

## A statistical method of forecasting the movement of cyclonic storms in the Bay of Bengal

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**ABSTRACT.** Arakawa (1963) established a set of regression equations to forecast the movement and the central pressure of typhoons in the western North Pacific. In the present paper, using a similar scheme, we have fitted two sets of regression equations, one each for the pre-monsoon and post monsoon seasons (1963 to 1970). These equations have been tested during the storm season of 1971 and the results, which are encouraging, are presented in this paper.

### 1. Introduction

Forecasting the tracks and intensities of storms in the Bay of Bengal is a very important problem. Lot of property and human lives could be saved by accurately forecasting the future behaviour of a storm. A number of subjective, semi-objective and numerical techniques have been adopted in India and elsewhere to forecast the movement of storms. Although synoptic forecasters can predict the movement of the storm to a fair degree of accuracy with their skill and many years of experience, there is still a need for objective schemes to track the cyclonic storms as accurately as or more accurately than a synoptic forecaster. Recently Gupta and Datta (1971) used a storm analogue technique to forecast the storm movements in the Bay of Bengal. This scheme makes use of all the climatology of the storms from 1891-1970 for picking out the storm analogues. This scheme is being tested on real time basis for its utility in the day-to-day forecasting. A similar scheme given by Sikka and Suryanarayana (1971) uses 50 per cent persistence and 50 per cent climatology for predicting the movement of the storms. Another promising alternative scheme for prediction seems to be the statistical method of forecasting the tropical storms as suggested by Vignès and Miller (1959) for Atlantic hurricanes. Similar schemes have been used by various other workers for different oceanic areas. Arakawa (1963) has used it for North-western Pacific and Chaepyo Cook (1963) for all storms hitting Korea and its neighbourhood. In the present paper this statistical technique has been used for forecasting movement of Bay of Bengal storms.

The storm season in India is divided into two periods—(1) the pre-monsoon (months of April, May and early June) and (2) the post monsoon (late September to early December). Separate regression equations for forecasting storm movements have been developed for the two seasons.

### 2. Data used

The surface weather charts of 00 and 1200 GMT for the storm season of 1963 through 1970, plotted and analysed at Northern Hemisphere Analysis Centre, New Delhi were made use of for the above study. There were in all 17 storms, 8 in the pre-monsoon and 9 in the post monsoon seasons which have been used for fitting the above equations. These storms together with the number of days for which data has been used for each of these storms are given in Table 1. A 5-degree moving co-ordinate grid centred on the centre of the cyclonic storm has been used. The grid map is shown in Fig. 1. The area chosen was between Long. 85° to 105°E and Lat. 5° to 25°N so as to include most of the storm which are likely to hit the east coast of India or Bangla Desh. The total number of grid points are 25 and pressure values were picked up for all points keeping the centre of the grid at the centre of the cyclonic storm. The data were collected for the various storms from the period it is declared as a storm to the day of striking the coast or its weakening, whichever was earlier. The other parameters taken were the latitude, longitude and central pressure of the storm 12 and 24 hour earlier to chart time and the latitude, longitude and central pressure of the storm at the chart time, as indicated below. Thus

TABLE 1  
Storms fitting the regression equations

Pre-monsoon season		Post monsoon season	
Date	No. of day for which data was picked up	Date	No. of day for which data was picked up
26 May 1963	6	31 Dec 1965	6
7 May 1964	3	29 Sept 1966	5
10 May 1965	4	18 Nov 1966	7
30 May 1965	3	8 Dec 1966	9
16 May 1967	4	20 Oct 1968	13
8 May 1968	5	9 Nov 1968	11
14 May 1969	12	4 Nov 1969	8
5 May 1970	4	20 Oct 1970	8
		9 Nov 1970	9
Total	41		76

a total of 33 predictors were used—

- $\lambda_{-12}$ ,  $\phi_{-12}$ ,  $p_{-12}$  Central latitude, longitude and central pressure of the cyclonic storm 12-hr prior to chart time.
- $\lambda_{-24}$ ,  $\phi_{-24}$ ,  $p_{-24}$  Central latitude, longitude and central pressure of the cyclonic storm 24-hour prior to chart time.
- $\lambda_0$ ,  $\phi_0$  Latitude and longitude of the cyclonic storm centre at chart time.
- $X_1, X_2, X_3, \dots, X_{25}$  Pressure values at all grid points at chart time.

