

# Lunar atmospheric tides in surface winds at six Indian stations\*

K. N. RAO and S. JEEVANANDA REDDY

Meteorological Office, Poona

(Received 21 April 1971)

**ABSTRACT.** Using hourly data for 8 to 10 years, the lunar semi-diurnal tides—amplitude ( $l_2$ ) and phase ( $\lambda_2$ ) in surface winds at six Indian stations have been determined. In working out the vector probable error, the improved method of Malin and Chapman (1970) has been used. The values obtained for amplitude in the case of eastward ( $v$ ) component are in good agreement with the theoretical value (1 cm/sec), but in the case of southward ( $-u$ ) component, the calculated values are somewhat less. Except in a few cases, the sense of rotation of the wind vector is contrary to that predicted by the theory. But, however, the probable error values obtained are in reasonable limits, even though the data used are for a short period.

The increase with latitude, of the amplitude here inferred is in accordance with theory. However earlier studies (e.g., Haurwitz and Cowley) do not indicate such variation. The amplitudes obtained for eastward ( $v$ ) component (lying in the range 0.8-1.3 cm/sec) of wind for the six stations are greater than the southward ( $-u$ ) component (lying in the range 0.4-0.9 cm/sec) of wind and the probable errors follow this trend. The amplitude in J-season is greater than in D-season at all the six stations.

## 1. Introduction

Chapman (1948) evaluated lunar tides at Mauritius in the Southern Hemisphere and Haurwitz and Cowley (1968) for four American stations in the Northern Hemisphere. In meteorological elements, the lunar semi-diurnal tide is found to be significant. They obtained the amplitude of the lunar semi-diurnal wave as 1 cm/sec and the rotation with time of wind vector is opposite in the two hemispheres, which are both in agreement with theory. But in both the hemispheres, the sense of rotation of the calculated wind vector is contrary to that expected from theory.

In the present paper lunar tide ( $L_2$ ) in surface winds at six Indian stations in different latitudes have been studied. The data considered are the mean hourly values over a period of 8 to 10 years, as detailed in Table 1.

## 2. Analysis

The amplitude and phase are calculated using Chapman and Miller (1940) method and the vector probable error evaluated by a method due to Malin and Chapman (1970). The data have been analysed for the three seasons and annual, i.e., J = May to August, D = November to February, E = March, April, September & October and Y = Yearly. In case of meteorological data stormy days are very few. The data is therefore, not analysed according to calm and disturbed days as in the case of geomagnetic data. The computations were made for J, D, E and Y on IBM 1620 computer at Poona and on CDC 3600 computer at Bombay.

The solar and lunar tides are obtained in the form,

$$S = \sum_{p=1}^4 s_p \sin (pt + \theta_p) \quad (1)$$

$$\text{and } L = \sum_{n=1}^4 l_n \sin [t(n-2) + 2\tau + \lambda_n] \quad (2)$$

where,  $S$  = solar tide,  $s_p$  and  $\theta_p$  ( $p=1, 2, 3, 4$ ) represent amplitudes and phases respectively of the first four harmonics, i.e., 24, 12, 8 and 6 solar hourly waves in solar factor (24 solar hours = 1 solar day).  $L$  = Lunar tide,  $l_n$  and  $\lambda_n$  ( $n=1, 2, 3, 4$ ) represent amplitudes and phases respectively of the first four harmonics, i.e., 24, 12, 8 and 6 lunar hourly waves in lunar tide (24 lunar hours = 1 lunar day),  $t$  = time in degrees; increases from  $0^\circ$  to  $360^\circ$  from one local transit of the sun (local mid night) to the next, and  $\tau$  = time in degrees; increases from  $0^\circ$  to  $360^\circ$  from one local transit of the moon to the next (one lunar day = 1.03505 solar days).

As the lunar semi-diurnal wave alone is significant in meteorological elements, the above equation for lunar tides can be written as —

$$L = l_2 \sin (2\tau + \lambda_2) \quad (3)$$

and correspondingly its solar semi-diurnal tide is —

$$S = s_2 \sin (2t + \theta_2) \quad (4)$$

Necessary corrections are made for  $\lambda_2$  and  $\theta_2$ .

