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Generalized relationship between maximum rain depth, area and return period for major rainstorms

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ABSTRACT. A generalised relationship between maximum rain depth, area and return period has been determined using Karoly and Alexander technique. For this purpose major rainstorms which affected Bihar region during the 65-year period from 1897 to 1961 were analysed and their depth-area-duration data utilized. The 3-day relationship has been found to be of the form $d = -2 \cdot 76 \log A \log T - 4 \cdot 08 \log A + 16 \cdot 03 \log T + 23 \cdot 73$. Based on this relationship a nomogram has been prepared with the help of which maximum 3-day rain depths can be obtained for different areas and return periods. Conversion factors have also been worked out for obtaining maximum rain depths for one and two-day durations for different areas and return periods.

1. Introduction

An attempt has been made in this study to determine a generalised relationship, for a given duration, between the three variables (*viz.*, maximum rain depth, rain area and return period) which define the storm rainfall over an area. The technique developed by Karoly and Alexander (1960) for studying storm rainfall has been used in this study. This method has the advantage that for a fixed duration, maximum rainfall magnitudes for different areas and return periods can be obtained with the help of a single equation or a generalized depth-area-return period chart. In this study a generalised chart has been prepared on the basis of major rainstorm data of Bihar region. With the help of this chart maximum rain depths for different return periods and areas can be derived for a 3-day storm duration. Suitable ratios have also been derived which can be used to derive maximum 1 and 2-day rain depths for different return periods from maximum 3-day rainstorm depths.

2. Spatial distribution of tropical disturbances over and near Bihar region

While studying storm rainfall over Bihar region, Raman and Dhar (1966) have observed that practically all the major rainstorms over Bihar region are associated with the passage of tropical disturbances over and near this region. After crossing the Bengal-Orissa coast these disturbances give fairly widespread rainfall on either side of the storm track with heavier falls of rain to the south of the track especially over left front quadrant (Pisharoty and Asnani 1957; Ramaswamy 1967; Dhar and Mhaiskar 1970). Occasionally during mosoon months land depressions after their formation over Bengal-Orissa region, move over Bihar region and cause exceptionally heavey falls of rain. Some of these tropical disturbances after reaching east Madhya Pradesh and neighbourhood recurve and instead of moving in a westerly or northwesterly direction move in a northerly or northeasterly direction. It has been observed that often these recurving disturbances cause intense rainfall over Bihar region. Spatial distribution of these tropical disturbances, drawn on the basis of their occurrence in each one-degree Lat.-Long. grid squares during the 80-year period from 1891 to 1970 over and near Bihar region is shown in Fig. 1. It is evident from this figure that entire Bihar region comes under the influence of these disturbances although maximum storm activity is mainly confined to south Bihar.

3. Karoly-Alexander technique

Karoly-Alexander (1960) technique of obtaining a general relationship between the three rainstorm parameters has been utilised in this study of major rainstorms of Bihar region. The technique is briefly summarised below.

3.1. *dTA and dAT series*

The three variables (rainstorm parameters) which define storm rainfall for a fixed duration are (*i*) maximum rain depth (d), (*ii*) rainstorm area (A) over which maximum rain depth (d) has occurred in a given duration, and (*iii*) return period (T).

From a detailed depth-area-duration analysis of major rainstorms over a long period of years for a given region, a series of maximum rain depths for different sizes of area (say, 500, 1,000, 2000 etc sq. miles) and durations (say, 1, 2, 3, etc observational-days) are obtained whose return periord

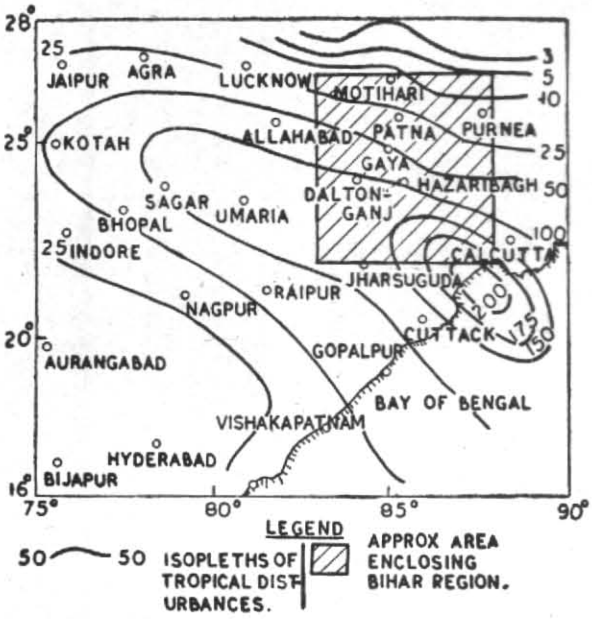

Fig. 1. Spatial distribution of tropical disturbances(de partmental storms) over and near the Bihar region during 1891 to 1970

can be worked out using the conventional methods. From this data two types of series can be obtained which can be used for estimating maximum rain depths for different areas and return periods. One of the series is called *dTA* series and the other dAT series. The *dAT* series can also be derived from the *dAT* series by plotting the latter on a semi-log paper with maximum rain depth (d) as ordinate on the arithmetic scale against return period (T) on a logarithmic scale fror eich fixed area (A), say 500, 1000 etc sq. miles. Straight lines are then fitted to the data by eye for each *dTA* set of values following the trend of the group of points with little or moderate scatter and giving due weight to the position of the top ranking points in the series.

