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The unsaturated downdraught

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ABSTRACT. The basic element in the phenomenon of the "humidity dip" observed and analyzed by the U.S. Thunderstorm Project is an unsaturated downdraught. As observed, the phenomenon is rather paradoxical since it is at the epoch of the heaviest thunderstorm rainfall that the surface relative humidity at a station decreases from near saturation to values as low as 60 or 70 per cent. Physically, since the surface rainfall is determined by the rate of downward transport of water, the factors contributory to an intense rainfall will be (*i*) a strong downdraught, (*ii*) high liquid-water content in the downflowing air, (*iii*) low evaporation in the downdraught, and (*iv*) high terminal velocity of drops, the last factors needing large drops. The first of these factors leads to a high rate of adiabatic compression of the downcoming air and allows less time for the drops to evaporate into it. On the other hand, the large drops, only a small number of which can lead to a high liquid-water content, are comparatively inefficient in providing water vapour to the downflowing air, the net result being that the downdraught reaching ground is unsaturated.

The physical picture envisaged above necessitates a radical departure from the pseudo-adiabatic form of the thermodynamic equation. The difference between the resulting new equation and the classical pseudoadiabatic form is examined by assuming a steady state and computing the resulting distribution of temperature and humidity in the downdraught for different raindrop sizes. It is found that the larger the sizes of the drops into which the liquid water is dispersed and the stronger the downdraughts, the less is the humidity and higher the temperature lapse in the downdraught, the limit being set by dry-adiabatic conditions.

1. Introduction

One of the most important of contributions of the U. S. Thunderstorm Project is a thorough observational analysis of the downdraught which develops in the mature stage of the thunderstorm and, on reaching ground, gives rise to the surface phenomena named by Byers and Braham (1949) as the "pressure nose and dome," the "temperature break," and the "humidity dip". These authors have given plausible explanations of the formation of the downdraught as well as the accompanying surface phenomena. However, no quantitative formulation embodying all the observed phenomena has yet been achieved although Das (1963, 1964) has roughly modelled the formation and progress of the downdraught resulting from the drag of the raindrops. The numerical computations made by Das strongly pointed towards the mechanism of the "temperature break" by revealing a large negative buoyancy in the downdraught all of which was not explained away by the suspended water. However, his model based on the classical form of the thermodynamic equation, ruled out the humidity dip. It is this last phenomenon to which this paper is addressed.

As observed by the Thunderstorm Project the phenomenon of the humidity dip is rather paradoxical since it is at the epoch of the heaviest rainfall that the surface relative humidity decreases from near saturation to values as low as 60 or 70 per cent. In other words, the "downward flowing air becomes unsaturated as it descends, *even* in the presence of large concentrations of liquid water" indicated by the accompanying heavy rain (Byers and Braham, 1949). According to Byers and Braham two processes may account for the apparent anomaly of the unsaturated downdraught. The *first* suggested process is that the downdraught air is desiccated

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by the cold precipitation particles. If the precipitation particles, such as rain or hail, are sufficiently colder than the ambient air, the water vapour pressure near the surface of these particles will be lower than that of the surroundings, thus resulting in a water-vapour flux directed toward the particles. These will grow at the expense of the water vapour in the ambient air and thereby reduce its relative humidity. The *second* process suggested to explain the lack of saturation is that the downdraught is unable to remain saturated because the rate of evaporation of the water drops is too slow to provide for the increase in the saturation mixing ratio as the air descends to lower levels. In that case the air of the thunderstorm downdraught would be heated at a rate between that of the moist and the dry-adiabatic and would reach the ground in an unsaturated state.

It may be noted that both processes mentioned above envisage rain temperatures lower than air temperatures – the first as the *cause* of the humidity dip and the second as its *effect*, but the former sees a much lower rain temperature than the latter. The measurements of Byers *et al.* (1949) have shown that the rain-water temperature is generally lower than the air temperature, but in only a few instances is it low enough to initiate the first process. Thus while the first process cannot be ruled out, being in fact operative in some cases, the second process seems to provide a more general physical basis for the explanation of both the humidity dip and the rain and air-temperature anomaly.

The physical mechanism envisaged in the present study centres around a downdraught which, being fairly strong, causes a high rate of adiabatic compression of the downcoming air and allows only a short time to its liquid-water content to evaporate in it. On the other hand, a given liquid-water content composed of a small number of large drops evaporates much less than when the same consists of a large number of small drops. Consequently a strong downdraught carrying its liquid-water content in the form of large drops will tend to remain unsaturated not only when the liquid water itself is small, thereby causing a shower of low intensity, but also when a large liquid-water content transported by an intense downdraught results in an intense thunderstorm rainfall. The purpose of this paper is to demonstrate the validity of these conclusions by a numerical integration of the thermodynamic equation of Das (1969).

It may be pointed out that the study reported here was started by one of us as early as 1964 but was postponed. The work was resumed in 1966 and after it was completed in 1967 we found that Kamburova and Ludlam (1966) had already published the results of a similar investigation. These authors used the same physical picture as ours but did not formulate the thermodynamics of the problem as systematically as we have attempted. Further, in computing the evaporation from drops they were restricted by a formula valid for the spherical drops only and by that they identified the drop-surface temperature with the wetbulb temperature of the ambient air.

The early version of this work has already been reported briefly at the International Conference on Cloud Physics (1968) at Toronto, Canada (Das and Subba Rao 1968). The study presented here has improved numerical accuracy.