The results of the analysis are shown in Table 2. In the course of computations of  $L_2$ , solar tide,  $S_2$ , has also been determined and these are given

\*Accepted for presentation at the X<sup>th</sup> Assembly, I.U.G.G., in August 1971, Moscow

TABLE 1  
List of stations and other details of data

Station	Latitude (N)	Longitude (E)	Altitude (m)	Period of record	No. of days
Bangalore C.O.	12 58	77 35	921	1951-60	2700
Madras	13 00	80 11	16	1951-58	2470
Poona	18 32	73 51	559	1951-60	2840
Calcutta	22 32	88 20	6	1951-58	2620
Jodhpur	26 18	73 01	224	1951-58	2560
Jaipur	26 49	75 48	390	1951-58	2400

TABLE 2  
Lunar tidal variations of surface winds

Sea- son	Southward component (-u)			Eastward component (v)		
	Ampli- tude ( $l_2$ ) (cm/sec)	P.E. ( $r_2$ ) (cm/sec)	Phase ( $^\circ$ )	Ampli- tude ( $l_2$ ) (cm/sec)	P.E. ( $r_2$ ) (cm/sec)	Phase ( $^\circ$ )
<i>Bangalore (C.O.)</i>						
D	0.58	0.20	20	1.02	0.35	280
E	0.52	0.27	300	1.01	0.40	340
J	0.76	0.28	340	1.10	0.38	270
Y	0.62	0.20	260	1.02	0.36	310
<i>Madras</i>						
D	0.61	0.30	200	1.10	0.40	270
E	0.41	0.21	330	1.00	0.36	300
J	0.66	0.24	350	1.11	0.31	260
Y	0.60	0.30	300	1.04	0.35	340
<i>Poona</i>						
D	0.60	0.27	210	0.91	0.32	340
E	0.43	0.20	20	1.14	0.35	250
J	0.80	0.22	320	1.17	0.40	310
Y	0.66	0.17	280	1.11	0.30	340
<i>Calcutta</i>						
D	0.60	0.22	80	0.80	0.32	100
E	0.54	0.31	200	1.12	0.41	300
J	0.84	0.26	320	1.26	0.30	280
Y	0.74	0.24	300	1.16	0.32	350
<i>Jodhpur</i>						
D	0.72	0.25	310	0.98	0.36	340
E	0.79	0.23	210	1.26	0.42	20
J	0.88	0.30	340	1.20	0.38	270
Y	0.86	0.24	280	1.10	0.34	20
<i>Jaipur</i>						
D	0.66	0.30	340	1.12	0.40	30
E	0.71	0.34	270	1.19	0.36	350
J	0.92	0.22	300	1.25	0.33	340
Y	0.82	0.26	210	1.18	0.31	320

TABLE 3  
Solar tidal variations of surface winds

Sea- son	Southward component (-u)		Eastward component (v)	
	Amplitude $s_2$ (cm/sec)	Phase $\theta_2$ ( $^\circ$ )	Amplitude $s_2$ (cm/sec)	Phase $\theta_2$ ( $^\circ$ )
<i>Bangalore (C.O.)</i>				
D	4.00	340	4.70	200
E	2.85	320	4.20	170
J	2.58	40	3.26	90
Y	3.29	20	4.16	80
<i>Madras</i>				
D	2.85	300	4.20	320
E	2.96	210	4.80	160
J	2.60	60	4.60	90
Y	3.00	80	4.70	100
<i>Poona</i>				
D	3.52	260	5.56	160
E	3.00	70	5.72	80
J	3.43	50	3.62	90
Y	3.05	40	4.65	90
<i>Calcutta</i>				
D	3.62	200	5.50	30
E	4.00	120	5.30	20
J	4.22	70	4.60	100
Y	4.60	60	5.24	40
<i>Jodhpur</i>				
D	4.50	300	5.80	30
E	3.90	260	6.20	30
J	5.85	340	6.58	350
Y	4.95	320	6.42	10
<i>Jaipur</i>				
D	3.80	80	5.02	320
E	4.60	100	6.46	270
J	5.81	340	7.20	310
Y	5.62	60	6.80	80

