*Indian J. Met. Geophys.*(1974),25, 3 & 4, 449-452

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# Airflow in a city

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*(Received 31 May 1972)*

ABSTRACT. The most imporant factor influencing the atmospheric pollution is wind. In the present article a theoretical model is described to obtain the wind flow inside a city area.

A horizontally homogeneous stationary turbulent layer is considered and the constructions like buildings, etc. are considered as obstacles in the fluid flow and are further assumed to be uniformly placed. The air is considered to be thermally neutral. The equations for the above said fluid flow are derived and a solution is obtained. The important results of theoretical calculations, namely, the variation of wind velocity with the area and type of obstruction and with the momentum flux entering from the ground flow are illustrated by means of diagrams.

## 1. Introduction

The most important meteorological parameter influencing the atmospheric pollution is wind. An accurate assessment of wind vector, facilitates a quantitative assessment of pollution from factory chimneys as well as from transport.

The regime of wind and turbulent exchange inside a city area, inside forests and hill regions attracts to itself the attention of meteorologists as well as specialists in other fields like botanists and agriculturists. Hence considerable attention has been given to this problem recently (Euone 1963, Laikhtman 1970, Vittal Murthy 1971).

In the present article a simple theoretical model is presented to obtain the airflow inside a city area.

## 2. Mathematical formulation of the problem

Mathematically speaking the problem of assessment of airflow inside a city area comes within the frame of surface layer of the atmosphere. The surface layer is a layer inside the atmosphere extending from the earth's surface to a height of some tens of metres (maximum upto 50 m approximately). The meteorological regime inside a surface layer is different from that of free atmosphere due to additional forces like turbulence coming into play. The accurate assessment of meteorological regime inside a surface layer is dealt by the author elsewhere (Vittal Murthy 1971).

The airflow inside a city area is similar to the flow inside a surface layer, but differs from it due to the obstructions to the general flow caused by buildings. The obstructions produce two effects : (1) There is loss of kinetic energy in the general flow due to friction, and (2) They generate vortices and thereby increase the turbulence in the flow. The kinetic energy of the flow inside a city depends on the extraction of energy from the general flow at higher levels. This extraction depends on the internal structure of the air stream in which the city is immersed. The downward eddy momentum flux, which determines the three dimensional motion of the atmospheric layer inside a city area, monotonically decreases towards the ground due to loss of energy caused by turbulence. Let us consider a thermally neutral, horizontally homogeneous surface layer of the atmosphere. Let us assume for simplicity that buildings are uniformly placed. Let us direct the vertical axis downwards. Neglecting the relatively small components of synoptic scale horizontal pressure gradient and Coriolis force, the equation of motion and the equation of turbulent energy can be written as

$$\rho \frac{du}{dt} = -\frac{d\tau}{d\zeta} - f \quad (1)$$

$$\rho \frac{d}{dt}\left(\frac{u^2}{2}\right) = -\frac{d}{d\zeta}(u\tau) + \tau\frac{\partial u}{\partial \zeta} - fu \quad (2)$$

where

$\rho$ = density of air $g/M^3$

$u$ = velocity of the flow

$\tau$ = surface stress (defined in Eq. 6).

Here $f$ is the loss of momentum in unit volume per unit time caused by friction. This is equivalent to some additional force. Following Eq. (1) we can say that the acceleration of a given mass of air is less than the momentum flux, as some part of the regular momentum flux will be appearing as fluctuating one. In Eq. (2) for energy $[\tau(\partial u/\partial\zeta) - fu]$ is the amount of energy of average flow which is changing to energy of perturbations.

It is natural to assume that $f$ depends

(*i*) on the momentum flux entering into a unit volume of the city $\tau$ and

(*ii*) on the total surface area of a obstructions within each unit volume $= (\delta n.d\zeta/Sd\zeta)$

where,

$\delta$ denotes perimeter of individual obstruction,

$n$ is the number of obstructions in the volume $S\,d\zeta$ and

$S$ is the total ground area of the city.