2. The Basic Equations

The basic equations of the study result from a new thermodynamical system in which the phase changes between the water substances are described by the microphysical processes rather than being specified *a priori* as is done in the classical moist-adiabatic approach. The important ingredients of this system have been discussed by Das (1963, 1969). It has been shown that the distribution of water in its different phases is an essential part of this system, and since the liquid water is distributed in the form of droplets and drops, which generally do not move with the cloudy air, equations of continuity for the water substances and for the concentrations of the hydrometeors are required for a complete quantitative description of the system.

The present study is directed toward the limited objective of understanding the unsaturated downdraught in relation to its strength and to the sizes of drops contained in it. Consequently, it is not necessary to write the governing equations for the general case of cumulus draughts. However, little additional work is involved if one starts from a three-dimensional frame and specializes to the one-dimensional draughts with one-dimensional motions of drops. This approach has the advantage of indicating how the special problem discussed in the present study is related to the more general aspects of the dynamics of the cumulus clouds.

(a) *The equation of continuity of the water substances* — Considering the cloud to be a mixture of dry air, water vapour, and liquid water (dispersed in the form of droplets and drops), we let ρ_d, ρ_v, ρ_l, respectively, be their masses per unit volume of the cloudy air. In addition, let $\mathbf{V}$ be the velocity of the gaseous phase and $\mathbf{V}_l$ be that of the liquid phase relative to the gaseous phase. Then, applying the principle of conservation of mass to the cloudy air, we can write,

$$\frac{\partial}{\partial t}(\rho_d+\rho_v+\rho_l)+\nabla.[(\rho_d+\rho_v)\mathbf{V}+\rho_l(\mathbf{V}+\mathbf{V}_l)]=0 \quad (1)$$

In the above, ρ_l is implicitly considered as composed of drops of a single size which, of course, is a function of space and time. However, since ρ_l is dispersed in the cloud in the form of drops of different sizes, one really should write a more general form of (1) as,

$$\frac{\partial}{\partial t}(\rho_d+\rho_v+\rho_l)+\nabla.[(\rho_d+\rho_v+\rho_l)\mathbf{V}+ + \int_0^\infty m_r\, n_r\, \mathbf{V}_r\, dr]=0 \quad (1a)$$

where, m_r and $n_r\,dr$ are, respectively, the mass and the number concentrations of drops of radii lying between r and $r+dr$ and $\mathbf{V}_r$ is the velocity of the drops of radius r relative to the gaseous phase. However, in the present study we shall be concerned only with (1).

Since the mass of the dry air is conserved independently of the water substances, equation (1) can be split into two parts. Thus for the dry-air component we can write,

$$\frac{\partial \rho_d}{\partial t}+\nabla.(\rho_d\mathbf{V})=0 \quad (2)$$

In writing (2) in the customary form,

$$\frac{d\rho_d}{dt}+\rho_d\nabla.\mathbf{V}=0, \quad (3)$$

we define the substantial time derivative,

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\mathbf{V}.\nabla, \quad (4)$$

which is the basic substantial derivative of the system and denotes changes observed by an observer moving with the gaseous phase.

On account of the possibility of phase changes between water vapour and liquid water, the two have always been treated together. The equation for the change of water substances is obtained by subtracting (2) from (1), so that we have,

$$\frac{\partial}{\partial t}(\rho_v+\rho_l)+\nabla.[(\rho_v+\rho_l)\mathbf{V}+\rho_l\mathbf{V}_l]=0$$

Together with (4) this becomes,

$$\frac{d}{dt}(\rho_v+\rho_l)+(\rho_v+\rho_l)\nabla.\mathbf{V}+\nabla.(\rho_l\mathbf{V}_l)=0 \quad (5)$$

Using (3), we can write (5) as,

$$\frac{d}{dt}\left(\xi_v+\xi_l\right)=-\frac{1}{\rho_d}\nabla.(\rho_l\mathbf{V}_l), \quad (6)$$

where, ξ_v $(\equiv \rho_v/\rho_d)$ and ξ_l $(\equiv \rho_l/\rho_d)$ are, respectively, the mixing ratios of water vapour and liquid water in the cloud.

(b) *The thermodynamic equation* — Following Das (1963) we write the thermodynamic equation as a conservation of entropy principle:

$$\frac{d}{dt}(\rho_d\phi_d+\rho_v\phi_v+\rho_l\phi_l)+(\rho_d\phi_d+\rho_v\phi_v+ +\rho_l\phi_l)\nabla.\mathbf{V}+\nabla.(\rho_l\phi_l\mathbf{V}_l)=0 \quad (7)$$

where, ϕ_d, ϕ_v, and ϕ_l are, respectively the entropy of a unit mass of dry air, water vapour and liquid water.

