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Representation of circulation patterns by orthogonal polynomials with application to specification of weather from them

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ABSTRACT. A technique for representation of atmospheric circulation patterns as a linear combination of simple patterns given by a set of Tschebyscheff orthogonal polynomials has been examined. This enables 5-day mean 700 mb circulation systems over Indian region to be parameterised in terms of a limited number of circulation indices which can be utilised for developing objective statistical relationships between atmospheric circulation systems and the weather.

1. Introduction

Wadsworth and Bryan (1948), in the course of an investigation of analogue forecasting, developed a technique to express a pressure or contour map in form of a mathematical equation involving a limited number of parameters. Such representation was found useful in establishing relationships between circulation patterns and weather by workers at Massachusetts Institute of Technology and elsewhere (Friedman 1955, Malone 1956, Jorgensen 1959). In the present study the efficiency of technique developed by Wadsworth and Bryan has been examined for representation of 5-day mean 700 mb flow patterns over Indian area with an aim of utilising such representation for specifying 5-day rainfall from 5-day mean 700 mb charts.

2. Mathematical representation of charts

The procedure for mathematical representation of a pressure or contour height chart consists of fitting a two dimensional surface of a desired degree to the given chart. The fitting is done by weighted addition of a family of primary hypothetical surfaces orthogonal to each other over the field of map. Such primary surfaces obtained in terms of Tschebyscheff orthogonal polynomials fitted to a network of regularly spaced points. The contour height or pressure value of a point having coordinates (x_i, y_j) , can thus be represented exactly by a functional relation of the form —

$$\hat{h}_{ij} = \vec{h} + \sum_{l=1}^{L-1} a_l P_l (x_i) + \sum_{m=1}^{M-1} b_m P_m (y_j) + \sum_{l=1}^{L-1} \sum_{m=1}^{M-1} C_{lm} P_{l_m} (x_i, y_j)$$
(1)

where L, M = number of grid points along x and y directions.

 \overline{h} = mean value for the chart (area)

 $P_l(x_i)$, $P_m(y_j) := i^{\text{th}}$ and j^{th} terms of Tschebyscheff orthogonal polynomials of degree l and m in x and y respectively and,

$$P_{lm}(x_i, y_j) = P_l(x_i) \cdot P_m(y_j)$$

With the arrays L and M of the network fixed, the orthogonal polynomials are fixed and can be determined once for all. Representation of values at different grid points therefore involves determination of parameters a_l . $b_m C_{lm}$ and \overline{h} . The values at different points can be approximated with sufficient degree of accuracy by reducing the degree of terms appreciably. After some experimentation a truncated series consisting of 14 orthogonal polynomials representing a surface of degree 4 in x and y was found suitable for the present study. The value at point (x_i, y_j) in terms of this series is approximated as —

$$\hat{h}_{ij} = \bar{h} + \sum_{l=1}^{4} a_l P_l(x_i) + \sum_{m=1}^{4} b_m P_m(y_j) + \sum_{l=1}^{3} \sum_{m=1}^{3} C_{lm} P_{lm}(x_i, y_j)$$

$$l + m \leq 4$$
(2)

Also it is convenient to use normalised polynomials in the above relation so that the variance explained by each polynomial is given by the square of its coefficient. A schematic representation of these polynomials is given in Fig 1.

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Schematic representation of 14 Tschevyscheff orthogonal ploynomials used in the study. The arrows indicate the geostrophic winds associated with simple pressure patterns represented by respective orthogonal polynomials



Grid network of 111 points used for fitting of Tschebyscheff orthogonal polynomials and calculation of their coefficients. The grid points over Tibetan region are indicated by X

3. Application of procedure to 700 mb charts

This procedure of objective representation of circulation patterns was applied to 5-day mean 700 mb contour height and anomaly charts for monsoon months (June to September) for ten years 1958-67. The unnormalised polynomials were calculated from the recurrence formula (Fisher and Yates 1963) as —

$$P_{r+1} = P_1 P_r - \frac{r^2 (n^2 - r^2)}{4 (4r^2 - 1)} P_{r-1}$$
(3)

with $P_0 = 1$ and $P_1 = x$ or y

where, r = degree of polynomial

n = number of data points along x or y on which polynomials are based.

