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Mathematical distribution of rainfall in arid and semi-arid zones of Rajasthan

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ABSTRACT. The mathematical distribution of accumulated rainfall for 2 pentads, 4 pentads......20 pentads commencing from the onset of monsoon has been studied in respect of a typical arid zone station, *viz.* Jodhpur and semi-arid stations, namely, Jaipur in Rajasthan. In case of Jaipur, distribution beyond a month is normal while for Jodhpur, all accumulated pentads including the entire season is not normal at all. Probability charts using these theoretical distributions have been prepared and their utility discussed.

1. Introduction

Rajasthan reveals a variety of climatic zones ranging from extremely arid zone in Jaisalmer district to humid zone in Sirohi district. Delineation of various climatic zones in Rajasthan and their variation have been discussed by Krishnan (1968). Major portion of the area in the State, however, falls under arid and semi-arid zones, the respective coverage to the total area of the State being 58 and 36 per cent respectively. In these hot and dry regions, the rainfall is by far the most important factor limiting crop production. An interesting feature in the seasonal distribution of rainfall in these regions is that 85 to 95 per cent of the annual rainfall occurs during southwest monsoon season which commences in the semi-arid zone by the end of June and in the arid zone by the first week of July. The withdrawal of the monsoon occurs by the second or third week of September. Thus, the Kharif season (July to October) is the main cropping season with *Bajra*, *Til* and pulses like *Moong*, *moth* and *guar* etc as predominant crops in arid zone and *Jawar* in the semi-arid zone. Areas with irrigation are very limited. In view of these factors, a knowledge of the distribution of rainfall during the crop season and evaluation of drought hazards at various periods by computing probabilities of different amounts of rainfall during the growing season are extremely helpful for agricultural operations. The object of this paper is, therefore, to study in detail the distribution of the rainfall in different periods from the commencement of rainy season in arid and semi-arid zones, *viz.*, Jodhpur and Jaipur respectively. Attempt is also made to construct a probability chart for rainfall.

2. Review of literature

Pramanik and Jagannathan (1953) studied the rainfall distribution of a number of stations in India and showed that the skewness is small except in the arid region of Rajasthan and the semi-arid regions of the Deccan.

On the basis of assumption of normality in distribution, Sreenivasaiah and Yegnanarayanan (1959) produced maps showing probability of occurrence of different amounts of rainfall during the year as well as during the monsoon season. The percentage frequencies of daily rainfall exceeding specified intensities in Rajasthan have been shown to be successive terms of logarithmic series by Jagannathan and Raghavendra (1964). Mallik and Godbole (1966) presented merely weekly frequency distribution of rainfall in west Rajasthan during the monsoon without any attempt to study their theoretical distribution.

3. Data utilised

The daily rainfall data for sixty years (1901—1960) in respect of Jodhpur and Jaipur were utilised in this study. The rainfall totals for each pentad (five-day period) in the rainy season (June to September) were worked out. The serial numbering of pentads commences from 1 January and thus 31st pentad corresponds to 31 May to 4 June, 54th pentad corresponds to 23 to 27 September and so on.