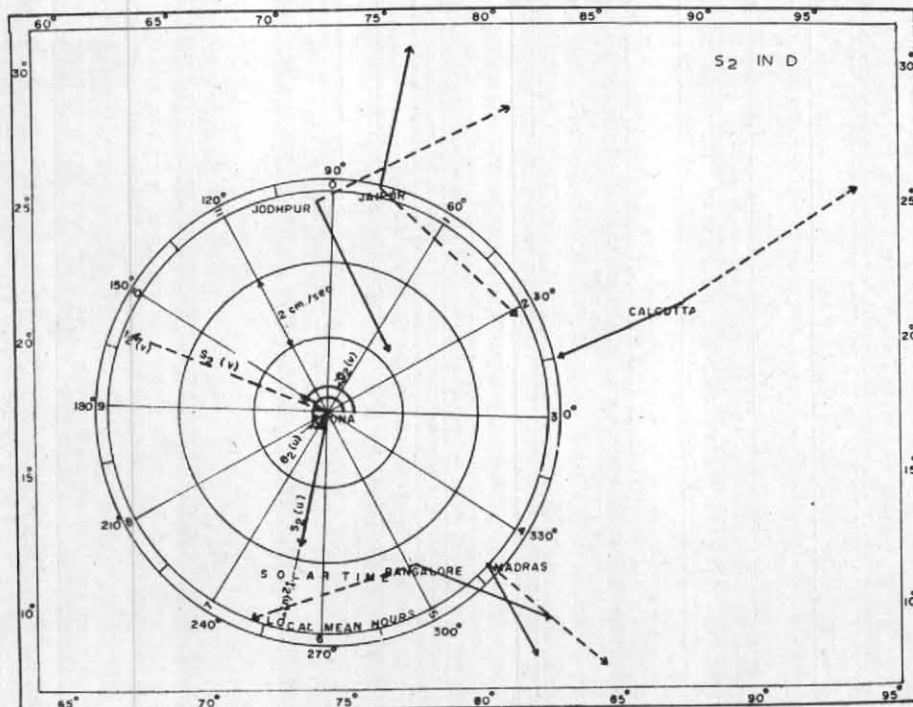


Fig. 1(a)

in Table 3. The relationship of  $L_2$  with  $S_2$  is also studied as both are semi-diurnal tides.

3. Discussion

*Lunar Tides* — Tidal theory shows that with the observed lunar tidal variations of the atmospheric pressure, the amplitudes of the wind components should be of the order of 1 cm/sec (Haurwitz and Cowley 1968). The results in Table 2 show agreement with this conclusion. Another point observed in the present analysis is that the amplitude of southward ( $-u$ ) component is comparatively less than the eastward ( $v$ ) component, at all the six stations. Such a feature has not been noted either at Mauritius studied by Chapman or at four American stations studied by Haurwitz and Cowley. The amplitude in  $-u$  and  $v$  components respectively lie in the ranges 0.4–0.9 cm/sec and 0.8–1.3 cm/sec respectively. The difference in amplitude of  $-u$  and  $v$  is wide in the case of stations nearer to the coast, viz., Madras, Calcutta and Poona than for inland stations like Jodhpur and Jaipur.

The amplitudes of both components increase generally towards the poles according to theory (Chapman and Bartels 1940), which also suggests that the phase constant for  $v$  should be  $90^\circ$  larger than for  $-u$  and the wind vector should rotate in the same direction as the tide generating body

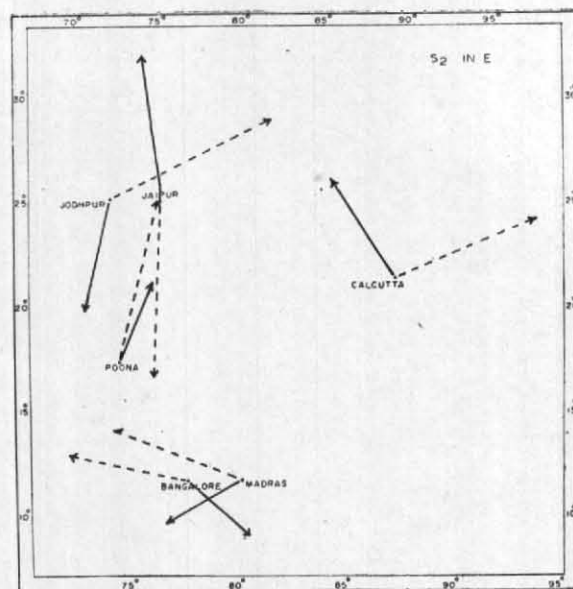


Fig. 1(b)

Figs. 1 (a)-(d). Harmonic dial representation of lunar semi-diurnal variation in surface winds (E, D, J seasons and Y annual) at six Indian stations

- Southward component
- .....→ Eastward component
- Amplitude — Length between the origin and arrow head
- P.E. — Length between the origin and first dot
- Phase — Angle between east direction and arrow line measured anticlockwise direction from east