Consistent with the data network the values of n for polynomials in x (along E-W) and y (along N-S) were 17 and 13 respectively.

The contour height and anomaly data at 111 points of the diamond type grid network (Fig. 2)

were used for determining the coefficients of polynomials for each chart. The data over Tibetan region necessary for calculating these coefficients were obtained by extending the analysis over extended area charts to this region. These coefficients can be calculated easily and independently by using orthogonal property of the polynomials —

$$\sum\limits_{i} P_{l}\left(x_{i}
ight)$$
 . $P_{m}\left(x_{i}
ight) = \sum\limits_{j} P_{l}\left(y_{j}
ight)$. $P_{m}\left(y_{j}
ight) = \delta_{lm}$

and

$$\sum_{i} \sum_{j} P_{lm}(x_i, y_j) \cdot P'_{lm'}(x_i, y_j) = \delta_{lm} \delta'_{lm'}$$

where, δ_{lm} is Kronecker delta having value 1 for l = m and 0 for $l \neq m$.

The coefficients a_l and b_m were calculated from the marginal means of data over the chart and coefficients C_{lm} were obtained using individual 111 values over the grid network.

The orthogonal polynomials being fixed, the parameters determining a circulation pattern are the coefficients of these polynomials and the chart mean. These parameters can therefore be effectively used in place of circulation patterns for any statistical studies. The polynomial coefficients were also normalised by standard deviation of the chart, so that their squares give directly the ratio of variance explained by the respective polynomials. In such a case standard deviation of the chart is one more parameter determining a circulation pattern.

For convenience, the orthonormal polynomials will hencefore be represented by notations P-1to P-14 as indicated in Fig. 1. It was found that the contour height as well as anomaly patterns of 5-day mean 700 mb charts could be parameterised by the above procedure to a high degree of accuracy. Table 1 gives the percentage frequency of contour height and anomaly charts which could be represented with different degrees of over-all closeness of fit as given by percentage reduction in total variance. It may be seen that representation of contour height charts for June to August has been very good when such representation could explain more than 90 per cent of variance for about 90 per cent of charts. During September about 65 per cent of charts could be represented with such an accuracy and of the remaining charts about 30 per cent explained variance between 80 and 90 per cent. This gives coefficient of correlation between actual and fitted charts as high as 0.90 in more than 90 per cent of cases during all the four months. The closeness of overall fitness for anomaly charts also, though not of such a high degree of accuracy, is satisfactory. During June about 90 per cent

CIRCULATION PATTERNS BY ORTHOGONAL POLYNOMIALS

Total variance explained (%)		He	ight	Anomaly				
	Jun	Jul	Aug	Sep	Jun	Jul	Aug	Sep
0.0-4.9								
$5 \cdot 0 - 9 \cdot 9$								
$10 \cdot 0 - 14 \cdot 9$								
15.0-19.9								
$20 \cdot 0 - 24 \cdot 9$								
25.0-29.9						0.8	e	
30.0-34.9								
35.0-39.9					0.8	3.0		
40.0-44.9						3.0		1.6
$45 \cdot 0 - 49 \cdot 9$						2.2	0.8	2.
50.0-54.9						3.0	0.8	0.
55.0-59.9					0-8	5.3	0.8	4.
$60 \cdot 0 - 64 \cdot 9$					0.8	2.2	2.2	4.
65.0-69.9		0.8		0.8	6-2	3.8	3.8	4.
70.0-74.9	0.8			0.8	2.3	6.8	9.0	7.
75.0-79.9	0.8		0.8	3.1	12.4	11.3	14.2	15.
80.0-84.9	2.3		1.5	11.7	18.6	15.0	15.0	18.
85.0-89.9	7.7	4.4	6.0	18.7	27.1	21.8	27.1	18.
90.0-94.9	31.0	26.4	35.3	43.8	24.8	15.8	21.8	16.
95.0-100.0	57.4	68.4	56.4	21.1	6.2	6.0	4.5	4.

