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Estimates of vertical velocity by numerical and analytical methods — A comparison

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ABSTRACT. Numerical computations with the complete ω-equation for quasi-geostrophic adiabatic motion have been made for comparison with earlier computations with a simplified form of ω-equation. Also an attempt has been made to check the magnitudes of vertical velocity computed by the simplified ω-equation, with the values obtained from Kuo's analytical expression for ω. The extent to which there is agreement has been discussed.

1. Introduction

While making a diagnostic study of a monsoon depression by geostrophic baroclinic model, the authors calculated the vertical velocity field which is associated with a monsoon depression, by using a simplified form of ω- equation given by —

$$\sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} \approx f_0 \frac{\partial \mathbf{V}}{\partial p} \cdot \nabla (2\zeta + f) \quad (1)$$

(Rao and Rajamani 1970). It was mentioned therein that this form was derived from the ω-equation for quasi-geostrophic adiabatic motion given by,

$$\sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} = f_0 \frac{\partial}{\partial p} [\mathbf{V} . \nabla (\zeta + f)] - \nabla^2 \left[\mathbf{V} . \nabla \left(\frac{\partial \phi}{\partial p} \right) \right] \quad (2)$$

In combining the two terms of the right hand side of Eq. (2), to get a single term on the right hand side of Eq. (1), the terms like,

$$2 \left[\frac{\partial \mathbf{V}}{\partial x} \cdot \nabla \left(\frac{\partial^2 \phi}{\partial x \partial p} \right) + \frac{\partial \mathbf{V}}{\partial y} \cdot \nabla \left(\frac{\partial^2 \phi}{\partial y \partial p} \right) \right]$$

were omitted, considering their contribution to the value of the vertical velocity might be small. In order to assess the contribution of the terms omitted it has now been attempted to calculate the vertical velocity field by making use of the complete Eq. (2) for the same situation, *i.e.*, 1200 Z of 25 July 1966. (*See* Fig. 1). Also, the magnitudes of vertical velocity computed by the simplified expression for ω have been compared with the values obtained from an analytical expression for ω.

2. Comparison of the results obtained by the simplified and the complete ω-equation

If the vertical velocity computed from the simplified Eq. (1) is denoted by ω_I and that computed from the complete Eq. (2) by ω_{II}, the difference in the values computed by the two methods, *viz.*, $\omega_{II} - \omega_I$ will represent the contribution to the vertical velocity by the terms :

$$2 \left[\frac{\partial \mathbf{V}}{\partial x} \cdot \nabla \left(\frac{\partial^2 \phi}{\partial x \partial p} \right) + \frac{\partial \mathbf{V}}{\partial y} \cdot \nabla \left(\frac{\partial^2 \phi}{\partial y \partial p} \right) \right]$$

which have been omitted, and thus represent the error in the use of the simplified form.

Vertical velocity was calculated by both the methods. The values of vertical velocity ω_I computed by the simplified form and the difference $\omega_{II} - \omega_I$ in the values computed by the two methods were evaluated in respect of the isobaric surfaces 850, 700, 500 and 300 mb. But these quantities in respect of 700 mb and 300 mb only are presented in Figs. 2 (a) to 2(d), as 700 mb and 300-mb surfaces represent the lower and upper tropospheric conditions respectively. From Figs. 2(b) and 2(d) it is seen that the difference $\omega_{II} - \omega_I$ which is of the order of 2×10^{-4} mb/sec or 2 mm/sec is small. At the grid point 18°N, 80°E where the computed upward vertical velocity has the maximum value of $24 \cdot 1 \times 10^{-4}$ mb/sec at 700 mb, the percentage value of this difference $\omega_{II} - \omega_I$ is about 4 only, whereas the percentage difference becomes large towards the peripheral points. The term :