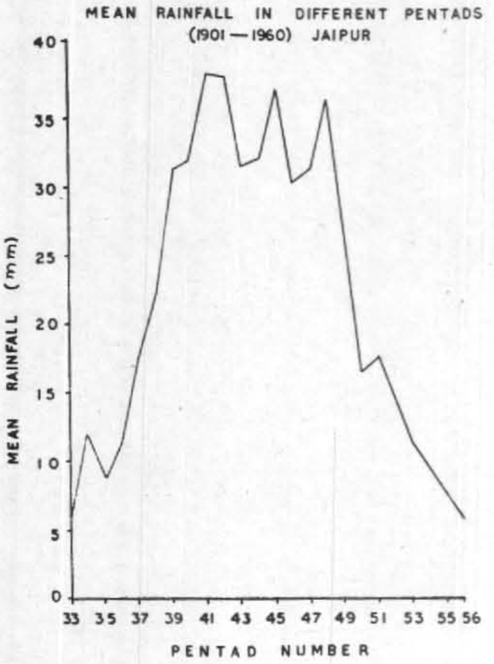

Fig. 1. Mean rainfall in different pentads (1901-1960) in Jaipur

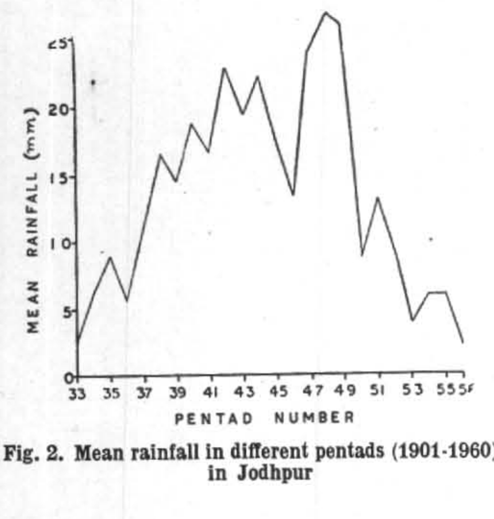

Fig. 2. Mean rainfall in different pentads (1901-1960) in Jodhpur

4. Method used

The normal rainfall as well as its standard deviation for each of the pentads from 33rd pentad to 56th pentad (10 June to 7 October) were computed on the basis of the data for the period 1901 to 1960. Normal accumulated rainfall in 2 pentads, 4 pentads to 20 pentads commencing from the 37th pentad were worked out for both the stations and histograms of their actual frequencies for the period 1901-1960 were drawn.

Attempt was first made to test the normality of these frequency distribution in order to find out how far normal curve can represent the actual observed distribution. The moments and cumulants upto fourth order were calculated from the observed frequency distribution after applying Sheppard's corrections for grouping. The values of g statistics and their standard errors were calculated for different accumulated rainfall series of Jodhpur and Jaipur by the Fisher's (1948) method.

An examination of the frequency histograms of those cases which do not follow normal distribution indicated that they may follow Gamma distribution. Since negative rainfall is not possible, the distribution has a zero lower bound and is unlimited on the right. The equation of the distribution is as follows —

$$f(x) = \frac{1}{\beta^{\gamma}\Gamma(\gamma)} x^{\gamma-1} e^{-x/\beta} \qquad (1)$$

$$\beta > 0 \text{ and } \gamma > 0$$

where, the mode of distribution is at $\beta(\gamma-1)$ if $\gamma > 1$ and at zero when $0 < \gamma \leqslant 1$. In the latter case, the distribution is J shaped. The amount of skewness of the distribution depends on the shape factor γ.

The procedure suggested by Thom (1958) has been used in the present study for fitting Gamma distribution to the observed rainfall data. This procedure has recently been elaborated in *Tech. Note No. 81* of World Meteorological Organization. For those cases which come under normal distribution, the usual method of fitting normal curve by finding out the probability density from standard tables corresponding to various rainfall amounts was used.

For obtaining the probability of rainfall exceeding certain values, the area under the probability curve for the concerned interval along the x axis has to be computed. For normal curves, ready made tables are available giving the area in various ranges of x. For Gamma distribution, however, the integral over a partial range of x is designated as incomplete Gamma function. The probability that an individual rainfall amount falls between 0 and any given value of x is given by the integral,

$$\int_0^x \frac{1}{\beta^{\gamma}\Gamma(\gamma)} x^{\gamma-1} e^{-x/\beta}\, dx \qquad (2)$$

