300 mb. The latest is about twice the observed wind, as a confirm of the large errors which one could make if he assumes the wind to be given by the geostrophic approximation. Even after the balancement, the difference remain almost the same, as the balancement only regards the vorticity field (and thus the circulation), and not the wind field itself.

4. Conclusions and suggestions

This study shows that it is interesting for the synoptic scale to balance the geopotential and the wind fields varying their relative importance as a continuous function of the latitude. However, when willing to use this scheme for a smaller scale model, it will be necessary to take into account also the initial divergence of the wind field, for instance through the ω-equation and the continuity equation to determine ω and the potential χ of the wind field. As regards the small scale, this scheme is also very attracting, as far as it allows to adjust the initialization procedure according to the parameter which one is more interested to predict, simply giving a suitable weight to the different observed parameters which enter the balance equations.

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