Percentage frequency distribution of 5-day mean 700 mb contour height and anomaly patterns with different degree of overall accuracy of fit by orthogonal polynomials

TABLE 1

of charts explained 75 per cent or more of variance whereas during July to September about 70 to 80 per cent of charts had so accurate a fit. Thus in case of anomaly charts also the correlation coefficient between actual and fitted charts is 0.85.

Table 2 gives relative contribution of different polynomials towards overall fitness of charts. In case of contour height charts polynomial P-5 contributes maximum towards fitness followed by P-6 during all the four months both for contour height as well as anomaly charts. For contour height charts these two polynomials alone explain 68 per cent of variance during July and more than 50 per cent during June and August. For September charts their contribution towards explained variance is 44 per cent. For anomaly charts these two polynomials together contribute about 37 to 40 per cent of variance. Next in order of importance for fit are polynomials P-1 and P-2, each of which on the average contributes more than 10 per cent of variance for the contour height charts. Mean variance contributed by them towards fitting of anomaly charts is about 10 and 4 per cent respectively. Contribution of other polynomials towards representation of contour charts on the average is about 16 per cent and for anomaly charts 30 per cent. This is due

to the fact that contour height charts generally consist of simpler and smoother patterns than the patterns of anomaly charts. On the average the fitted charts account for about 94 per cent of variance in case of contour height patterns and about 81 per cent for anomaly patterns, larger part of it being contributed by the polynomials P-1 and P-2 representing simple zonal patterns and polynomials P-5, and P-6 representing simple meridional patterns.

4. Seasonal variations of polynomial coefficients

It is interesting to examine the variations of coefficients of orthogonal polynomials with the advance of season. Fig. 3 gives such variations for the coefficients of polynomials P-1, P-2, P-5and P-6 for normal 5-day mean charts of 25 standard pentads for June to September together with variations of mean and standard deviation of the contour height values of these charts. The variations of coefficients of other polynomials having small contribution towards overall representation of charts are not shown.

The chart mean of contour height with a value of 3122 gpm in the beginning of June has a steady fall during the month registering its lowest value of 3098 gpm towards the beginning of July. But