By putting $x/\beta = V$ this integral becomes,

$$\frac{1}{\Gamma(\gamma)} \int_0^V V^{\gamma-1} e^{-V}\, dV \qquad (3)$$

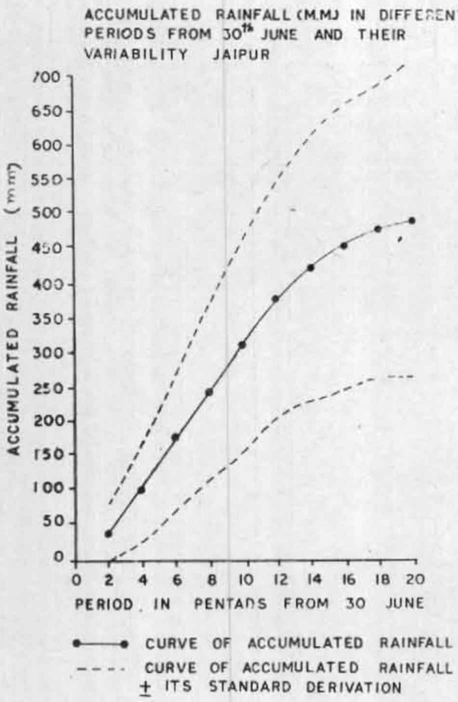

Fig. 3. Jaipur

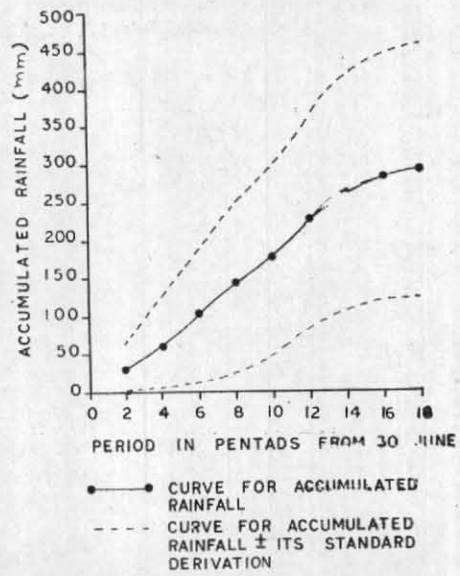

Fig. 4. Jodhpur

Accumulated rainfall (mm) in different periods from 30 June and their variability in Jodhpur

The values of this integral were computed using tables given by Abramovitz and Stegun (1965). Probability charts using normal and Gamma probability tables were prepared for Jodhpur and Jaipur.

5. Results and Discussion

Figs. 1 and 2 show the normal pentad rainfall of Jaipur and Jodhpur respectively for the period from 33rd pentad to 56th pentad (10 June to 7 October). While normal rainfall of Jaipur in each of these pentads exceeds 5 mm, the period during which each pentad receives a rainfall of 5 mm or more at Jodhpur extends from 15 June to 2 October with the exception of the period from 18 to 22 September. Mean pentad rainfall values in Jaipur show a steady increase in mid season and exceed 28 mm during the period 39th to 49th pentad (10 July to 2 September) whereas values at Jodhpur for the same period range only 13 to 27 mm. The highest pentad rainfall of 39 mm at Jaipur is during 20 to 29 July while Jodhpur has the highest value of 27 mm during 24 August to 2 September.

The 37th pentad (30 June to 4 July) is marked clearly by a sharp increase in the normal pentad rainfall and actually denotes the commencement of the rainy season. Accordingly, mean accumulated rainfall ($\bar{x}$) for 2 pentads, 4 pentads......20 pentads commencing from 37th pentad are presented in Figs. 3 and 4. Since the rainy season ends in Jodhpur earlier, accumulated rainfall upto 18 pentads only was worked out for that station. The standard deviation (σ) values were also worked out for rainfall for 2 pentads, 4 pentads etc and the values of $\bar{x}\pm\sigma$ are also presented in Figs. 3 and 4. Sharp increase in the accumulated rainfall is noticed in respect of Jaipur. However the rate of increase becomes smaller after 12 pentads. In case of Jodhpur the rate becomes smaller after 14 pentads. The coefficients of variation ($\sigma/\bar{x}$) expressed in percentage for various periods commencing from 30 June are given Table 1.

Jodhpur evidently shows greater variability in rainfall. Both the stations indicate little variation in the values beyond 12 pentads thereby showing that the subsequent rainfall is small enough to affect the values of coefficient of variation for longer periods.