### 3. Regression equations

From these predictors regression equations have been derived for  $(\lambda_{12}, \phi_{12})$  and  $(\lambda_{24}, \phi_{24})$  at Lat. ( $^{\circ}$ N) and Long. ( $^{\circ}$ E), 12 and 24-hour after chart time. As a first step no screening procedure has been adopted and all the 33 predictors have been used for deriving the above equations, whereas in the next section we have modified the set of the regression equations by using screening procedure.

Separate equations have been derived for the pre-monsoon and post monsoon seasons. Regression equations before screening and after screening have been given in the Appendix.

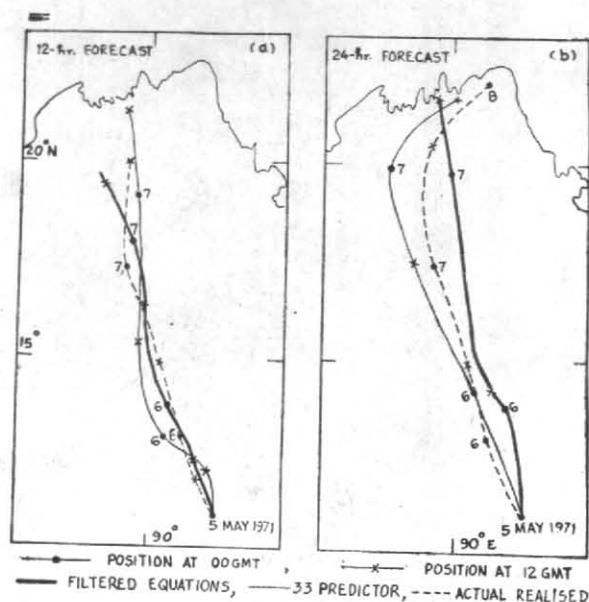


Fig. 1

(a) 12-hr and (b) 24-hr forecast positions based on 5 May 1971

The different sets of regression co-efficients for the two seasons for equations without screening are given in Tables 2 and 3, whereas co-efficients with screening are given along with the equations.

### 4. Independent testing

The above prediction equations were used to predict the movement of storms in the two seasons, pre-monsoon (2 storms) and post monsoon (1 storm) for the year 1971.

In Figs. 1 to 3, we have presented the forecasts positions based on the regression equation with and without screening. For the purpose of comparison we have indicated the actual realized position.

Qualitatively it can be noticed that the forecasts by screening are as good or better than one based on 33 parameters. Forecasts positions also agree fairly well with the actual realized.

### 5. Discussion of results

(a) *When the regression equations were used without screening*—We will be discussing first the 12-hour forecast positions for each of the four storms and then the 24-hour forecast positions.

#### 12-hr Forecast

In Table 4 we present the results of 12-hour forecast positions issued by the use of the above