		Height					Anomaly				
		Jun	Jul	Aug	Sep	Mean	Jun	Jul	Aug	Sep	Mean
X-Polyno	mials							÷. ·			
$P_1(\mathbf{X})$	P-1	$19 \cdot 0$	$4 \cdot 7$	$6 \cdot 0$	$13 \cdot 7$	$10 \cdot 9$	$10 \cdot 6$	$11 \cdot 7$	8.8	$8 \cdot 3$	$9 \cdot 9$
$P_2(\mathbf{X})$	P-2	$5 \cdot 4$	8.6	$15 \cdot 5$	$14 \cdot 6$	$11 \cdot 0$	$5 \cdot 2$	$2 \cdot 9$	$2 \cdot 9$	4.0	$3 \cdot 7$
$P_{3}(\mathbf{X})$	P-3	$1 \cdot 4$	$0 \cdot 5$	$0 \cdot 4$	$1 \cdot 3$	$0 \cdot 9$	2.5	1.6	$2 \cdot 0$	$1 \cdot 7$	$1 \cdot 9$
$P_4(\mathbf{X})$	P-4	1 - 1	$0 \cdot 3$	$0 \cdot 3$	0.6	0.6	$1 \cdot 7$	0.9	1.0	1.4	$1 \cdot 3$
Sum		$26 \cdot 9$	$14 \cdot 1$	$22 \cdot 2$	$30 \cdot 2$	$23 \cdot 4$	$20 \cdot 0$	$17 \cdot 1$	14.7	$15 \cdot 4$	16.8
Y-Polyno	mials										
$P_1(\mathbf{Y})$	P_{*5}	$41 \cdot 9$	$45 \cdot 7$	$34 \cdot 8$	$22 \cdot 8$	$36 \cdot 3$	$25 \cdot 5$	$19 \cdot 9$	$25 \cdot 0$	$21 \cdot 5$	$23 \cdot 0$
$P_{3}(\mathbf{Y})$	P-6	10.3	$22 \cdot 3$	$24 \cdot 6$	$21 \cdot 0$	$19 \cdot 5$	$12 \cdot 2$	$17 \cdot 2$	$15 \cdot 7$	$18 \cdot 2$	$15 \cdot 8$
$P_{3}(\mathbf{Y})$	P-7	$1 \cdot 6$	$2 \cdot 2$	$2 \cdot 0$	$3 \cdot 0$	$2 \cdot 2$	$5 \cdot 3$	$4 \cdot 1$	$4 \cdot 5$	$5 \cdot 1$	4.7
$P_4(\mathbf{Y})$	P-8	$1 \cdot 3$	$2 \cdot 1$	$2 \cdot 0$	$2 \cdot 0$	$1 \cdot 9$	$2 \cdot 9$	$3 \cdot 6$	$3 \cdot 9$	$3 \cdot 4$	$3 \cdot 5$
Sum		$55 \cdot 1$	$72 \cdot 3$	$63 \cdot 4$	$48 \cdot 8$	$59 \cdot 9$	$45 \cdot 9$	$44 \cdot 8$	$49 \cdot 1$	$48 \cdot 2$	$47 \cdot 0$
XY-Polyr	omials										
$P_{11}(\mathbf{X},\mathbf{Y}$) P-9	$7 \cdot 0$	$3 \cdot 3$	$2 \cdot 5$	3.3	$4 \cdot 0$	$5 \cdot 9$	4-4	6.0	$4 \cdot 7$	$5 \cdot 3$
P 11 (X,Y) P-10	$0 \cdot 9$	$1 \cdot 4$	3.3	3.7	$2 \cdot 3$	$2 \cdot 6$	$2 \cdot 5$	$2 \cdot 8$	$3 \cdot 1$	$2 \cdot 7$
$P_{81}(\mathbf{X},\mathbf{Y})$) P-11	$0 \cdot 3$	$0 \cdot 4$	0.5	0.6	0.5	1.1	1.0	$1 \cdot 1$	$0 \cdot 9$	$1 \cdot 0$
P18 (X,Y)) P-12	$1 \cdot 9$	$2 \cdot 3$	1.4	1.6	1.8	4.3	4.4	$5 \cdot 0$	$3 \cdot 1$	$4 \cdot 2$
Pas (X,Y) P-13	1.0	0.8	1.0	1.4	1.1	2.8	$1 \cdot 6$	$2 \cdot 4$	2.1	$2 \cdot 2$
P18 (X,Y)	P-14	$1 \cdot 0$	0.8	0.5	1.0	0.8	$2 \cdot 1$	2.0	$2 \cdot 2$	2.0	$2 \cdot 1$
Sum		$12 \cdot 1$	9.0	$9 \cdot 2$	$11 \cdot 6$	$10 \cdot 5$	$18 \cdot 8$	$15 \cdot 9$	19.5	$15 \cdot 9$	17.5
Total v	ariance explained	$94 \cdot 1$	$95 \cdot 4$	94.8	90.6	$93 \cdot 7$	$84 \cdot 7$	77.8	83.3	79.5	81.3

Percentage of total variance explained by orthogonal polynomial representation of 5-day mean 700 mb contour height and anomaly patterns. Figures in table are averages based on 10 years (1958 to 1967) data