The frequency distribution of accumulated rainfall in respect of both the stations were tested for normality. In case of Jodhpur g values were significant in all cases indicating that the distribution is not normal for all pentad values including those beyond 14 pentads when the histograms tend to become comparatively less skew. In Jaipur also, distribution is not normal in case of accumulated pentads upto 4. But g values are not significant for accumulated pentads beyond the 6 pentads which follow normal distribution as indicated by Table 2.

TABLE 1

Coefficient of variation of the accumulated pentad rainfall

Station	Length of period in pentads from 30 June									
	2	4	6	8	10	12	14	16	18	20
Jaipur	98	70	55	51	48	44	46	45	44	46
Jodhpur	131	113	85	78	71	61	61	58	57	—

TABLE 2

'g' statistics and their significance for Jaipur

	Period in pentads from 30 June							
	6	8	10	12	14	16	18	20
g_1	0·627	0·531	0·195	0·021	0·149	0·367	0·322	0·416
g_2	+0·050	—0·167	—0·734	—0·792	—0·608	—0·313	—0·087	—0·028
t for g_1	2·10	1·78	0·65	0·07	0·50	1·23	1·08	1·39
t for g_2	0·08	—0·27	—1·21	—1·30	—1·00	—0·51	—0·14	—0·09
	t value for significance				2·00 (5 per cent)			2·66 (1 per cent)

TABLE 3

Parameters of Gamma distribution

Period in pentads

Parameter	Jodhpur									Jaipur	
	2	4	6	8	10	12	14	16	18	2	4
γ	0·803	0·950	1·336	1·527	1·726	2·160	2·430	2·633	2·661	1·099	1·428
$1/\beta$	0·533	0·395	0·330	0·266	0·247	0·245	0·235	0·237	0·232	0·690	0·346

TABLE 4

χ^2 values for testing the goodness of fit of distributions fitted by Gamma functions

	Period in pentads								
	2	4	6	8	10	12	14	16	18
JODHPUR									
Actual χ^2	2·332	0·288	1·138	6·334	2·578	8·251	2·625	3·281	1·939
χ^2 value for significance 5 per cent	3·841	3·841	5·991	7·815	9·488	9·488	12·592	11·070	12·592
JAIPUR									
Actual χ^2	4·710	3·523							
χ^2 value for significance 5 per cent	5·991	7·815							