TABLE 2  
Regression coefficients for premonsoon season

S. No.	Regression coefficients for			
	$\lambda_{12}(a_i)$	$\phi_{12}(b_i)$	$\lambda_{24}(c_i)$	$\phi_{24}(d_i)$
1	-0.24046	-0.45694	0.00513	-1.04777
2	0.40171	0.30930	0.70525	0.59564
3	-0.15315	-0.52454	0.47135	-1.43770
4	-0.52552	0.77628	-2.25771	1.74133
5	0.30912	0.35726	1.16144	0.19900
6	-0.04907	-0.20777	-0.48840	-0.07860
7	-0.44170	0.30750	-0.97365	0.35696
8	0.30259	-0.09153	0.83293	-0.03919
9	0.24540	-0.12248	0.02135	-0.34813
10	-0.08553	0.08283	0.55084	0.39254
11	-0.01336	-0.15153	-0.01485	-0.36854
12	-0.06442	-0.07715	0.04835	-0.06059
13	0.00598	-0.01846	0.01352	-0.05458
14	-0.05088	-0.19645	0.02385	-0.58703
15	0.48964	0.06114	0.42500	0.73568
16	-0.09808	-0.01441	0.53379	-0.04399
17	0.28672	-0.10730	-0.01021	0.02996
18	0.03781	-0.08925	0.35555	-0.09595
19	-0.17060	0.19040	-0.65291	0.15889
20	0.10891	-0.01082	-0.21826	-0.14918
21	-0.05084	0.02167	-0.02704	-0.03760
22	-0.04735	0.45672	-0.65289	0.89043
23	-0.14509	-0.68313	0.56774	-1.05227
24	-0.02977	0.1057	-0.42122	0.19226
25	-0.04794	0.00749	0.29508	-0.05535
26	1.30462	-0.22369	1.85550	-0.31968
27	-0.11344	1.30244	-0.07374	1.43831
28	-0.01581	-0.16583	-0.40790	-0.25952
29	0.06991	0.26293	0.30802	0.09252
30	0.00820	0.04324	-0.21052	0.12288
31	-0.51908	0.13768	-1.01819	0.16030
32	-0.15945	-0.74768	-0.56267	-0.88267
33	-0.02963	0.07314	0.02761	0.09481

$a_0 = 68.39491$   $b_0 = -125.50839$   $c_0 = -75.27107$   
 $d_0 = -18.72465$

TABLE 2  
Regression coefficients for post monsoon season

S. No.	Regression coefficients for			
	$\lambda_{12}(a_i)$	$\phi_{12}(b_i)$	$\lambda_{24}(c_i)$	$\phi_{24}(d_i)$
1	0.25934	-0.13815	-0.26045	-0.19327
2	-0.00372	0.10674	0.71677	0.40353
3	0.23343	0.30470	-0.80346	0.17088
4	-0.36560	-0.13673	-0.05736	-0.10452
5	-0.05780	-0.03815	-0.37803	-0.50737
6	-0.33158	0.20726	0.13119	0.34655
7	0.05004	-0.05272	-0.10878	-0.17578
8	-0.13727	-0.04506	-0.16408	0.39309
9	0.08701	-0.36298	0.15946	-0.88517
10	0.10023	0.21399	0.11567	0.68576
11	0.17307	-0.31219	-0.37174	-0.65909
12	-0.20356	0.19860	0.06049	0.42852
13	0.03463	0.00880	0.06049	-0.01744
14	0.05756	0.22287	0.18054	0.09006
15	-0.05267	0.01912	-0.02891	0.03051
16	-0.09696	-0.06880	-0.10686	0.25670
17	0.22908	0.01415	0.50247	0.21119
18	-0.05560	-0.07898	-0.18339	-0.41767
19	0.04720	-0.22336	0.09788	0.05667
20	0.00334	-0.03982	-0.07044	-0.03712
21	-0.13619	0.01488	-0.28420	-0.16859
22	-0.01375	0.00742	-0.02704	-0.35930
23	0.03286	0.00746	0.02948	0.50061
25	0.09861	0.10265	0.11626	-0.09538
25	-0.03542	0.05065	0.06370	-0.03565
26	1.23288	0.18516	1.34933	0.44495
27	0.18259	1.34662	0.18613	1.74115
28	-0.14858	0.06186	-0.13254	0.30798
29	-0.30962	-0.26433	-0.48514	-0.46046
30	0.00748	-0.02855	-0.03341	-0.03508
31	-0.14685	-0.14923	-0.30219	-0.48328
32	0.12239	-0.20867	0.26026	-0.36901
33	-0.04709	0.01797	-0.07485	0.06067

$a_0 = 125.95313$   $b_0 = 86.24079$   $c_0 = 372.69960$   $d_0 = 61.67589$

regression equations. The actual location and the movement of the tropical cyclone are also given. In order to study how far the present scheme compared with the conventional forecast we have also included in the table the direction of movement as predicted by the Meteorological Office, Poona. It may be noticed from the table that generally the 12-hour forecast positions are within  $\pm 1.0^\circ$  Lat./Long. of actual for the pre-monsoon cases. In the following we discuss each storm separately.