Referring the entropies to a suitable basic state and stipulating that we restrict our consideration to vapour-liquid transition alone, we shall write the following expressions for entropies:

$$\phi_d(T)=c_p\ln\theta_d, \quad (8)$$

$$\phi_v(T)=\phi_l(T)+L/T, \quad (9)$$

$$\phi_l(T)=c\ln T, \quad (10)$$

where, T is the temperature of the mixture on the Kelvin scale, θ_d is the potential temperature of the dry air, again on the Kelvin scale, c and c_p are, respectively, the specific heat of water and the specific heat of dry air at constant pressure, and L is the latent heat of vaporization of water. Using (3), (6) and (9), we can write (7) as—

$$\frac{d\phi_d}{dt}+\xi_v\frac{d\phi_v}{dt}+\xi_l\frac{d\phi_l}{dt}+\frac{L}{T}\frac{d\xi_v}{dt}+ +\xi_l\mathbf{V}_l.\nabla\phi_l=0 \quad (11)$$

This would be the same as the basic equation for the pseudoadiabatic process if we replaced $d\xi_v/dt$ by $d\xi_{v_s}/dt$, where ξ_{v_s} is the saturation mixing ratio corresponding to the temperature and pressure of the parcel. However, the mixing ratio is determined by the microphysics of condensation on, and evaporation from centres such as nuclei, water droplets, and drops (and other hydrometeors). Hence, (11) should give a better representation of the state of affairs in a cloud than the traditional pseudoadiabatic equation, provided that $d\xi_v/dt$ is properly related to the physics of phase change of the water substances. In our case the only centres considered will be the drops.

In our search for an expression for $d\xi_v/dt$, we rewrite (6) as—

$$\frac{d\xi_v}{dt}=-\frac{d\xi_l}{dt}-\frac{1}{\rho_d}\nabla.(\rho_l\mathbf{V}_l) \quad (12)$$

and recall that this assumes the liquid water at any particular level to be composed of drops (or droplets) of the same mass. If the drops have a mass, m, at a concentration of n per unit volume, we can write,

$$\xi_l = nm/\rho_d,$$

or, if we want to treat the concentration of the drops as the number mixed in a unit mass of dry air,

$$\xi_l = Nm, \tag{13}$$

where, N $(= n/\rho_d)$ is the number of drops contained in a unit mass of dry air. Differentiating (13) we have,

$$\frac{d\xi_l}{dt} = m\,\frac{dN}{dt} + N\,\frac{dm}{dt}, \tag{14}$$

which is to be supplemented by expressions for dm/dt and dN/dt.

The derivative dm/dt does not represent all the mass change of a drop due to condensation or evaporation but only the part of the change that will be observed by an observer moving with the gaseous parcel. On the other hand, the actual change of mass due to condensation or evaporation, which is really the "substantial change" of the mass of the centre, is the change observed by an observer moving with the centre. We denote this substantial derivative of m as Dm/Dt, which is related to dm/dt through :

$$\frac{Dm}{Dt} = \frac{dm}{dt} + \mathbf{V}_l \,.\, \nabla m. \tag{15}$$

It is to be noted that the physical formulae for mass changes due to evaporation and condensation apply directly to Dm/Dt, so that in actual applications of (15) one should determine dm/dt as $Dm/Dt - \mathbf{V}_l.\nabla m$.

To find an expression for dN/dt, we use a continuity equation for n. A simple form of the continuity equation results if we assume that there is no coalescence or splitting of the drops. This assumption obviously is unrealistic but is consistent with our earlier assumption of m being a function only of height and time. Under these assumptions the continuity equation can be written,

$$\frac{\partial n}{\partial t} + \nabla \,.\, [n(\mathbf{V} + \mathbf{V}_l)] = 0 \tag{16}$$

Substituting $n = \rho_d N$ in (16), we have,

$$N\left[\frac{\partial \rho_d}{\partial t} + \nabla \,.\, (\rho_d \mathbf{V})\right] + \rho_d\left(\frac{\partial N}{\partial t} + \mathbf{V}.\nabla N\right) = -\nabla \,.\, (n\mathbf{V}_l),$$

which with (2) and (4) becomes,

$$\frac{dN}{dt} = -\frac{1}{\rho_d}\nabla \,.\, (n\mathbf{V}_l). \tag{17}$$

If in (14) we substitute for dm/dt from (15) and for dN/dt from (17), we get,

$$\frac{d\xi_l}{dt} = -\frac{m}{\rho_d}\nabla \,.\, (n\mathbf{V}) + N\left(\frac{Dm}{Dt} - \mathbf{V}_l.\nabla m\right)$$

$$= -\frac{1}{\rho_d}\nabla \cdot (nm\mathbf{V}_l) + N\frac{Dm}{Dt},$$

or, since $\rho_l = nm$,

$$\frac{d\xi_l}{dt} = -\frac{1}{\rho_d}\nabla \cdot (\rho_l\mathbf{V}_l) + N\frac{Dm}{Dt} \tag{18}$$

If we go back to (12) with (18), we obtain,

$$\frac{d\xi_v}{dt} = -N\frac{Dm}{Dt} \tag{19}$$

Since, as already implied, Dm/Dt represents the mass change of a drop due to condensation alone, (19) becomes the central equation in our thermodynamic system.

As is apparent, the most unpleasant feature of (19) is its mixing of two substantial derivatives. But on a closer study one will be convinced that this is unavoidable in a cloud where the products of condensation, in general, move with velocities different from that of the air. Moreover, since Dm/Dt relates directly to microphysical changes in the mass of a drop, its presence in the above equation offers considerable convenience in its practical manipulation when precipitation hydrometeors take part in the condensation-evaporation process. Indeed the data on the evaporation of drops given by Kinzer and Gunn (1951) can be applied directly only to this form of the moisture equation.