TABLE 2



Variations during monsoon season of 5-day 700 mb normal contour height chart mean (M), standard deviation (S) and coefficients of orthogonal polynomials P-1, P-2,P-5 and P-6.

for a secondary minimum (3101 gpm) occurring in pentad centred on 1 August the mean continues rising thereafter and attains a value of 3129 gpm by the end of September. The standard deviation of contour height of chart remains between about 18 and 22 gpm till the beginning of August and falls thereafter attaining its lowest value of 10 gpm by the end of September. There is a tendency for higher (lower) values of standard deviation being associated with lower (higher) values of chart mean. This is in conformity with general synoptic feature that prominent weather systems occur during the period of overall low pressure fields.

As indicated by P-5 curve, the north-south gradient (with low towards north) in contour height field has quite a high value in the beginning of June and rises further attaining its first maximum value in pentad centred on 17 June. Thereafter till the beginning of August value of coefficient of P-5 oscillates between 0.7 and 0.8 with another prominent maximum in pentad centred on 12 July and another weak maximum at pentad centred on 1 August. Except during first half of September when gradient remains nearly steady, there is a general fall in meridional gradient during August and September bringing its value to zero by the end of season. The maximum around 17 June appears to be associated with general intensification of heat low over the northern parts of the country and that around 12 July with general shift and intensification of trough with the advancement of monsoon over north India. The value of coefficient of polynomial P-6 rises steadily from near zero in the beginning of July. The rise during the period closely follows the fall in the value of chart mean. Thereafter this coefficient oscillates with an overall rising tendency till the beginning of September when it attains highest value of 0.6. This is followed by sharp fall in coefficient value till end of September. The variations in the values of coefficients of polynomials P-5 and P-6 are found to be generally out of phase with each other perhaps due to the fact that intensification or shift of trough over the central parts of the country decreases overall north-south gradient whereas its weakening in central part of country or movement to north results in increase in overall meridional gradient.

The coefficient of polynomial P-1 has large negative value (-0.65) in the beginning of June indicating marked east-west gradient in contour height field with higher contour heights in the western and lower in the eastern parts of the region. With the advance of season this gradient decreases attaining a minimum value by the beginning of July. This fall in east-west gradient closely follows the fall in chart mean of contour height values. A very weak east-west gradient prevails from the beginning of July to the middle of August during which period its variations follow the variations in the coefficient of polynomial P-5 indicating that positioning and intensification of seasonal trough over central parts of country decreases overall east-west gradient in contour height field and vice-versa. After middle of August this gradient again builds up and continues intensifying till the end of season. But for small oscillations, the coefficient of polynomial P-2 steadily increases from a small value of less than 0.2 in the beginning of June to about 0.6 by the end of September. Such type of variations of this coefficient indicate that with the advancement of season the contour height values continue to fall over the warm central continental area of the region in comparison to comparatively cold oceanic regions of Bay of Bengal and Arabian Sea. This feature is a general reflection of influence of land and sea contrast on the general circulation of the region.

Examples of representation of 700 mb chart by orthogonal polynomials and specification of rainfall from it

Figs. 4(a) and 4(b) show 5-day mean 700 mb actual contour height and anomaly patterns for the pentad 30 June to 4 July 1964. The contour height and anomaly patterns as arrived at by fitting of

TABLE 3

Normalised coefficients of orthogonal polynomials fitted to 5-day mean 700 mb contour height and anomaly patterns for pentad June 30 to July 4, 1964 together with variance explained by each of the polynomials