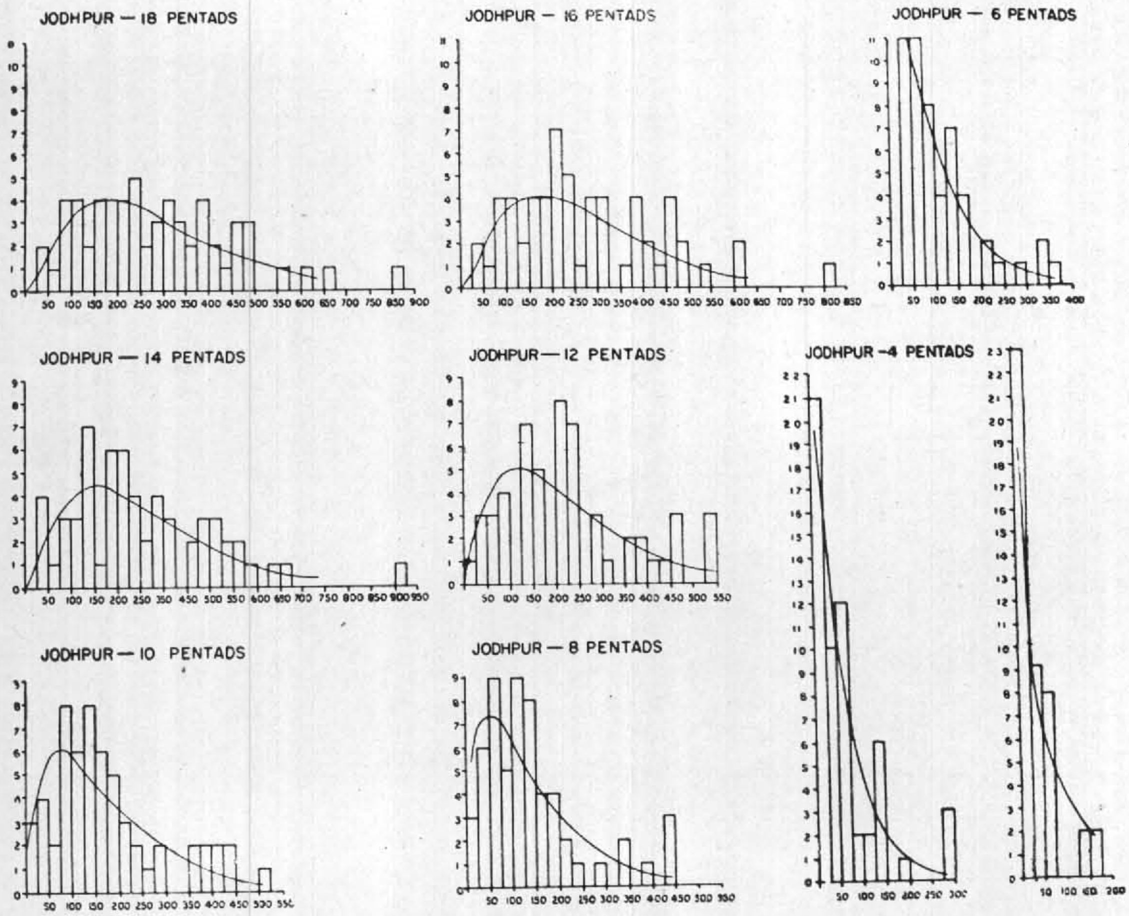

Fig. 5

Histograms of actual frequency and theoretical frequency curves of cumulative pentad rainfall from 30 June during 1901-1960

Thus the distribution of rainfall in various periods beyond a month from the commencement is normal in respect of this typical semi-arid zone station, *viz.*, Jaipur while the same in various periods in respect of the typical arid zone station, *viz.*, Jodhpur including the entire season is not normal at all. It is interesting to note that there seems a basic difference between these two climatic zones not only in the actual amounts of rainfall but also in their types of theoretical distribution.

Table 3 gives the parameters of the Gamma distribution fitted to those cases where the distribution is not normal.

The values of γ increase with duration while those of $1/\beta$ decrease. However, $1/\beta$ associated with late periods exhibits a tendency to decrease less rapidly with extended duration than do the estimates for periods occurring earlier in the season.

The fitted Gamma curves are shown in Figs. 5 and 6 as continuous lines. **From Table 3,** it is obvious that the distribution is J shaped (γ being less than 1) in case of 2 and 4 pentads at Jodhpur whereas it is highly skew but not J shaped (γ being greater than 1) even for 2 pentads in Jaipur. It may be concluded in general that there is a transition from J shaped distribution to normal distribution through skew distributions as we go from small durations to large durations. This transition is very fast for semi-arid zone station (Jaipur) and very slow for arid zone station (Jodhpur). This may be seen from the fact that in case of Jaipur, the distribution of 2 pentads is highly skew and that of 4 pentads is skew, while that of 6 pentads has already become normal. One may probably expect that if we go for lower period like one pentad or less, the distribution may become 'J shaped'. In the case of Jodhpur upto 4 pentads, the distribution is J shaped and for all higher pentads upto 18, the distribution is skew, with the skewness decreasing. Probably, one can only expect a trend towards normality which is