In the *dAT* series the maximum rain depths are plotted on the arithmetic scale of the semi-log paper while areas are plotted on the log scale for different return periods, say 50, 100 years, etc. It has, however, been observed that in a meteorologically homogeneous region *dAT* lines are nearly straight with maximum rain depth (d) decreasing as the area increases. The slope of these lines become steeper as the return period increases. For most purposes the *dAT* series are preferable as in these series area is treated as a continuous variable while only selected values of return period are generally used.

3·2. *Relationship between dTA and dAT series*

Karoly and Alexander have shown that when both these series are plotted on a semi-log paper with maximum rain depth (d) on arithmetic seale and return period (T) and area (A) on log scale respectively, the following conditions hold good—

(*i*) *dTA* series are straight lines, and

(*ii*) for the same return period (d_2-d_1) is proportional to $(\log A_2-\log A_1)$, then

(*iii*) *dAT* series will also be straight lines and

(*iv*) lines of each series will converge to points each with the same average rain depth.

Using the above relationships, Karoly and Alexadender developed a generalised relationship expressing maximum rain depth (d) as a function of return period (T) and area (A).

3·3. *Generalised relationship between maximum rain depth and return period*

The equations of the straight lines representing *dTA* and *dAT* series respectively may be written as :

$$dTA \text{ series}: \quad d = a + c \log T \qquad (1)$$

$$dAT \text{ series}: \quad d = b - k \log A \qquad (2)$$

where a, b, c and k are the coefficients and d is the maximum rain depth over an area A and T is return period. It follows from Eq. (1) and (2) that in the case of *dTA* series, *i.e.*, Eqn. (1) maximum rain depths increase proportionately with the increase of the logarithm of the return period for a given area. Similarly in the case of *dAT* seiries, *i.e.*, Eq. (2), the maximum rain depth decreases proportionately with the increase of the logarithm of the area for a given return period.

Using the above relationships between *dTA* and *dAT* series, Karoly and Alexander obtained the following functional relationship between the three variables, *viz.*, maximum rain depth (d) return period (T) and area (A) :

$$d = mxu + nu + rx + s \qquad (3)$$

wihere m, n, r and s are the coefficients to be determined from the data and where,

$$x = \log T \qquad (4)$$

$$\text{and } u = \log A \qquad (5)$$

Eq. (3) represents a second order ruled surface in which *dTA* and *dAT* series are projections in (T—d) and (A—d) planes respectively. By rotating the x and u axes through an angle of 45° and shifting the origin, Eq. (3) can be reduced to

$$2\, d' = x'^2 - u'^2 \qquad (6)$$

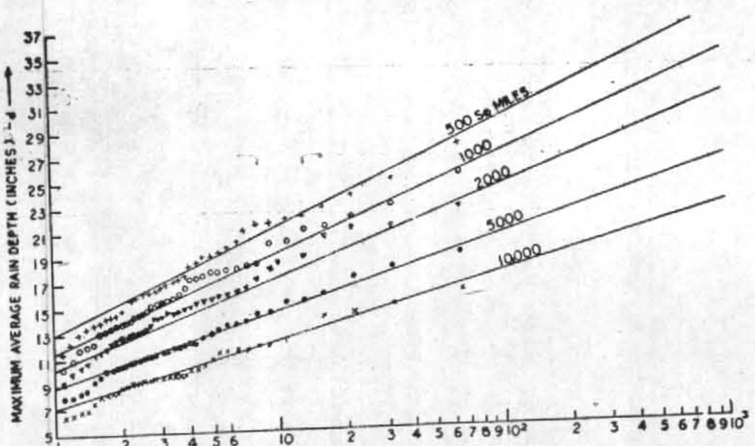

Fig 2.

Relationships between maximum rain depth (d) and return period (T) for certain fixed areas (A)

where x' and u' are the new rotated axes. Eq. (6) indicates that the surface represented by this equation is a hyperbolic paraboloid (Apostol 1962) where

$$x' = \frac{1}{\sqrt{2}}\left(u+x+\frac{n+r}{m}\right) \tag{7}$$

$$u' = \frac{1}{\sqrt{2}}\left(u-x-\frac{n-r}{m}\right) \tag{8}$$

$$d' = \left(d+\frac{nr}{m}-s\right)/m \tag{9}$$

The above technique has been applied to 3-day maximum rainstorm data of Bihar region in order to obtain a generalised relationship between maximum rain depth, area and return period.