(c) *The case of a steady one-dimensional draught*—The thermodynamic system described by (11), (15), (17) and (19) eliminates the necessity that the air inside an adiabatic cloud should always be treated as just saturated. However, the system now has become more elaborate, in that it involves a description not only of temperature and humidity but also of the size and concentration of the hydrometeors. If the above equations were solved with a set of initial and boundary conditions, they would give a description of the fields of ϕ_d, ξ_v, m, and N in both space and time. However the idealizations adopted in the development of these equations do not warrant such as general study. All we intend to do is to study the physical content of these equations in so far as they apply to a steady one-dimensional downdraught.

For a steady one-dimensional convective draught of vertical velocity, w (which is not necessarily a constant), one can write $d/dt=wd/dz$. Substituting the expression for ϕ_d from (8) into (11), using the familiar expression for potential temperature and assuming hydrostatic balance, we have,

$$-\frac{dT}{dz}=\frac{g}{c_p}+\frac{L}{c_p}\frac{d\xi_v}{dz}, \qquad (20)$$

where, g is the acceleration due to gravity.

In writing a steady-state form of (15), we consider only the change in m that is due to condensation (or evaporation) so that we can treat Dm/Dt as a function of temperature, pressure, ξ_v, and m, which, in turn, are functions of z. In other words we can write—

$$\frac{Dm}{Dt}=f(T,p,m,\xi_v), \qquad (21)$$

where,

$$T\equiv T(z),\ p\equiv p(z),\ m\equiv m(z), \text{ and } \xi_v\equiv\xi_v(z).$$

In addition we shall approximate $\mathbf{V}_l$ by $-\hat{k}V_T$, where V_T is the terminal velocity of a freely falling drop and $\hat{k}$ is a unit vector in the positive direction of the **z** axis. Thus (15) can be written as—

$$\frac{dm}{dz}=\frac{f}{w-V_T} \qquad (22)$$

The steady-state one-dimensional forms of (17) and (19) can easily be seen to be—

$$\frac{dN}{dz}=\frac{1}{w\rho_d}\frac{d}{dz}(nV_T), \qquad (23)$$

$$\text{and}\quad \frac{d\xi_v}{dz}=-\frac{N}{w}f, \qquad (24)$$

respectively. In our study we shall solve (20), and (22) through (24) with values of f obtained from the experimental data of Kinzer and Gunn (1951) and those of ρ_d (rather ρ_a, the air density) and p determined from the hydrostatic equation.

3. The Numerical Procedure

In the numerical integration of (22) through (24) we assume w to be constant, $w=-w_D$, where, w_D is the (constant) strength of the downdraught. This assumption obviously is not realistic (nor are many other assumptions made earlier). However, as already indicated, the aim of this work is to understand the physical content of the new thermodynamic system in the background of an idealized downdraught, rather than simulate a natural downdraught which is certainly more complicated. The assumption of a constant downdraught keeps the physical framework at its utmost simplicity and allows a transparent view of the physics of our problem.

The finite-difference forms adopted for the work are:

$$T_{j+1}=T_j-(g\triangle z/c_p)\,[\xi_v)_{j+1}-\xi_v)_j], \qquad (25)$$

$$\xi_v)_{j+1}=\xi_v)_j+(N_j\triangle z/2w_D)\,(f_j+f_{j+1}), \qquad (26)$$

$$N_{j+1}=N_j-\frac{[n_{j+1}V_T)_{j+1}-n_jV_T)_j]}{(w_D\rho_a)_j} \qquad (27)$$

$$m_{j+1}=m_j-\frac{(\triangle z/2)\,(f_j+f_{j+1})}{[w_D+V_T)_j]} \qquad (28)$$

$$n_j=\rho_a)_jN_j,\ \rho_l)_j=n_jm_j \qquad (29)$$

where, the subscripts j and $j+1$ refer to the levels to which the subscripted quantities belong. These equations are supplemented by a finite-difference form of the standard hydrostatic equation.

In computing with equations (25) through (29), we specify a temperature, a pressure, a liquid-water content, and a drop-size at the cloud base where the air is assumed to be saturated. This information is used to determine the air density, drop concentration, mixing ratio of water vapour, and the terminal velocity of the drops. Once the quantities at the cloud base are known, the quantities at the next lower level are determined by a fairly straightforward routine which includes an iterative procedure very similar to one required to produce a moist adiabat. The procedure runs as follows—

At first the temperature and mixing ratio are extrapolated dry-adiabatically. These quantities give the virtual temperature for determining the pressure and the relative humidity, which together with the drop-size information give f at the new level. These are now used to determine a new value of ξ_v from (26) and of T from (25). Obviously these new values would be different from the old ones, so the outlined procedure is repeated with the new values. The process is continued until the latest value of the temperature differs from that obtained in the immediately preceding iteration by less than a certain limit of accuray (0°·01K in the present study). The corresponding last value of ξ_v is taken as the mixing ratio at the new level. This iterative procedure also includes a continuous updating of the value of the pressure as more accurate

TABLE 1

Vertical distributions of thermodynamic and hydrometeoric quantities in constant downdraughts below cloud base