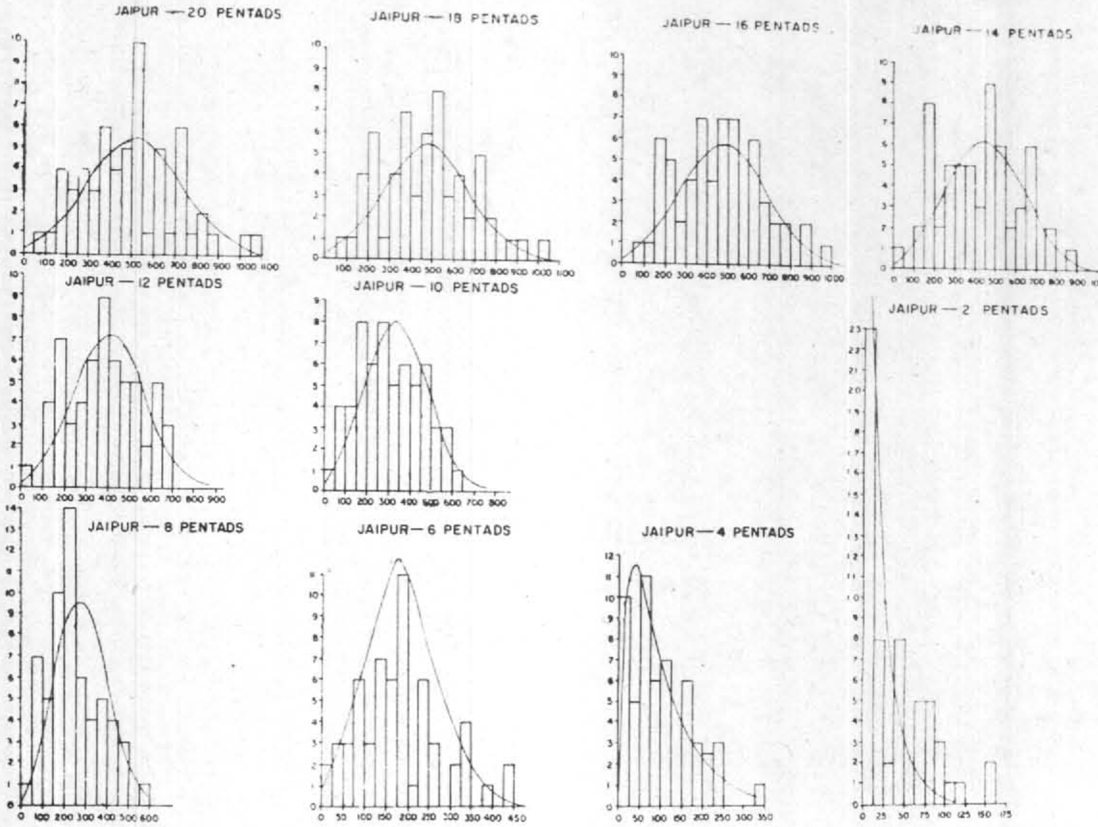

Fig. 6

Histograms of actual frequency and theoretical frequency curves of cumulative pentad rainfall from 30 June during 1901-1960

never achieved in this case because the duration of rainfall does not extend beyond 18 pentads.

In order to test the goodness of fit χ^2 test was applied. Results are presented in Table 4.

It is seen that none of the χ^2 values are significant even at 5 per cent showing the goodness of fit in all cases.

The normal distribution was fitted to the rainfall of 6 to 20 pentads in case of Jaipur by the usual method described earlier. These functions are also shown in Fg. 5 as continuous curves.

6. Probability charts

The probabilities of occurrence of different amounts of rainfall were worked out using Gamma distribution upto 4 pentads and normal curve thereafter for Jaipur and Gamma distribution for all pentads for Jodhpur. The curves so drawn are indicated as probability charts for the two stations (Figs. 7 and 8). These charts give the likelihood of different rainfall amounts for consecutive accumulated pentads and could be used