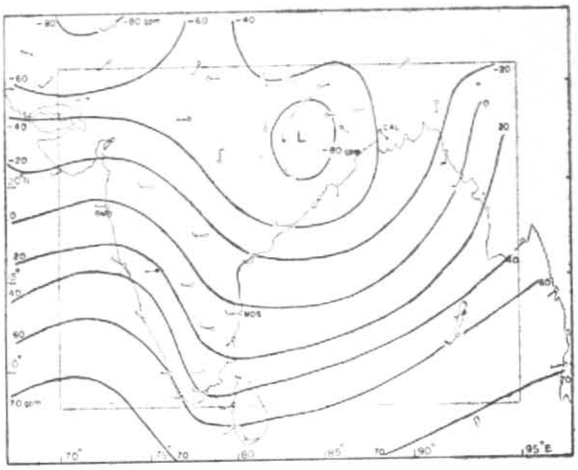

Fig. 1(a)

1000 mb chart at 1200 GMT on 25 July 1966

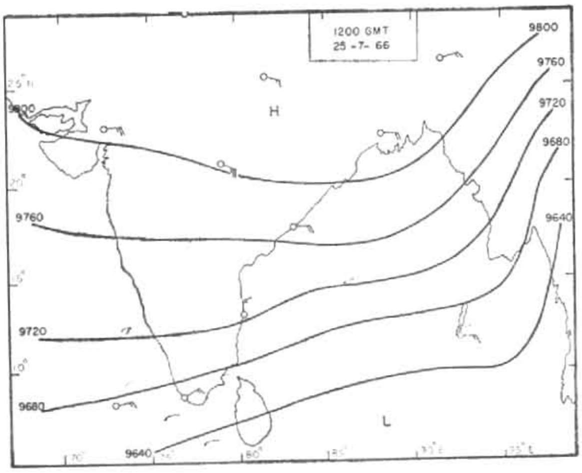

Fig. 1(b)

300 mb chart at 1200 GMT on 25 July 1966

TABLE 1

Values of Γ_p and α at different pressure surfaces

Parameter	850 mb	700 mb	500 mb	300 mb
Γ_p (°A cb^{-1})	—0·43	—0·60	—0·71	—0·74
α*	12·33	17·03	20·31	20·97

*nondimensional quantity

TABLE 2

Maximum upward and downward vertical velocities at 700-mb level obtained by the three models ω in mm/sec

Method	Maximum vertical velocity	
	upward	downward
ω_I	24·1	15·5
ω_{II}	23·2	17·9
ω_k (Kuo's Method)	22·9	22·6

$$2\left[\frac{\partial \mathbf{V}}{\partial x}\cdot\nabla\left(\frac{\partial^2\phi}{\partial x\partial p}\right)+\frac{\partial \mathbf{V}}{\partial y}\cdot\nabla\left(\frac{\partial^2\phi}{\partial y\partial p}\right)\right]$$

can be shown to be,

$$\frac{2g}{f\triangle p}\left[\frac{\partial^2 h}{\partial x\partial y}\left(\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}\right)-\right.$$
$$\left.-\frac{\partial^2\phi}{\partial x\partial y}\left(\frac{\partial^2 h}{\partial x^2}-\frac{\partial^2 h}{\partial y^2}\right)\right]$$

where h denotes the thickness of an isobaric layer and it represents an effect of the deformation of the two fields ϕ and h. As pointed out by Eliassen (1956) it is difficult to interpret this term. According to Eq. (1) large values of vertical velocity are associated with large values of the gradient of vorticity. In regions where vorticity $[(\partial v/\partial x) - (\partial u/\partial y)]$ values are large, the shear deformation $[(\partial v/\partial x) + (\partial u/\partial y)]$ is likely to be small, and the effect of the above expression is also likely to be small. However, where vorticity values are small, the shear deformation may be large, and the difference $\omega_{II} - \omega_{I}$ representing the effect of the deformation of height is greater. It may be concluded that the terms omitted do not contribute significantly, expecially at grid points where the values of the vertical velocity ω_I are large. Further, computation by the complete Eq. (2) takes up $1\frac{1}{2}$ hours with IBM 1620, while computation with the simplified form takes 30 minutes only. In view of this, it is felt that the use of the simplified equation will suffice for diagnostic purposes, when the computor facilities are limited, commensurate with the errors in observation, analysis and numerical methods of computation.