	Heig	ht	Anomaly			
Poly- nomial	Coefficient value	Variance explained (%)	Co efficient value	Variance explained (%)		
P-1	-1.602	2.6	-1·238	$1 \cdot 5$		
P-2	2.146	4.6	-0.072	0.0		
P-3	0.392	0.2	0.404	$0 \cdot 2$		
P-4	0.330	0.1	0.432	$0 \cdot 2$		
P-5	5.798	33.6	-0.085	0.0		
P-6	6 • 493	42.2	7.825	$61 \cdot 2$		
P-7	-0.624	0.4	0.018	0.0		
P-8	$-2 \cdot 347$	5.5	-3.449	$11 \cdot 9$		
P-9	1.673	2.8	1.393	1.9		
P-10	-1.668	2.8	-2.658	7.1		
P-11	-0.194	0.0	-1.063	1.1		
P-12	-0.980	1.0	-0.902	0.8		
P-13	-0.372	0.1	0.771	0.6		
P-14	-0.033	0.0	0.008	0.0		
Total		95.9		86.5		

orthogonal polynomials together with the distribution of error of fit are also shown. The values of coefficients of different polynomials and their contributions towards overall representation in terms of variance explained by each are shown in Table 3. Total variance explained by the polynomial representation is about 96 per cent in case of contour height chart and 87 per cent for the anomaly chart. The four polynomials P-5, P-6, P-8, and P-2 explain about 86 per cent of variance for contour height patterns, with their individual contribution of about 34, 42, 5 and 5 per cent of variance respectively. Among the remaining polynomials, P-1, P-9 and P-10 together contribute another about 8 per cent of variance. Thus 94 per cent of variance is explained by these 7 polynomials leaving only about 2 per cent for the remaining seven.

In the case of anomaly chart maximum contribution towards variance of 61 per cent is by polynomial P-6 followed by P-8 and P-10 which contribute about 12 and 7 per cent of variance respectively. Of the remaining polynomials, P-1, P-9 and P-11 together contribute another 4.5 per cent of variance leaving only about 2 per cent of variance being contributed by rest of the eight polynomials. Thus we see that though the overall representation of these charts is in terms of 14 polynomials only a few of them are necessary for the actual representation of these particular

TABLE	4	
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Percentage of variance contributed by circulation indices included in the regression equations

June		J	uly	Au	gust	September		
Circulation index	Contributed variance (%)	Circulation index	Contributed variance (%)	Circulation index	Contributed variance (%)	Circulation index	Contributed variance (%)	
\mathbf{H}_8	43.4	A_8	42.9	A ₈	$34 \cdot 1$	He	47.9	
H_1	17.4	H_7	$8 \cdot 2$.4 2	7.5	Ha	6.7	
S_H	4.1	A 13	$4 \cdot 7$	H12	3.1	Ha	4.8	
A_{5}	$2 \cdot 4$	A_1	1.1	H_{11}	3.4	H ₇	2.7	
H14	1.5	A 12	$4 \cdot 3$	H_2	1.6	HA	3.2	
		H 2	2 3	A_4	1.4	As	1.9	
				H14	1.1	A_3	1.6	
				.4,	1.0	A_1	1.8	
						A_{0}^{2}	1.5	
Total	68.7		$63 \cdot 5$		$53 \cdot 2$		$72 \cdot 1$	