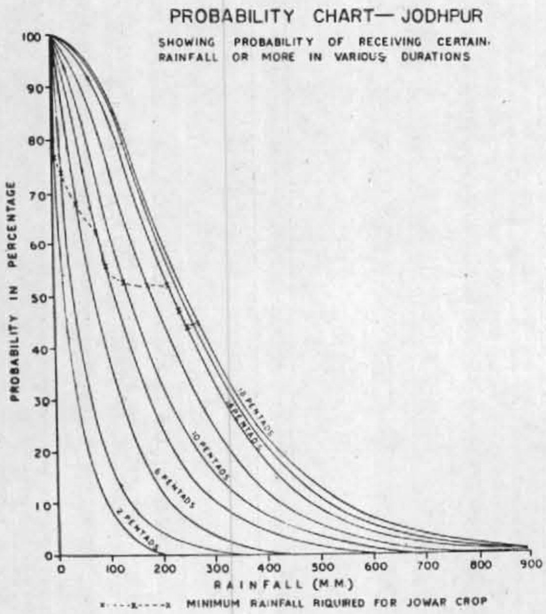

Fig. 7

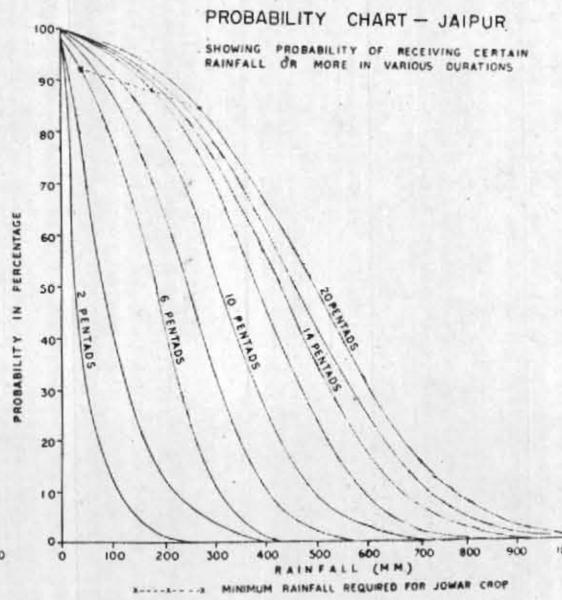

Fig. 8

by agricultural and irrigation engineers etc for their planning purposes, since they can compute for a fixed per cent of confidence (say 80 per cent) how much rainfall can be expected in various periods during the growing season. Further, by computing the drought-base rainfall for various pentads in respect of crops like *Bajra*, pulse, *Jowar* etc, *viz.*, the minimum rainfall required in various periods within the growing season for not allowing the yields of the particular crops in the region to fall very much below normal, it is possible to know the probability of drought occurrence with the help of the above charts. Similar work in respect of corn crop has been done by Barger and Thom (1949 b) in U.S.A. Once the critical excessive rainfall associated with floods has been defined, it is possible to compute the probability of these excessive rainfall amounts also from these charts as readily as the probability of drought. Since the rainfall distribution in arid zones is marked by skewness, the mean is not the most probable value as in the case of normal distribution. The mode which is the most likely amount is usually less than mean in these cases. Thus the probability charts based on the actual mathematical distribution obtained from the observed data are helpful for assisting agricultural planning.

7. Illustration for finding out likelihood of droughts from the probability charts

An illustration of using probability charts (Figs. 7 and 8) for determining probability of droughts in various periods during the growing season is presented below. Krishnan (1969) gave consumptive use of water during successive fortnights from sowing of *Jowar* crop in Jodhpur and Jaipur. By assuming that the crop will suffer from drought if even half of these consumptive use values are not received as rainfall during various periods in the growing season, the drought-base values were plotted on the probability curve of the concerned pentad. Now, the likelihood of obtaining atleast the drought-base values of rainfall or above can be easily read from probability charts (Figs. 7 and 8). It is seen from the figures that the likelihood of receiving the required rainfall is high (exceeding 85 per cent) in case of Jaipur. Though the likelihood is very high for short period, it falls below 50 per cent for longer periods thereby showing how risky it is to grow *Jowar* in Jodhpur. Thus, these probability charts for various stations in arid and semi-arid zones can be helpful in evolving suitable cropping patterns for the regions.

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