Cloud baes liquid-water contents, 1 gm m^{-3}

Z (km)	ρ_l (gm m^{-3})	n (m^{-3})	r (mm)	I (cm hr^{-1})	T (°K)	H (%)	ρ_l (gm m^{-3})	n (m^{-3})	r (mm)	I (cm hr^{-1})	T (°K)	H (%)
(a) Strength of downdraught = 5 m sec^{-1})												
	Drop Radius of Cloud Base = 0·2 mm						Drop Radius at Cloud Base = 0·5 mm					
1·5†	1·000	29,840	0·200	2·383	290·0	100·0	1·000	1,910	0·500	3·251	290·0	100·0
1·0	0·783	30,830	0·182	1·867	294·1	83·7	0·970	1,954	0·491	3·152	294·7	79·7
0·5	0·328	31,960	0·135	0·729	297·7	74·2	0·847	2,001	0·466	2·645	299·0	66·2
0·0	0·000*	*	*	*	*	*	0·650	2,051	0·423	1·934	303·1	56·4
	Drop Radius at Cloud Base = 1·0 mm						Drop Radius at Cloud Base = 1·5 mm					
1·5†	1·000	238·7	1·000	4·136	290·0	100·0	1·000	70·73	1·500	4·702	290·0	100·0
1·0	1.002	243·0	0·995	4·142	294·8	78·7	1·006	71·85	1·495	4·732	294·8	78·5
0·5	0·973	247·4	0·979	4·023	299·5	63·4	0·995	72·99	1·482	4·676	299·6	62·8
0·0	0·914	251·9	0·953	3·716	304·0	52·1	0·965	74·13	1·460	4·497	304·2	51·1
(b) Strength of downdraught = 10 m sec^{-1}												
	Drop Radius at Cloud Base = 0·2 mm						Drop Radius at Cloud Base = 0·5 mm					
1·5†	1·000	29,840	0·200	4·183	290·0	100·0	1·000	1·910	0·500	5·051	290·0	100·0
1·0	0·878	30,950	0·189	3·673	294·5	81·2	0·994	1·967	0·494	5·020	294·8	78·9
0·5	0·552	32,150	0·160	2·220	298·5	69·4	0·919	2·026	0·477	4·642	299·4	64·1
0·0	0·142	33,430	0·100	0·547	302·3	60·1	0·791	2·088	0·449	3·893	303·8	53·1
	Drop Radius at Cloud Base = 1·0 mm						Drop Radius at Cloud Base = 1·5 mm					
1·5†	1·000	238·7	1·000	5·936	290·0	100·0	1·000	70·73	1·500	6·502	290·0	100·0
1·0	1·014	244·7	0·996	6·018	294·9	78·4	1·016	72·36	1·497	6·606	294·9	78·3
0·5	1·005	250·9	0·985	5·965	299·6	62·5	1·018	74·00	1·487	6·622	299·7	62·2
0·0	0·973	257·1	0·967	5·703	304·3	50·7	1·007	75·67	1·470	6·502	304·5	50·1
(c) Strength of downdraught = 15 m sec^{-1}												
	Drop Radius at Cloud Base = 0·2 mm						Drop Radius at Cloud Base = 0·5 mm					
1·5†	1·000	29,840	0·200	5·983	290·0	100·0	1·000	1·910	0·5	6·851	290·0	100·0
1·0	0·923	30,990	0·192	5·521	294·6	80·2	1·006	1·973	0·496	6·893	294·8	78·6
0·5	0·660	32,210	0·169	3·843	298·8	67·5	0·960	2·038	0·483	6·576	299·5	63·3
0·0	0·311	33,510	0·130	1·809	302·8	57·8	0·868	2·105	0·462	5·833	304·1	51·9
	Drop Radius at Cloud Base = 1·0 mm						Drop Radius at Cloud Base = 1·5 mm					
1·5†	1·000	238·7	1·000	7·736	290·0	100·0	1·000	70·73	1·500	8·302	290·0	100·0
1·0	1·020	245·6	0·997	7·892	294·9	78·3	1·022	72·64	1·497	8·480	294·9	78·2
0·5	1·023	252·7	0·989	7·912	299·7	62·2	1·032	74·58	1·489	8·571	299·7	62·0
0·0	1·006	259·9	0·974	7·708	304·8	50·2	1·031	76·55	1·476	8·563	304·5	49·8

*Drops evaporated before reaching ground †Cloud base

values of temperature and mixing ratio are obtained.

Once the pressure, temperature, mixing ratio, and f are determined in the iterative procedure described above (27), (28) and (29) are then used to obtain N, m and n for the new level.

When the quantities at the level next to the cloud base are determined, they form the initial values from which the quantities at the next lower level are computed. The process is continued until the ground level is reached.

The numerical values used in the work are:

Cloud-base height = 1500 m above ground.

Downdraught, speed,

$$w_D = 5, 10, \text{ and } 15 \text{m sec}^{-1},$$

Cloud-base liquid-water

content = 1, 3, and 5 gm m^{-3},

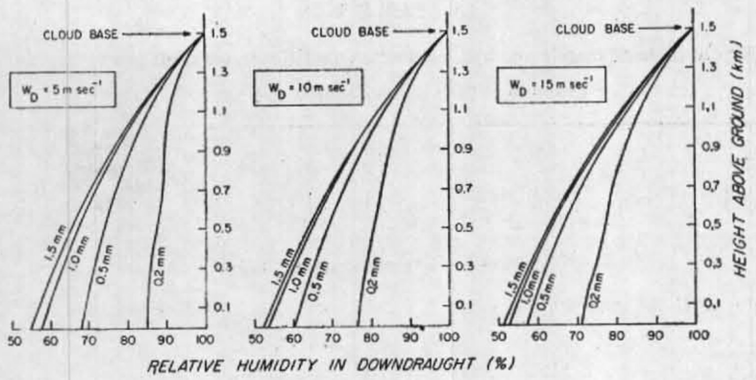

Fig. 1

Relative humidity in steady downdraughts of constant speeds, *WD*, starting at the cloud base with a liquid water content of 3 gm m^{-3} divided into drops of uniform size. Each curve is labelled in terms of radii of drops

Radius of the raindrops
at cloud base = $0 \cdot 2, 0 \cdot 5, 1 \cdot 0$, and $1 \cdot 5$ mm,

Temperature and pressure at
cloud base = 290°K, 850 mb,
$\triangle z = 20$ m.