TABLE 5

Comparison between observed and calculated rainfall

Calculated			<		Obs	erved ra	infall (en	a)			
rainfall (cm)		0·0- 2·0	$2 \cdot 1 - 4 \cdot 0$	$4 \cdot 1 - 6 \cdot 0$	$\begin{array}{c} 6\cdot 1 \\ 8\cdot 0 \end{array}$	$8 \cdot 1 - 10 \cdot 0$	$10 \cdot 1 - 12 \cdot 0$	$\begin{array}{c} 12\cdot 1\\ 14\cdot 0 \end{array}$	$14 \cdot 1 - 16 \cdot 0$	$16 \cdot 1 - 18 \cdot 0$	$18 \cdot 1 - 20 \cdot 0$
				(a) JUNE						
$0 \cdot 0 - 2 \cdot 0$		73	9								
$2 \cdot 1 - 4 \cdot 0$		4	23	6	1						
$4 \cdot 1 - 6 \cdot 0$		1	2	4	2	1		1			
$6 \cdot 1 - 8 \cdot 0$				1	1						
				(1	b) JULY						
0.0-2.0		19	2	1							
$2 \cdot 1 - 4 \cdot 0$		7	18	9							
4.1-6.0		2	8	14	9	2	1				
$6 \cdot 1 - 8 \cdot 0$	e		3	8	2	4	2	1			
8.1-10.0				2	2	5	ĩ	- î	1	1	,
$10 \cdot 1 - 12 \cdot 0$					1	2	2	1	1	1	1
12.1-14.0						1	~				
				(c) AUGU	ST		νE			
$0 \cdot 0 - 2 \cdot 0$		10	3	2							
$2 \cdot 1 - 4 \cdot 0$		7	15	9	-3	1		- 1			
$4 \cdot 1 - 6 \cdot 0$			14	21	1	2	1	A			
$6 \cdot 1 - 8 \cdot 0$			1	3	9	5	4	4			
$8 \cdot 1 - 10 \cdot 0$			1	1	4	3	3	÷	1		
$10 \cdot 1 - 12 \cdot 0$						3					
$12 \cdot 1 - 14 \cdot 0$						1					
				(d) SEPTE	MBER					
0.0-2.0		53	3	2	1						
$2 \cdot 1 - 4 \cdot 0$		8	5	6	3						
4.1-6.0		1	6	6	8	2	1				
6.1-8.0		1		1	4	ĩ	1	9			
8.110.0				1	2	4	2	4	5		
10.1-12.0				1	-	1	-		2		
1500 D 1000 100											

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Actual and fitted 5-day mean 700 mb contour height and anon aly (gpm) patterns for pentad 30 June to 4 July 1964. The dotted lines in fitted chart indicate distribution of error of fit (fitted minus actual values)

contour height and anomaly patterns. A comparison between actual and fitted patterns shows that broad features of the patterns of both the charts have been well brought out by the orthogonal fitting. Sharp gradients and deep systems have however been smoothened to some extent in both the charts. The trough in actual contour height chart has central closed contour of 3040 gpm, whereas in the fitted chart it is possible to draw central contour of value 3050 gpm only. As shown by error distribution field this smoothening has resulted in concentration of error of fit over trough and ridge areas where sharp gradients prevail.

As an example of specification of 5-day rainfall from 700 mb contour height and anomaly patterns, regression equations between area mean rainfall of west Madhya Pradesh (consisting of 8 stations, namely, Khandwa, Neemuch, Nowgang, Sagar, Hoshangabad, Seoni, Indore and Bhopal) and circulation indices consisting of chart mean, standard deviation and coefficients of orthogonal

polynomials for contour height and anomaly charts were fitted. Actual 5-day area rainfall distribution was found to be skew and so was normalised before fitting regression relationships. Out of different normalising factors tried, the square root transformation was found more suitable for normalising rainfall distribution. It was not necessary to use all the circulation indices in regression equation as many of them explained very little of the total variance. For selection of effective indices screening regression technique was applied for their successive selection in order of their contribution towards explained variance and only those which could explain at least 1 per cent of the total variance were included in the final regression equations. Regression equations from June to September based on 10 years (1958 to 1967) rainfall and contour height and anomaly data of overlapping pentads are given below:

June

 $\begin{array}{r} R_5^{\frac{1}{2}} = 0\cdot 24391 {-\!\!\!-\!}0\cdot 27117 \hspace{0.2cm} H_8 {+\!\!\!-\!}0\cdot 17370 \hspace{0.2cm} H_1 {+\!\!\!\!+\!}\\ + 0\cdot 07179 \hspace{0.2cm} S_H {-\!\!\!-\!\!\!-\!}0\cdot 04019 \hspace{0.2cm} A_5 {-\!\!\!-\!\!\!-\!\!\!-\!}0\cdot 10495 \hspace{0.2cm} H_{14} \end{array}$