For the flow without obstructions $n=0$, $\delta=\infty$

In general it can be assumed that

$$f = constant\ (\tau)^{\alpha}(\delta)^{\beta}$$

From theory of dimensions it can be deduced that $f = a^2(\tau/\delta)$ where $a$ is a non-dimensional coefficient, depending upon type and geometrical distribution of obstruction. Let us assume further for simplicity, a stationery surface layer. The turbulent regime inside a city area can be written as follows :

$$\frac{d\tau}{d\zeta} = -a^2\frac{\tau}{\delta} \tag{3}$$

$$\frac{d}{d\zeta}(u\tau) = c\rho\frac{b^2}{k} \tag{4}$$

$$k = l\sqrt{b} \tag{5}$$

$$\tau = k\rho\frac{du}{d\zeta} \tag{6}$$

where

$b =$ turbulent energy,

$l =$ mixing length.

Here Eq. (3) is the equation of motion and Eq. (4) is for turbulent energy. Expanding and rearranging, this can be written as

$$\tau\frac{du}{d\zeta} + a^2\frac{\tau}{\delta}u - c\rho\frac{b^2}{k} = 0 \tag{4'}$$

where the first two terms on the left hand side indicate the incoming turbulent energy from the general flow. The third term is the dissipation of turbulent energy (Laikhtman 1970).

The unknown quantities in the above set of equations are $\tau$, $u$, $b$, $k$, $l$. But the equations are 4. This is not a closed system. A closed syestem can be obtained, provided we can assign a proper expression for $l$. Following Karman (1930) we can write that $l$ is a function of $du/d\zeta$, $d^2u/d\zeta^2$, $d^3u/d\zeta^3$ and the higher derivatives of $du/d\zeta$

$$l = F\left(\frac{du}{d\zeta}, \frac{d^2u}{d\zeta^2}, \frac{d^3u}{d\zeta^3}, \ldots\right)$$

If we restrict to the first two terms in the expression on the right hand side, we have 3 dependent parameters and two independent dimensions $L$ and $T$. Applying $\pi$ theorem in dimensional analysis, we can show that there can exist only one combination of $L$ and $T$ for expressing this relationship. By the same argument we can show that

$$l = -\chi c^{\frac{1}{4}}\frac{du}{d\zeta}\bigg/\frac{d^2u}{d\zeta^2} \tag{7}$$

where $\chi$ and $c$ are empirical constants, This scale of turbulence has been derived on the assumption that the building structures (or obstruction in the general flow) are sparse.

If the density of the buildings is more, then it is convenient to express mixing length as

$$l = 2c^{\frac{1}{4}}m^2\delta \tag{8}$$

where $m$ depends on the type of obstruction.

Set of Eqns. (3) to (6) with either of Eq. (7) or (8) form a closed set.

### 3. Boundary conditions

As this is a boundary value problem we have to derive the proper boundary conditions. In the present article the following boundary conditions are considered

$$\tau\big|_{\xi\to 0} = \tau_0 \tag{9}$$

$$u\big|_{\xi\to\infty} = 0 \tag{10}$$

$$k\big|_{\xi\to\infty} = 0 \tag{11}$$

Boundary condition at Eq. (9) envisages that the momentum stress entering this layer from the free atmosphere is a constant. Boundary conditions at Eqns. (10) and (11) utilise the fact that $u$ and $k$ are zero at the ground (rigid surface).