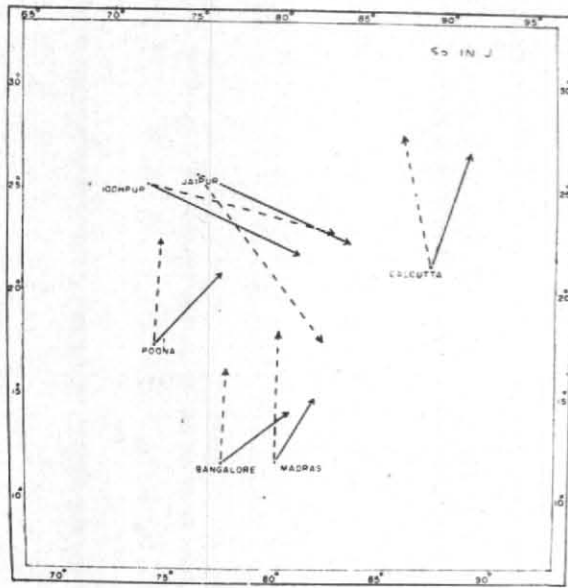


Fig. 1(c)

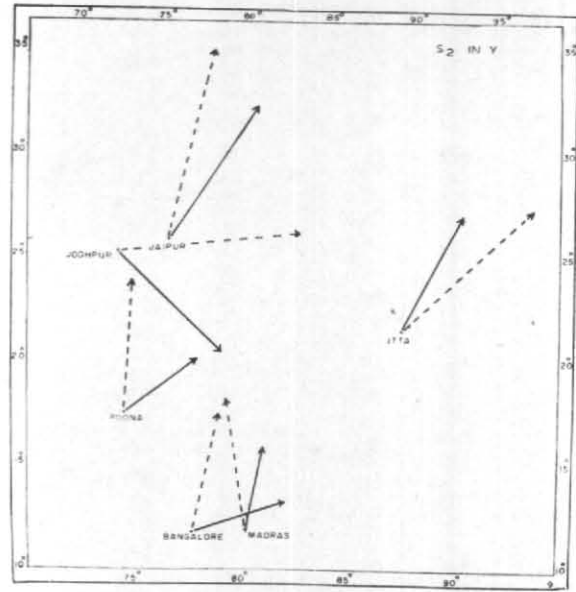


Fig. 1(d)

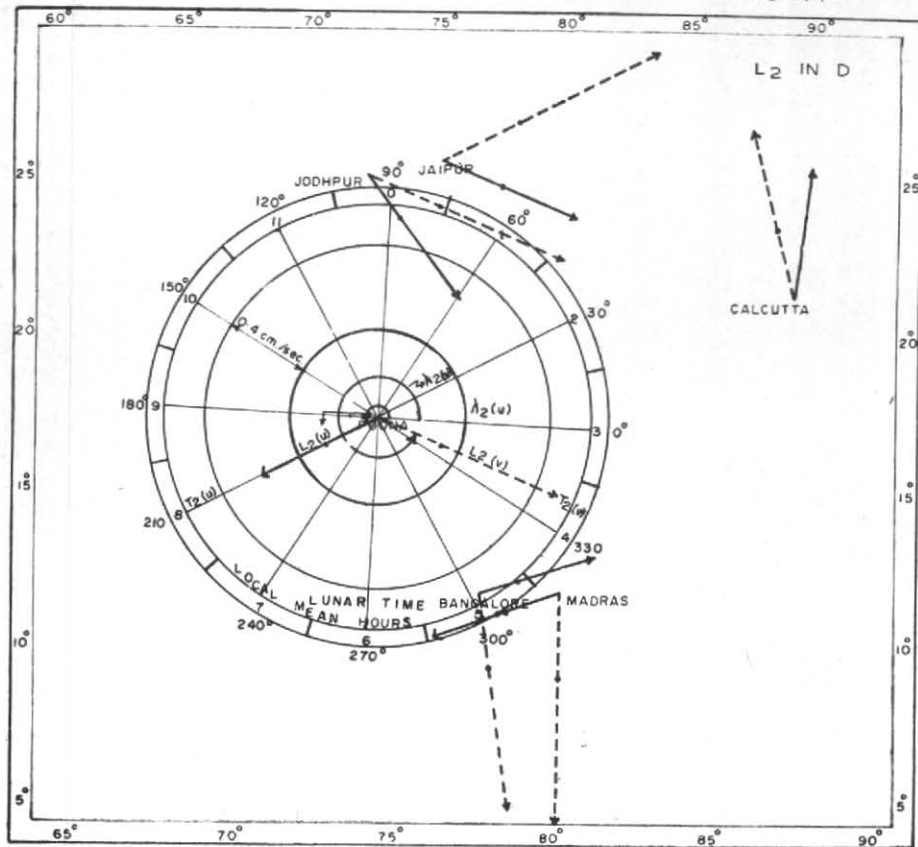


Fig. 2(a)