(i) 5 May 1971 storm—The movement based on the initial positions on 5th and 6th are within

$\pm 0.5^\circ$  of the actual positions of the storm but after 6th (1200 GMT) movement given by the present scheme, although more or less in the same direction as actual, is comparatively faster. Comparison of the direction of movement, to the one given by the conventional forecaster would indicate that the former is either better or the same as compared to the latter.

(ii) 4 June 1971 storm—The forecast positions for this storm are almost within  $\pm 0.5^\circ$  of the actual storm and the direction of movement given by the present scheme is almost on all occasions nearer to the actual and better than that given by



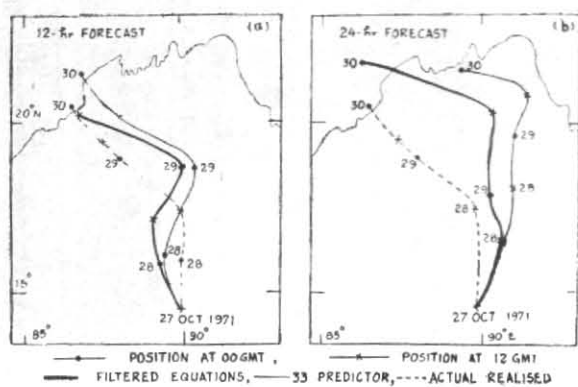


Fig. 2  
(a) 12-hr and (b) 24-hr forecast position  
based on 27 October 1971

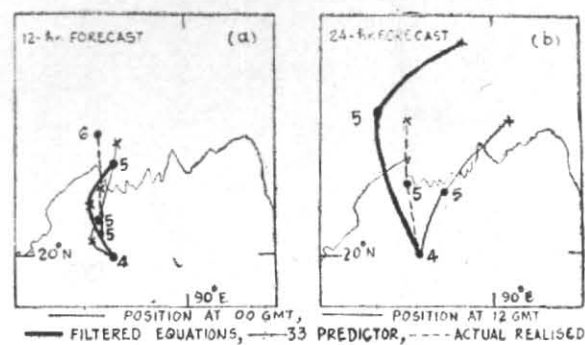


Fig. 3  
(a) 12-hr and (b) 24-hr forecast positions  
based on 4 June 1971

TABLE 4  
12-hour forecast position of storms

Date	Time (GMT)	Initial position		Forecast direction of movement (deg)		Realised direction of movement (deg)	Forecast position		Actual position	
		Lat. (°N)	Long. (°E)	Stat. method	Conventional		Lat. (°N)	Long. (°E)	Lat. (°N)	Long. (°E)
5 May 1971	00	11.0	92.0	350 (340)	315	340	12.1 (12.4)	91.8 (91.4)	12.0	91.5
	12	12.0	91.5	320 (335)	315	335	13.0 (13.8)	90.6 (90.7)	13.0	91.0
6 May 1971	00	13.0	91.0	335 (340)	335	345	15.5 (16.4)	89.8 (89.9)	15.0	90.5
	12	15.0	90.5	350 (345)	335	340	19.2 (18.1)	89.7 (89.7)	17.5	89.5
7 May 1971	00	17.5	89.5	360 (340)	360	360	21.3 (19.7)	89.4 (88.8)	20.0	89.5
4 Jun 1971	00	20.0	88.5	030 (060)	315	050	20.3 (21.1)	87.8 (87.8)	20.5	88.0
	12	20.5	88.0	360 (015)	315	360	20.7 (22.0)	88.0 (88.4)	21.5	88.0
5 Jun 1971	00	21.5	88.0	025 (020)	360	360	22.4 (23.4)	88.4 (88.8)	22.5	88.0
	12	22.5	88.0	Entered Inland			22.7 (24.1)	85.8 (87.1)	23.0	88.5
27 Oct 1971	12	14.5	90.0	340 (335)	315 to 360	360	16.1 (15.8)	89.5 (89.2)	16.0	90.0
28 Oct 1971	00	16.0	90.0	360 (325)	315 to 360	360	17.5 (17.3)	90.0 (89.1)	17.5	90.0
	12	17.5	90.0	360 (020)	360	310	18.7 (18.8)	90.4 (90.1)	19.0	88.0
29 Oct 1971	00	19.0	88.0	360 (320)	335	320	20.2 (20.3)	88.0 (86.7)	19.5	87.5