Further, L, c_p, V_T, and f were taken from the *Smithsonian Meteorological Tables* with proper interpolation where necessary. The saturation vapour pressure over water was determined from the expression :

$$e_s = 6 \cdot 107 \ \exp \ [a(T - b)/(T - c)]$$

where, e_s is the saturation vapour pressure in millibars at the temperature of T° K, and $a = 17 \cdot 2693882$, $b = 273° \cdot 16$ K, and $c = 35° \cdot 86$ K. This formula has been given by Murray (1967).

4. Results and Discussion

The results of the study are summarized in Tables 1, 2, and 3. The values tabulated are those of ρ_l, the liquid-water content; n, the concentration of drops (per unit volume); r, the radius of the drops; T, the temperature, and H, the relative humidity, in the downdraught ; and I, the intensity of rainfall. The last quantity is computed from —

$$I = (w_D + V_T)\,\rho_l/\delta_l,$$

where δ_l is the density of liquid water (assumed 1 gm cm^{-3}). The tabulated values are shown at four levels, namely, at the ground, and at 500 1000, and 1500 m above ground, the last level being the cloud base.

Some of the results also are presented graphically in Figs. 1, 2, and 3. The figures provide more detailed information on the nature of the vertical distributions of humidity, temperature, and drop size as functions of the strengths of the (constant) downdraught. Fig. 1, showing the vertical distribution of relative humidity, has three sets of curves, each set relating to one value of the downdraught. In a set there are four curves, each labelled in terms of the assumed drop radius at the cloud base.

The solid curves in Fig. 2 showing the vertical distributions of temperature in the downdraught correspond, label for label, to those in Fig. 1. The dotted curves in the figure give temperature distributions in the downdraught as would be obtained if the parcels were lowered either wholly dry-adiabatically or wholly moist-adiabatically.

The curves in Fig. 3 show the extent to which the drops evaporate as they are carried down wards in the unsaturated downdraught. This information is useful in that it can lead to an idea of what percentage of the water available at the cloud base would reach ground as rainfall.

From a study of the tables and the figures one immediately can see that even in the presence of large quantities of water (which, under suitable circumstances, are capable of giving rain intensities as high as 400 mm hr^{-1}), the downdraught tends to remain unsaturated. The subsaturation is greater with smaller water content, larger drop sizes, and stronger downdraughts. The temperatures in the downdraught lie between those to be obtained in the dry-adiabatic process on the one hand and the moist-adiabatic process on the other. The circumstances that cause greater subsaturation tend to take the temperature distribution closer to the dry-adiabatic.

The other important physical fact obvious from this study is that a given liquid water divided into a larger number of smaller drops is

TABLE 2

Vertical distributions of thermodynamic and hydrometeoric quantities in constant downdraughts below cloud base
Cloud base liquid water content 3 gm m^{-3}