3. Comparison with Kuo's model

Although the region of upward vertical velocities computed with the simplified ω-equation agreed well with the region of precipitation in association with the depression, it was considered desirable to check the magnitudes of vertical velocity computed by the simplified expression, with the values that can be obtained from some analytical expression for ω. Kuo (1953) had obtained an analytical expression from the same set of equations, *viz.*, the vorticity and the thermodynamic equations for quasi-geostrophic motion, although they were linearised. In deriving this expression Kuo considered the motion which is composed of a basic current that increases linearly with decreasing pressure, but is uniform in the horizontal directions, and a wave disturbance. Further he assumed ω to be zero at the ground and at the top of the atmosphere and represented ω by the function, $p\ (p-p_0)\ M\ (x,\ y,\ p,\ t)$ where, M varies slowly with p.

The expression obtained by Kuo is :

$$\omega=\frac{k^2}{2f_o}\ \frac{p(p_o-p)}{1+2\,(\alpha/p_o)(p_o-p)}\ \times$$
$$\times\ \left[2v\ \frac{\partial u}{\partial p}-\frac{\beta}{k^2}\ \frac{\partial v}{\partial p}\right] \qquad (3)$$

where,

$$\left.\begin{aligned}\alpha &= -\,(k^2 R\Gamma_p\ /\ 4f_o^2)\ p_o\\ \Gamma_p &= \frac{\Gamma}{\theta}\ \frac{\partial\theta}{\partial p}\\ k^2 &= 4\pi^2/L^2,\ \beta=\partial f/\partial y\end{aligned}\right\} \qquad (4)$$

and the other symbols have their usual meanings.

For any pressure surface p, the factor before the bracket on the right side of expression (3) becomes a constant, so that ω at the pressure surface p is proportional to the two terms in the bracket. For short and moderate waves (having wavelength less than 6000 km) the second term in the bracket is generally small, compared with the first term. In the monsoon case as $\partial u/\partial p$ is positive, upward motion (ω negative), can be expected only in regions where v is negative, *i.e.*, where there is northerly component of wind. Thus in a monsoon situation, upward motion can take place only in the region from the wedge line to the trough line.

For the synoptic situation, (Fig. 1) a wavelength of 2000 km was assumed and value of Γ_p and α from 850, 700, 500 and 300 mb are given in Table 1. The values of the vertical velocity, denoted by ω_k have been evaluated at 850 700, 500 and 300 mb surfaces. The extreme values of the vertical velocity at 700 mb surface computed by the three met ods are given in Table 2. It may be stated that all the three methods give the maximum upward vertical velocity at the same grid point, *viz.*, 18°N, 80°E and the maximum downward vertical velocity also at the same grid point, *viz.*, 18°N, 86°E. As far as the upward motion at grid point 18°N, 80°E is concerned, there is very close agreement, while for downward motion at 18°N, 86°E the agreement in the values computed by the simplified equation and the complete equation is good, but Kuo's model gives a higher value. It is to be kept in mind that values of vertical velocity computed by simplified or complete ω-equation are obtained by three-dimensional relaxation, while in the case of Kuo's model, the values are obtained