4. Application of Karoly-Alexander technique

In this study the 3-day maximum rain depth data obtained by analysing all the major rainstorms during the 65-year period from 1897 to 1961 were utilised in order to obtain a generalised relationship of the type given in Eq. (3). To achieve this, all the rain depth for the standard areas of 500, 1000, 2000 etc. sq. miles were first arranged in descending order and ranks where assigned to each item of the series. For each series return periods were then worked out using the formula $T=(N+1)/m$, where N is number of years of record and m is the rank number. Maximum rain depths were then plotted on arithmetic scale and return periods on log scale of the semi-log paper. Best eye fit dTA lines were drawn through the points for fixed areas of 500, 1000,.......... 10,000 sq. miles. Fig. 2 shows the five eye fit dTA lines for 500, 1000, 2000, 5000 and 10,000 sq. miles. The equations of two dTA lines for areas of 1000 and 10,000 sq. miles are

Area $A_1=1000$ sq. miles, $d=11\cdot50+7\cdot75\log\,)$ (10)

Area $A_2=10{,}000$ sq. miles, $d=7\cdot40+5\cdot00\log T$ (11)

TABLE 1

Area (A) (sq. miles)	Maximum rain depths (d) in inches		
	T=10 yr.	T=100 yr.	T=1000 yr.
1000	19·25	27·00	34·75
10,000	12·40	17·40	22·40

From Eqns. (10) and (11) maximum rain depths for $T = 10$, 100 and 1000-year return periods were computed and the same are give in Table 1.

In the same way, dAT lines were plotted on semi-log paper for different return periods. Equations of two dAT lines for return periods of 100 and 1000 years are —

T=100 years, $d=55\cdot80-9\cdot60\log A$ (12)

T=1000 years, $d=71\cdot80-12\cdot35\log A$ (13)

The numerical values of the coefficients in Eq. (3) for the 3-day duration were then determined by using some of the values given in Table 1. Substituting these values in Eq. (3) a set of imulaneous equations were obtained which on solving gave the values of the coefficients as follows :

$m=-2\cdot76$, $n=-4\cdot08$, $r=16\cdot03$ and $s=23\cdot73$

TABLE 2

Area (sq. miles)	Percentage ratios of 1-day to 3-day rain depths					Percentage ratios of 2-day to 3-day rain depths				
	Return periods (yr)					Return periods (yr)				
	10	20	50	100	Average	10	20	50	100	Average
500	61	61	61	61	61	87	88	88	88	88
1000	59	60	60	60	60	86	86	86	86	86
2000	57	57	58	58	58	83	83	83	83	83
5000	54	55	56	56	55	80	80	80	80	80
10,000	50	50	50	50	50	79	79	79	78	79
15,000	48	48	48	48	48	79	79	77	77	78
20,000	42	42	42	42	42	74	73	72	72	73

On substituting these values in Eq. (3) a generalised relationship between maximum depth, area and return period was obtained for a 3-day duration for Bihar region. :

$$d = -2\cdot76 \log A \log T - 4\cdot08 \log A + 16\cdot03 \log T + 23\cdot73 \qquad (14)$$

Intersection of the surface represented by the above equation with planes perpendicular to the d-axis will give a family of curves representing a relationship between $\log T$ and $\log A$ for a fixed values of d. For fixed d we obtain from Eq. (14) the following—

$$\log A = \frac{16.03 \log T + 23.73 - d}{2.76 \log T + 4.08} \qquad (15)$$

Using the following substitution in Eq. (15)

$$\log A = u' + 5\cdot81$$

$$\log T = x' - 1\cdot48$$

the linear terms in $\log A$ and $\log T$ cancel out and we obtain

$$d = 2\cdot76\, u'x'$$

$$= 2\cdot76\,(5\cdot81 - \log A)\,(1\cdot48 + \log T) \qquad (16)$$

Eq. (16) represents a hyperbolic paraboloid surface and has been used to obtain a family of straight lines representing the relationship between area and return period for different maximum depths of rainfall. This relationship has been used in constructing a nomogram shown at Fig. 3 which shows the relationship between area and return period for different values of maximum rain depth.