	12	19.5	87.5	320 (010)	335	320	21.5 (20.4)	86.9 (87.7)	20.5	86.5
30 Oct 1971	00	20.5	86.5	015 (355)	335	360	22.0 (21.6)	86.9 (86.2)	21.5	86.5
	12	21.5	86.5	Entered Inland			23.1 (22.6)	86.1 (87.4)	21.8	87.5

TABLE 5  
24-hour forecast position of storms

Date	Time (GMT)	Initial position		Forecast direction of movement (deg)		Realised direction of movement (deg)	Forecast position		Actual position	
		Lat. (°N)	Long. (°E)	Stat. method	Conventional		Lat. (°N)	Long. (°E)	Lat. (°N)	Long. (°E)
5 May 1971	00	11.0	92.0	340 (355)	315	335	14.2 (13.8)	90.6 (91.6)	13.0	91.0
	12	12.0	91.5	340 (350)	315	340	17.6 (14.1)	89.0 (91.1)	15.0	91.0
6 May 1971	00	13.0	91.0	345 (350)	335	340	19.9 (19.8)	88.2 (90.0)	17.5	89.5
	12	15.0	90.5	355 (350)	335	350	21.6 (21.7)	90.1 (89.7)	20.5	89.5
7 May 1971	00	17.5	89.5	010 (355)	360	015	24.2 (24.2)	90.0 (88.9)	22.0	91.0
4 Jun 1971	00	20.0	88.5	340 (335)	315	360	23.3 (29.4)	87.2 (88.7)	21.5	88.0
	12	20.5	88.0	010 (030)	315	360	24.4 (28.9)	89.5 (90.5)	22.5	88.0
5 Jun 1971	00	21.5	88.0	055	360	310	21.8 (21.7)	88.5 (89.5)	23.0	86.5
	12	22.5	88.0	290	360	300	23.8 (22.7)	83.1 (87.2)	24.0	85.0
27 Oct 1971	12	14.5	90.0	015 (020)	315	360	18.1 (16.5)	91.1 (90.8)	17.5	90.0
28 Oct 1971	00	16.0	90.0	015 (010)	315	060	19.7 (17.9)	91.2 (90.3)	19.0	88.0
	12	17.5	90.0	025 (010)	315	040	20.9 (20.4)	91.6 (90.5)	19.5	87.5
29 Oct 1971	00	19.0	88.0	025 (060)	335	315	21.7 (21.9)	89.4 (86.2)	20.5	86.5
	12	19.5	87.5	Entd. Inland	335	335	24.2 (22.2)	87.8 (87.7)	21.5	86.5
30 Oct 1971	00	20.5	86.5	Entd. Inland	335	065	23.3 (23.4)	86.2 (86.2)	21.8	87.5
	12	21.5	86.5	Entered Inland						

NOTE: Figures within bracket give direction and position without screening.

the conventional method, e.g., the direction of movement based on 4 June 1971 (00 GMT) position as indicated by the present scheme is 30° compared to 315° given by conventional method and 50° the actual realised. Similarly, at the next synoptic hour this scheme has given the direction of movement as 360° and actual realised is also 360° compared to 315° given by conventional method. The movement next day however indicated by conventional forecast is slightly better being, 360° compared to actual 360° and statistical forecast of 025°.