Z (km)	ρ_l (gm m^{-3})	n (m^{-3})	r (mm)	I (cm hr^{-1})	T (°K)	H (%)	ρ_l (gm m^{-3})	n (m^{-3})	r (mm)	I (cm hr^{-1})	T (°K)	H (%)
(a) Strength of Downdraught=5 sec^{-1}												
	Drop Radius at Cloud Base=0·2 mm						Drop Radius at Cloud Base=0·5 mm					
1·5†	3·000	89,520	0·200	7·150	290·0	100·0	3·000	5,729	0·500	9·752	290·0	100·0
1·0	2·583	92,690	0·188	6·156	293·3	90·2	2·928	5,867	0·492	9·517	294·3	82·6
0·5	1·848	96,220	0·166	4·105	295·8	87·1	2·649	6,016	0·472	8·268	297·8	73·6
0·0	0·945	100,000	0·131	2·099	298·2	85·2	2·244	6,175	0·443	7·005	300·9	67·8
	Drop Radius at Cloud Base=1·0 mm						Drop Radius at Cloud Base=1·5 mm					
1·5†	3·000	716·2	1·000	12·41	290·0	100·0	3·000	212·2	1·500	14·10	290·0	100.0
1·0	3·008	729·3	0·995	12·44	294·6	79·9	3·020	215·6	1·496	14·20	294·7	79·3
0·5	2·935	742·9	0·981	12·14	298·9	66·8	2·991	219·1	1·843	14·06	299·2	65·1
0·0	2·792	757·0	0·958	11·35	302·8	57·7	2·917	222·6	1·462	13·59	303·4	54·9
(b) Strenth of Downdraught=10 m sec^{-1}												
	Drop Radius at Cloud Base=0·2 mm						Drop Radius at Cloud Base=0·5 mm					
1·5†	3·000	89,520	0·200	12·55	290·0	100·0	3·000	5,729	0·500	15·15	290·0	100·0
1·0	2·728	93,000	0·191	11·41	293·9	85·8	2·989	5,903	0·495	15·10	294·6	80·5
0·5	2·044	96,860	0·171	8·218	296·7	80·7	2·806	6,088	0·479	14·17	298·6	68·5
0·0	1·256	101,000	0·144	5·049	299·6	76·0	2·500	6,283	0·456	12·30	302·3	60·1
	Drop Radius at Cloud Base=1·0 mm						Drop Radius at Cloud Base=1·5					
1·5†	3·000	716·2	1·000	17·81	290·0	100·0	3·000	212·1	1·500	19·50	290·7	100·0
1·0	3·042	734·3	0·996	18·06	294·8	79·0	3·049	217·1	1·497	19·82	294·8	78·7
0·5	3·022	753·1	0·986	17·94	299·3	64·3	3·059	222·1	1·487	19·89	299·5	63·4
0·0	2·939	772·3	0·968	17·24	303·6	53·8	3·030	227·2	1·471	19·57	304·0	52·2
(c) Strength of Downdraught=15 m sec^{-1}												
	Drop Radius at Cloud Base=0·2 mm						Drop Radius at Cloud Base=0·5 mm					
1·5†	3·000	89,520	0·200	17·95	290·0	100·0	3·000	5,729	0·500	20·55	290·0	100·0
1·0	2·820	93,100	0·193	16·87	294·1	83·8	3·023	5,920	0·496	20·71	294·7	79·7
0·5	2·222	97,050	0·176	13·30	297·3	76·9	2·899	6,121	0·484	19·91	298·9	66·4
0·0	1·552	101,200	0·154	9·032	300·4	70·7	2·674	6,332	0·465	17·97	302·9	57·0
	Drop Radius at Cloud Base=1·0 mm						Drop Radius at Cloud Base=1·5 mm					
1·5†	3·000	716·2	1·000	23·21	290·0	100·0	3·000	21·22	1·500	24·90	290·0	100·0
1·0	3·061	737·0	0·997	23·68	294·8	78·7	3·065	21·79	1·497	25·44	294·8	78·5
0·5	3·073	758·5	0·989	23·77	299·5	63·5	3·100	22·38	1·489	25·73	299·6	62·8
0·0	3·030	780·5	0·975	23·44	304·0	52·3	3·101	22·98	1·477	25·74	304·2	51·2

†Cloud base

more efficient in supplying moisture to the downdraught than the same liquid water divided into (a smaller number of) larger drops. Now, in reality, the liquid water in the downdraught is divided into a population of drops of varying sizes. It is easily conceivable, therefore, that the smaller drops and droplets evaporating into the downdraught which has just left the cloud base will tend to keep it well near saturation so that appreciable subsaturation will develop only after the downdraught has descended considerably below the cloud base.

Although the effect of a downdraught varying with height is outside the scope of this study one can draw qualitative inferences from the results presented. Let us assume, for example, a downdraught that increases in strength as it approaches the ground. In this case the rate of increase of subsaturation as the downdraught just descends from the cloud base would be rather small but as the downdraught strengthens this rate will increase rapidly. Again, near the ground, the rate of increase of subsaturation will slow down

TABLE 3

Vertical distributions of thermodynamic and hydrometeoric quantities in constant downdraughts below cloud base

Cloud base liquid-water content, 5 gm m^{-3}.