6. Use of the depth-area-return period nomogram

Suppose a dam has to be constructed over a certain river in Bihar region whose basin area up to the dam site is about 1000 sq. miles and in this connection maximum 3-day rain depth of 100-year return period is required by the design engineers for the estimation of spillway capacity of the dam. In order to obtain 100-year 3-day maximum rain depth over this basin the nomogram at Fig. 3 is entered at the desired return period T (*i. e.*, 100 years in the present case) and desired area A (*i. e.* 1000 sq. miles). The point of interesection of these two lines on a fixed rain depth line will give the maximum depth of rain that is likely to occur over that basin during a 100-year period. In the present example the 100-year and 1000 sq. mile lines interesect on the line representing $d=27$ inches. Thus the design rainfall for 1000 sq. mile area for a return period of 100 years is of the order of 7 inches during a duration of 3 days. Similarly, the maximum 3-day average depth over 10,000 sq. miles for a 200-year return period is of the order of 19 inches. The estimates of maximum rain depths obtained with the help of the nomogram have been checked up with the depths obtained by frequency analysis of actual rain depths and have been found to be in full agreement.

7. Relationship between one-day and 2-day maximum rain depths to 3-day rain depths

In this study 3-observational day maximum rain depths have been used instead of 1 or 2-day depths on account of the fact that a 3-day depth is very nearly equal to 72-hour maximum rainfall (Hershfield 1961, Miller 1964, Dhar and Ramachandran 1970). Ratios have also been determined upto 20,000 sq. miles for converting 3-day maximum rain depths of different return periods into 1 and 2-day rain depths. The percentage ratios for different return periods over Bihar region are given in Table 2.

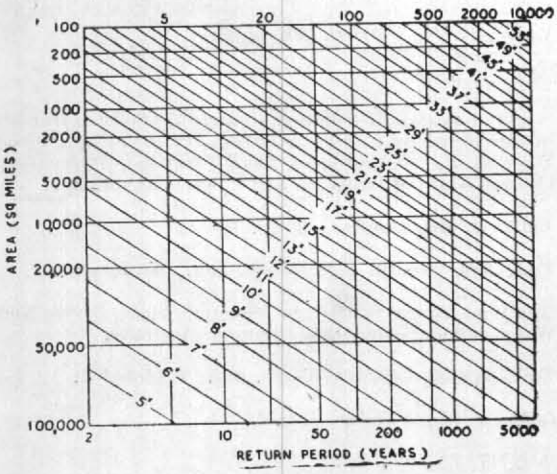

Fig 3.
Relationship betwen max. rain depth, area and return period for rainstorms of 3-day duration over Bihar region

TABLE 3

Percentage ratios of one and 2-day maximum rain depths to 3-day depths in the upper reaches of Krishna-Godaveri river systems

Basin	One day rainfall as %of 3 day*	2-day rainfall as % of 3-day*
(*i*) Bhima upto Ujjani dam site (5740 sq. miles)	54	79
(*ii*) Godaveri upto Gangapur (6000 sq. miles)	52	81
(*iii*) Krishna upto Sangali (3800 sq. miles)	56	80

*1, 2 and 3-day maximum rain depths refer to depth obtained by depth-duration envelope curve method whose return periods are of the order of 50 to 60 years (Dhar and Mhaiskar 1970).

Thus if we want to know one-day or 2-day maximum rain depths for a particular area for a given return period, this information can be obtained by first picking out the maximum rain depth from the nomogram at Fig. 3 for a 3-day duration for that area and the desired return period. This depth when multiplied by appropriate ratios from Table 2 will give one or 2-day maximum rain depths for that area and the given return period. For example, if the 3-day maximum rain depth for 1,000 sq. miles is of the order of 27 inches for the return period of 100 years, then 1 and 2-day rain depths for the same size area and the same return period can be obtained by applying the relevant ratios from Table 2 which are 0·60 and 0·86 respectively.

It is seen from Table 2 that for the same area the conversion factors for converting 3-day maximum rain depths into one-day or 2-day depths for different return periods are nearly of the same order. That is for converting 3-day maximum rain depth into one or 2-day depths for the same area in the same region, the ratios, for each day, for different return periods are almost identical. This was also noticed by Hariharan and Tripathi (1971) while comparing depth-duration data of different basins in the country. This only shows that *dAT* lines are not parallel to each other but do converge at one single point. The ratios obtained in Table 2 were compared with similar ratios obtained in a recent study for basins in the Krishna–Godavari upper reaches (Dhar and Mhaiskar 1970). As is evident from Table 3, these ratios are almost of the same order as those obtained in Table 2. However, detailed studies have to be carried out before it can be said with confidence whether these ratios change or remain same for different regions of the country.

7. Summary

(*i*) A three-dimensional relationship between maximum rain depths, area and return period has been worked out for 3-day duration for the major rainstorms of Bihar regions using the rainstorm data from 1897 to 1961 (Eq. 4). A nomogram has also been constructed on the basis of above equation for directly obtaining the values of maximum rain depths for different areas and return periods for a 3-day duration.

(*ii*) Conversion factors have been worked out for obtaining 1-day and 2-day maximum rain depths for different areas and return periods from 3-day rain depths.

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