(iii) 27 October 1971 storm—It is noticed from Table 4 that the forecast position was within a degree or less of the actual, except for the forecast positions based on 28 October 1971 (1200 GMT) when the storm was fast recurving. If we look for the direction of movement, then those given by conventional forecast seem to be better in a few cases.

But it may be seen that the conventional forecast had been from the beginning giving the movement between 315° to 360° although the storm in the initial stages was moving dead northwards and later northwest wards. Therefore the forecasts by conventional methods in the initial stages were not correct

24-hr Forecasts

24-hr forecast positions are given in Table 5.

(i) 5 May 1971 storm—The forecast positions for this storm differ from the actual positions by a maximum of 2.5°. The direction of movement given by the present scheme is much closer to the actual, better than conventional forecast and had been within 10° on all days except the forecast based on 5 May 1971 (1200 GMT) when both the position and the direction had been slightly out,

(ii) 4 June 1971 storm — The direction of movement given by the statistical model are generally better than the conventional forecasts except on 5 June 1971 (00 GMT) when position given by conventional method was better.

(iii) 27 October 1971 storm — Forecast positions based on the present scheme have been within  $1^\circ$  to  $1.5^\circ$  of the actual on all the days. Although the forecast direction of movement had given more bias towards a slight easterly component, i.e., about  $15^\circ$  compared to the actual movement of  $360^\circ$  to  $320^\circ$ . The conventional forecaster however continued to indicate the movement to be NNW to N.

(b) When the equations were used after screening

(i) 5 May 1971 storm — The positions were almost the same as given by those equations without screening upto 6 May 1971 (00GMT); later the position given by the second set of equations are nearer the actual positions for 12-hr as well as for 24-hr.

(ii) 4 June 1971 storm — The positions are almost similar to the one given by the second set of equations.

(iii) 27 October 1971 — The forecast positions in this case are better than the forecast positions with the second set of equations.

#### 6. Concluding remarks

(i) The direction of movement given by this model for the first 12-hr has been generally better than the conventional forecast. However, it may be noticed that the forecast given by the present scheme has given better results for premonsoon cases than that for the post monsoon cases.

(ii) The direction of movement for 24-hr has also been better as compared to the conventional forecast as far as the premonsoon storms are con-

cerned although the same cannot definitely be concluded for the post monsoon cases.

(iii) By using screening procedure (Vigas and Miller 1959) the number of variables entering in the regression equations is considerably reduced without changing the forecast positions to any appreciable extent. We propose to use these filtered equations for operational purpose.

(iv) It is seen from the filtered regression equations that the contribution of pressure values is very negligible. However, a change of 2 mb in the pressure value may at the most change the forecast positions by  $0.5^\circ$  but the forecast positions depend very much on the initial, 12-hr and 24-hr positions of the storms before chart time. Hence these positions have to be estimated to the utmost accuracy.

(v) The present scheme can be fruitfully utilised for finalising the storm track forecast in the manner suggested by Datta (1972).

(vi) The regression equation for both premonsoon and post monsoon cases were based on the sea level analysis done at N.H.A.C. on polar stereographic charts, but for test purposes the analysis for the pre-monsoon period was on the same projection whereas for the post monsoon case it was on the mercator projection, since N.H.A.C. had changed the projection to mercator from 1 July 1971. This may account for the comparatively higher differences in the forecast positions and movements for the post monsoon cases.