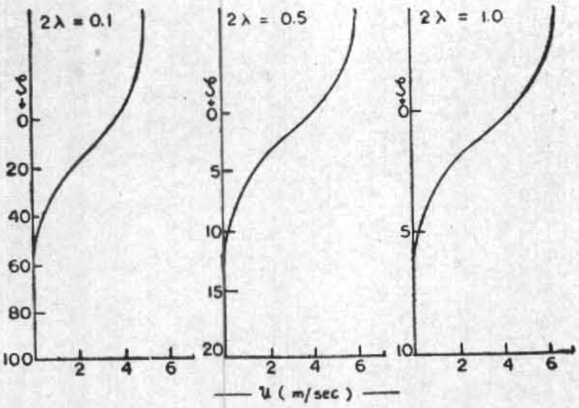


Fig. 1 Fig. 2 Fig. 3

Variation of vertical velocity profile with thickness of obstructions

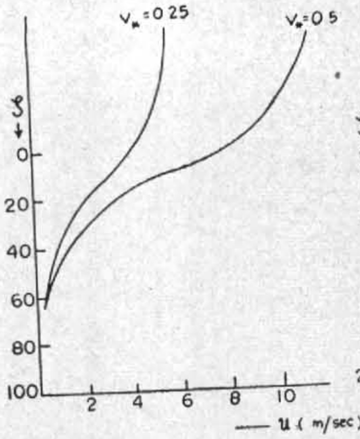


Fig. 4

Relationship between vertical velocity profile and dynamic velocity ($V^*$)

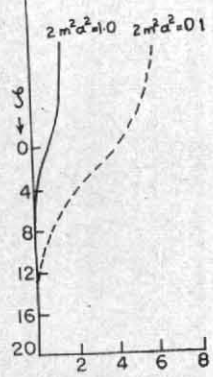


Fig. 5

Dependence of vertical velocity profile on type of obstructions

## 4. Solution of the problem

The set of equations with the above boundary conditions can be reduced to a single nonlinear differential equation.

Further from Eq. (3) we can assume that

$$\tau = \tau_0 \; e^{-a^2/\delta\zeta} \tag{12}$$

Let us denote $a^2/\delta$ by $2\lambda$ which probably is a function of the thickness of obstruction.

The solution to the set of Eqns. (3) to (6) with Eq. (8) and with the boundary conditions at Eq. (9) to (11) is

$$\tau = \tau_0 \, e^{-2\lambda\zeta} \tag{13}$$

$$u = \frac{\sqrt{\tau_0/\rho} \;\; e^{-\lambda\zeta}}{m^2 \, a^2 . 3^{1/4}} \tag{14}$$

$$\sqrt{b} = (\, 3/c^{\frac{1}{2}} \;)^{1/4} \sqrt{\;\tau_0/\,\rho\;} \; e^{-\lambda\zeta} \tag{15}$$

$$k = \frac{3^{\frac{1}{4}} \; m^2 \, a^2 \sqrt{\tau_0 / \rho}}{\lambda} . e^{-\lambda\zeta} \tag{16}$$

$$l = \;\; c^{1/4} \, \lambda^{-1} \tag{17}$$

As $l$ in the above equations is not dependent on the height, the solution with expression of $l$ as in Eq. (7) is similar to the above solution at Eq. (13) to (17) and can be obtained by substituting $X$ in place $m_2a^2$. *The profiles in either case however will* be similar. The parameter $\lambda$ in the above system can be calculated from wind observation by using the expression.

$$\lambda = \frac{l_n \; u_2/u_1}{\zeta_2 - \zeta_1} \tag{18}$$

This expression at Eq. (18) can be derived from Eq. (14).

## 5. Results and conclusions

In Figs. 1 to 5 some important results of the theoretical calculations are discussed.

Figs. 1 to 3 show the variation of wind profile with $\zeta$ for different values of $\lambda$ which represent the ability of the obstructions to create turbulence in the general flow across the city. It is observed that the velocity increases with height and decreases with the increase of $\lambda$. This can be explained by loss of momentum due to friction and due to the formation of vortices, when the flow is met with obstructions like buildings. Fig. 5 describes the dependence of wind velocity on the type of obstruction, which is characterised by parameter $2m^2 \, a^2$. The greater the value of $2m^2 \, a^2$ the lesser the wind velocity. Greater values of $2m^2 \, a^2$ corresponds to more rough obstructions.

Fig. 4 characterises the change of wind velocity with dynamic velocity $V_*$. It can also be considered that Fig. 4 depicts the change of wind velocity with momentum flux as momentum flux is directly proportional to dynamic velocity $V_*$. It is evident from the graph that the greater the momentum within the buildings, the greater the velocity in the later for a given thickness of obstruction.

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