moves across the sky. Except in a few cases, the corresponding results in Table 2 are contrary to the predicted values. Such a feature has also been observed in the results for American stations studied by Haurwitz and Cowley (1968). It was suggested by them that this disagreement seems to be

an indication of the poor determination of the lunar tidal motions. It is, however, seen from Table 2 that the vector probable errors in the present analysis are in reasonable limits suggesting that the calculations are satisfactory.

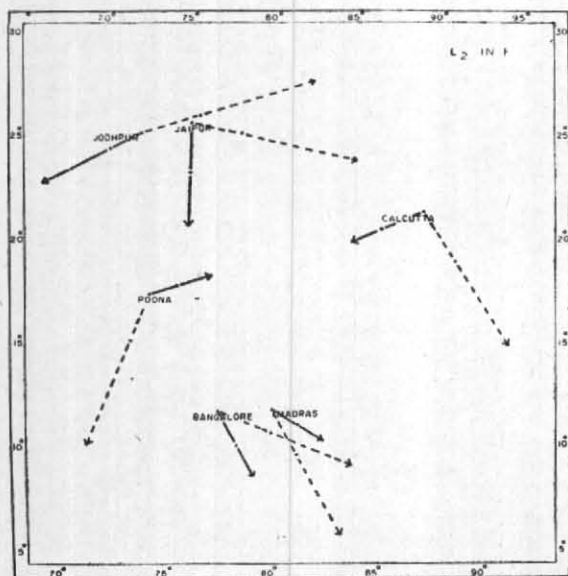


Fig. 2(b)

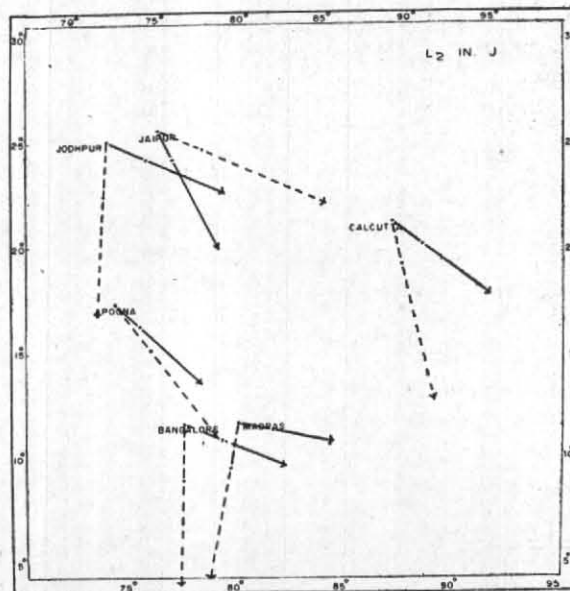


Fig. 2(c)

The amplitudes in J-season are greater than those in D-season at all the six stations, (both for  $-u$  and  $v$  components) [and also satisfies the finding that the amplitude should increase towards pole (but not in E-season). For all the seasons, the phase angles are not following any regular pattern, though yearly values of phases and amplitudes follow a regular variation in both  $-u$  and  $v$  components.

**Solar Tides**—A study of the results in Table 3 shows that the amplitudes of  $-u$  component are less than those of  $v$  component. The 'theoretical' phase for  $S_2(-u)$  should be  $248^\circ$  in Northern Hemisphere and  $68^\circ$  in Southern Hemisphere; for  $S_2(v)$  the phase should be  $338^\circ$  in all latitudes. These phase predictions do not agree well with the results in Table 3. The amplitudes are less than the predicted value of nearly 17 cm/sec at  $20^\circ$  latitude, but they increase towards poles as per theory. Half of the values follow the theoretical value of the rotation of the wind vector. The amplitude for J-season is greater than for D-season in case of Calcutta, Jodhpur, and Jaipur, but the reverse is true in case of Madras, Poona and Bangalore. No regular variation with latitude and season has been observed either in amplitude or in phase.