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APPENDIX

Regression equations before screening

$$\lambda_{12} = a_0 + \left(\sum_{z=1}^{25} a_i x_i\right) + a_{26} \lambda_0 + a_{27} \phi_0 + a_{28} \lambda_{-12} + a_{29} \phi_{-12} + a_{30} p_{-12} + a_{31} \lambda_{-24} + a_{32} \phi_{-24} + a_{33} p_{-24} \quad (i)$$

$$\phi_{12} = b_0 + \left(\sum_{z=1}^{25} b_i x_i\right) + b_{26} \lambda_0 + b_{27} \phi_0 + b_{28} \lambda_{-12} + b_{29} \phi_{-12} + b_{30} p_{-12} + b_{31} \lambda_{-24} + b_{32} \phi_{-24} + b_{33} p_{-24} \quad (ii)$$

$$\lambda_{24} = c_0 + \left(\sum_{z=1}^{25} c_i x_i\right) + c_{26} \lambda_0 + c_{27} \phi_0 + c_{28} \lambda_{-12} + c_{29} \phi_{-12} + c_{30} p_{-12} + c_{31} \lambda_{-24} + c_{32} \phi_{-24} + c_{33} p_{-24} \quad (iii)$$

$$\phi_{24} = d_0 + \left(\sum_{z=1}^{25} d_i x_i\right) + d_{26} \lambda_0 + d_{27} \phi_0 + d_{28} \lambda_{-12} + d_{29} \phi_{-12} + d_{30} p_{-12} + d_{31} \lambda_{-24} + d_{32} \phi_{-24} + d_{33} p_{-24} \quad (iv)$$

where  $a_i, b_i, c_i$  and  $d_i$ , the 33 regression coefficients for  $\lambda_{12}, \phi_{12}, \lambda_{24}, \phi_{24}$  and  $a_0, b_0, c_0, d_0$ , the intercepts for each of these equations and for both the season's are given in Tables 2 and 3.

Regression equations after screening

(a) Pre-monsoon Season

$$\lambda_{12} = 81.11827 + 1.33534 \lambda_0 - 0.49783 \lambda_{-24} - 0.09336 p_{-24} - 0.14442 X_{18} + 0.15848 X_9 \quad (\text{P.R.} = 93.3) \quad (i)$$

$$\phi_{12} = 103.43805 + 1.41728 \phi_0 - 0.40336 \phi_{-24} - 0.10401 X_{23} \quad (\text{P.R.} = 98.7) \quad (ii)$$

$$\lambda_{24} = 29.19434 + 1.33724 \lambda_0 - 0.96323 \lambda_{-24} - 0.12216 p_{-24} - 0.09896 X_{21} + 0.91454 X_{15} - 0.42219 \phi_{-24} - 0.67780 X_6 \quad (\text{P.R.} = 91.4) \quad (iii)$$

$$\phi_{24} = 199.17896 + 1.68670 \phi_0 - 0.64801 \phi_{-24} - 0.20132 X_{14} \quad (\text{P.R.} = 96.9) \quad (iv)$$

(b) Post monsoon Season

$$\lambda_{12} = 0.53554 + 1.40303 \lambda_0 - 0.39294 \lambda_{-12} \quad (\text{P.R.} = 96.9) \quad (i)$$

$$\phi_{12} = 80.78525 + 1.56330 \phi_0 - 0.65500 \phi_{-12} + 0.11610 \lambda_0 - 0.26133 X_{11} + 0.18779 X_{12} \quad (\text{P.R.} = 95.3) \quad (ii)$$

$$\lambda_{24} = 312.03394 + 1.47499 \lambda_0 - 0.24012 X_{24} - 0.53050 \lambda_{-12} - 0.06738 p_{-24} \quad (\text{P.R.} = 92.9) \quad (iii)$$

$$\phi_{24} = 132.56703 + 2.19053 \phi_0 - 1.27423 \phi_{-12} + 0.59373 \lambda_0 - 0.41131 \lambda_{-24} - 0.12689 X_{20} \quad (\text{P.R.} = 86.7) \quad (iv)$$

(P.R.—stands for percentage reduction in variance)