July

$$\begin{array}{rl} R_5^{\frac{1}{2}} &=& 1 \cdot 40830 - 0 \cdot 27844 \ A_8 \ -0 \cdot 20766 \ H_7 - \\ & 0 \cdot 04382 \ A_{13} + 0 \cdot 09429 \ A_1 + 0 \cdot 14160 \ A_{12} \ + \\ & 0 \cdot 15620 \ H_9 \end{array}$$

August

September

 $\begin{array}{l} R_5^{\frac{1}{2}} = 1\cdot 25738 + 0\cdot 10922 \ H_6 - 0\cdot 40728 \ H_3 - \\ 0\cdot 11322 H_8 - 0\cdot 12947 \ H_7 - 0\cdot 33177 \ H_4 - \\ 0\cdot 14816 \ A_8 + 0\cdot 25515 \ A_3 + 0\cdot 07223 \ A_1 - \\ 0\cdot 07783 \ A_{10} \end{array}$

In above regression equations $R_5^{\frac{1}{2}}$ stands for square root of 5-day rainfall, H_1 through H_{14} and A_1 through A_{14} stand for coefficients of contour height and anomaly polynomials respectively. M_H , M_A and S_H , S_A denote mean and standard deviation of contour height and anomaly charts.

Table 4 gives the percentage addition to the variance explained by circulation indices included in regression equations. We see that between 40 and 60 per cent of variance of square root of rainfall could be explained by first two circulation indices alone. In all 5 to 9 circulation indices are able to explain variance between 53 and 69 per cent. Additional variance explained by remaining indices was too small to justify their inclusion in regression equations.

A comparison between actually observed rainfall and the rainfall calculated from circulation indices through regression equations is given in Tables 5(a)to 5(c). The specification of rainfall is quite satisfactory even in terms of narrow rainfall intervals of 2 cm. The rainfall specified from the circulation indices agrees well for lower rainfall amounts, *i.e.*, upto about 8 cm. For higher rainfall amount, the specified rainfall is often an underestimate. This error in specification of higher rainfall amounts can perhaps be reduced to certain extent by developing a suitable scheme for graded inflation of calculated rainfall.

6. Conclusions

We see that general features of synoptic charts can be represented to a very high degree of accuracy by a linear combination of a limited number of simple patterns developed by fitting of Tschebyscheff orthogonal polynomials to the area grid network. Such representation however, smooths out sharp gradients and deep systems to some extent. A considerable amount of information about the circulation feature is contained in the coefficients of these orthogonal polynomials which can be used as useful circulation indices for developing statistical relationship between circulation patterns and weather. Only a limited number of such circulation indices from a particular level contribute effectively towards such relationships. It is therefore possible to develop regression equations between weather and circulation features at different levels and their anomalies and trends involving relatively small number of predictor variables. Such statistical relationships can be of particular importance in developing objective extended range forecasting techniques. Concurrent and lag relationships between 5-day mean 700 mb circulation patterns and the rainfall by above technique are being developed for different subdivisions of India and will be reported in due course.

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Fisher, R. A. and Yates, F.	1963	Statistical Tables for Biological, Agricultural and Medical Research, Sixth Ed., p. 33.
Friedman, D. G.	1955	J. Met., 12, 5, p. 428.
Jorgensen, D. L.	1959	Ibid., 16, 1, p. 21.
Malone, T. F.	1956	Application of Synoptic Climatology to weather prediction, Chap. 28 in Weather Analysis and Forecasting by S. Petterssen, Second Ed., Vol. II, p. 238.
Wadsworth, G. P. and Bryan, J. G.	1948	USAF, Air Weath. Service Tech. Rep. No. 105-38, p. 202.

REFERENCES

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