Z (km)	ρ_l (gm m^{-3})	n (m^{-3})	r (mm)	I (cm hr^{-1})	T (°K)	H (%)	ρ_l (gm m^{-3})	n (m^{-3})	r (mm)	I (cm hr^{-1})	T (°K)	H (%)
(a) Strength of Downdraught=5 m sec^{-1}												
	Drop Radius at Cloud Base=0·2 mm						Drop Radius at Cloud Base=0·5 mm					
1·5†	5·000	149,200	0·200	11·92	290·0	100·0	5·000	9,549	0·500	16·25	290·0	100·0
1·0	4·534	154,600	0·191	10·81	292·9	93·4	4·905	9,784	0·493	15·95	294·0	84·8
0·5	3·760	160,500	0·178	8·960	295·0	93·3	4·548	10,040	0·476	14·78	296·9	79·2
0·0	2·948	166,800	0·162	6·549	297·4	91·0	4·076	10,310	0·455	12·72	299·7	75·5
	Drop Radius at Cloud Base=1·0 mm						Drop Radius at Cloud Base=1·5 mm					
1·5†	5·000	1,194	1·000	20·68	290·0	100·0	5·000	353·7	1·500	23·51	290·0	100·0
1·0	5·018	1,216	0·995	20·76	294·5	81·0	5·036	359·4	1·496	23·68	294·8	80·0
0·5	4·917	1,239	0·982	20·34	298·4	69·8	4·996	365·3	1·484	23·49	298·8	67·2
0·0	4·723	1,263	0·963	19·19	301·9	62·6	4·892	371·4	1·465	22·79	302·7	58·5
(b) Strength of Downdraught=10 m sec^{-1}												
	Drop Radius at Cloud Base=0·2 mm						Drop Radius at Cloud Base=0·5 mm					
1·5†	5·000	149,200	0·200	20·92	290·0	100·0	5·000	9,549	0·500	25·25	290·0	100·0
1·0	4·658	155,200	0·193	19·49	293·5	89·0	4·994	9,843	0·495	25·22	294·4	81·9
0·5	3·860	161,800	0·179	16·15	295·8	87·4	4·744	10,160	0·481	23·96	298·0	72·3
0·0	3·035	168,800	0·163	12·20	298·4	84·1	4·338	10,500	0·462	21·35	301·2	66·1
	Drop Radius at Cloud Base=1·0 mm						Drop Radius at Cloud Base=1·5 mm					
1·5†	5·000	1,194	1·000	29·68	290·0	100·0	5·000	353·7	1·500	32·51	290·0	100·0
1·0	5·073	1,224	0·9964	30·11	294·7	79·6	5·082	361·9	1·497	33·04	294·8	79·1
0·5	5·048	1,256	0·9864	29·97	299·0	66·0	5·104	370·3	1·487	33·18	299·3	64·6
0·0	4·931	1,289	0·9702	28·92	303·0	56·6	5·065	378·9	1·472	32·71	303·6	54·2
(c) Strength of Downdraught=15 m sec^{-1}												
	Drop Radius at Cloud Base=0·2 mm						Drop Radius at Cloud Base=0·5 mm					
1·5†	5·000	149,200	0·200	29·92	290·0	100·0	5·000	9,549	0·500	34·25	290·0	100·0
1·0	4·765	155,300	0·194	28·51	293·8	86·4	5·045	9,871	0·496	34·56	294·5	80·7
0·5	4·053	162,100	0·181	24·25	296·4	82·8	4·884	10,210	0·485	33·46	298·5	69·2
0·0	3·250	169,300	0·166	18·92	299·1	79·2	4·560	10,580	0·469	30·65	302·0	61·6
	Drop Radius at Cloud Base=1·0 mm						Drop Radius at Cloud Base=1·5 mm					
1·5†	5·000	1,194	1·000	38·68	290·0	100·0	5·000	353·7	1·500	41·51	290·0	100·0
1·0	5·103	1,229	0·997	39·48	294·8	79·1	5·109	363·3	1·497	42·41	294·8	78·8
0·5	5·129	1,265	0·989	39·68	299·6	64·6	5·169	373·1	1·490	42·91	299·4	63·6
0·0	5·071	1,302	0·976	39·23	303·5	53·4	5·177	383·2	1·477	42·98	303·9	52·6

†Cloud base

on account of the slowing down of the downdraught.

The consequence of what has been said in the last two paragraphs is the following :

Given the strength and horizontal distribution of the surface gust resulting from a downdraught, one can, with suitable assumptions, make inferences on the strength of the downdraught somewhat above the ground. One also can make a measurement of the drop-size distribution in the shower that reaches ground. If, using these data, one infers the subsaturation to be observed at the ground, one is likely to come out with a value considerably higher than that which is observed. In other words direct application of the results of the present study will give a much lower humidity than that which is likely to be observed in reality. It is no wonder, therefore, that relative humidities observed in intense thunderstorm

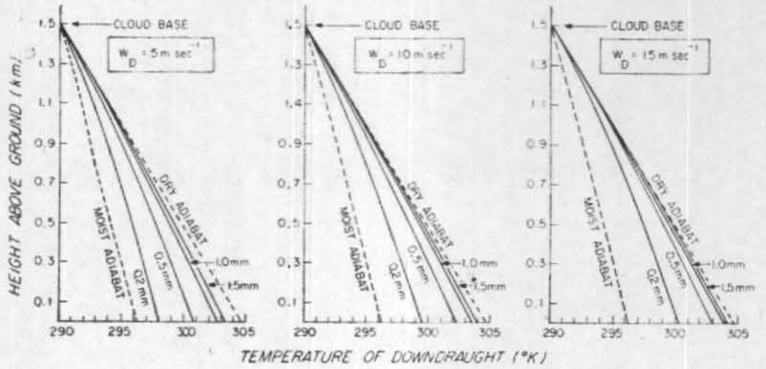

Fig. 2

Temperature of steady downdraughts under conditions of Fig. 1. The dotted curves (as marked) **show the** relevant dry adiabatic and moist adiabatic distributions of temperature

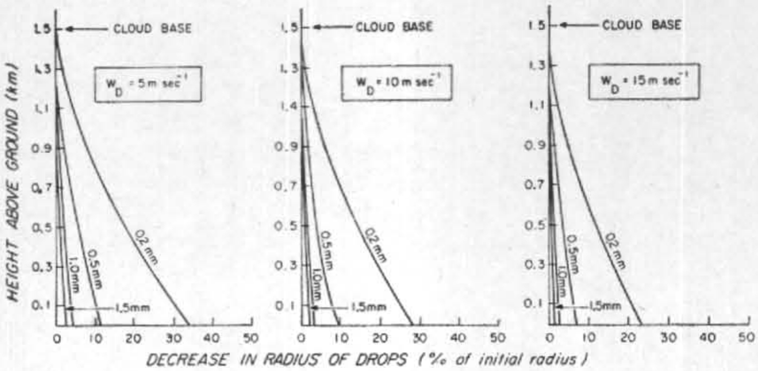

Fig. 3

The decrease in the radii of the drops as they descend from the cloud base to the ground under conditions of Fig. 1

downdraughts (with intense precipitation) are of the order of 70 per cent rather than 50 to 60 per cent as arrived at in the present computations.

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