**Harmonic dials**—The lunar and solar daily variations are represented by harmonic dials—Figs. 1 and 2 respectively.

In case of Mauritius studied by Chapman, the amplitudes of  $L_2(-u)$  and  $L_2(v)$  are similar though their phases are considerably different.

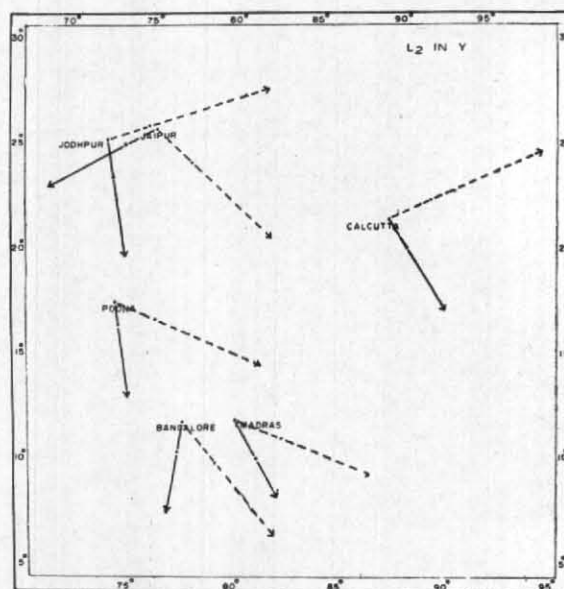


Fig. 2(d)

Figs. 2(a)-(d). Harmonic dial representation of solar semi-diurnal variation in surface winds (D, E, J seasons and Y annual) at six Indian stations

—————→ Southward component  
 .....→ Eastward component

Amplitude—Length between the origin and arrow head

Phase—Angle between east direction and arrow line, measured anticlockwise direction from east

For  $S_2$  the phases are nearly the same. However amplitude  $S_2(v)$  is twice of  $S_2(-u)$ . The ratios  $s_2/l_2$  in  $-u$  and  $v$  are 10 and 20 respectively. But in the present study, it is seen from the harmonic dials that the phases are considerably different in many cases both for  $S_2$  and  $L_2$ . In  $S_2$  and  $L_2$ ,  $v$  components are about one to one and half times the amplitudes of  $-u$ . The ratios  $s_2/l_2$  are approximately 5 and 4 in case of Bangalore, Madras and Poona, and 6 and 5 at Calcutta, Jodhpur and Jaipur respectively. These suggest  $s_2/l_2$  increases with latitude.

#### 4. Summary and conclusions

1. The amplitudes of  $-u$  and  $v$  components are in accord with theory and increase with latitude. Amplitude  $-u$  is less than amplitude  $v$ . The rotation of wind vector is also observed to be generally in accord with theory.

2. The differences in amplitude between  $-u$  and  $v$  components are more near coastal stations

than at inland stations.

3. The amplitudes in J-season are greater than those in D-season at all the six stations.

4. The seasonal phase angles do not appear to follow any regular pattern. However, yearly values of phase and amplitude show a regular variation in both  $-u$  and  $v$  components.

5. The ratio of  $s_2/l_2$  increases with latitude and is more for  $-u$  than for  $v$ .

*Acknowledgements*—The authors are thankful to Shri B.N. Bhargava, Director and Shri D.R.K. Rao, Assistant Meteorologist, Colaba and Alibag Observatories, for the valuable discussions. Thanks are also due to the members of staff of the Investigation and Development Section of the Office of the Deputy Director General of Observatories (Climatology & Geophysics), for the help in the computational work.

#### REFERENCES

- |                                  |      |   |
|----------------------------------|------|---|
| Chapman, S.                      | 1948 | <i>Verb. Met. Un. geod. geophys. Int.</i> , Oslo.         |
| Chapman, S. and Bartels, J.      | 1940 | <i>Geomagnetism</i> , 2, 766, Clarendon Press, Oxford.    |
| Chapman, S. and Miller, J. C. P. | 1940 | <i>Mon. Not. R. astr. Soc., geophys. Suppl.</i> , 4, 649. |
| Haurwitz, S. and Cowley, A. D.   | 1968 | <i>Geophys. J. R. astr. Soc.</i> , 15, 103.               |
| Lindzen, R. S. and Chapman, S.   | 1969 | <i>Space Sci. Rev.</i> , 10(A).                           |
| Malin, S. R. C. and Chapman, S.  | 1970 | <i>Geophys. J. R. astr. Soc.</i> , 19, 15.                |