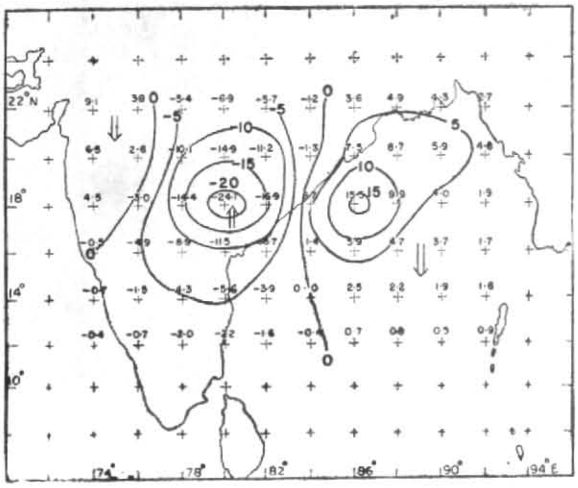

Fig. 2 (a). 700 mb

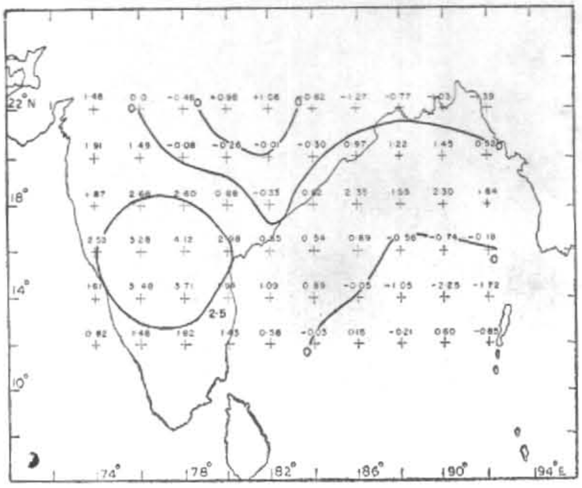

Fig. 2(b). 700 mb

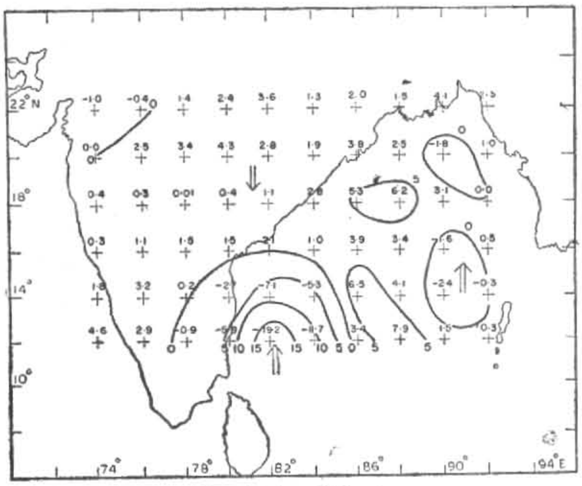

Fig. 2(c). 300 mb

Fig. 2(a) and 2(c). ω_I vertical velocity at 700 and 300 mb levels at 1200 GMT on 25 July 1966 (unit : 10^{-4} mb/sec)

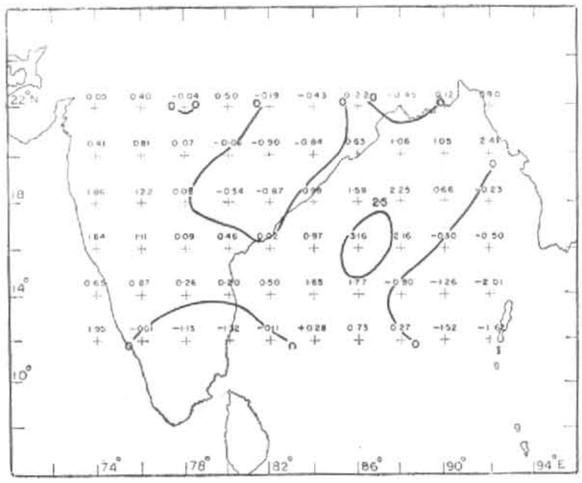

Fig. 2(d). 300 mb

Fig. 2(b) and 2(d). $\omega_{II}-\omega_I$ difference in vertical velocities computed by the two methods at 700 and 300 mb levels (units : 10^{-4} mb/sec)

as spot values, without being smoothened in the horizontal or in the vertical.

The vertical velocities computed from Kuo's expression for 700 mb and 300-mb surfaces are given in Figs. 3 (a) and 3 (c). On a comparison of Figs. 3 (a) and 3 (c) with 2(a) and 2(c), it is seen that there is a close agreement in the pattern and also in the order of magnitude of the values as far as 700 mb surface is concerned, but the agreement is not good in the case of 300-mb surface.

Figs. 3 (b) and 3 (d) give the difference $\omega_k-\omega_I$ in the values computed by Kuo's expression and the simplified ω-equation. In Fig. 3 (b) the difference is of the order of 5×10^{-4} mb/sec over the land region, while the differences are larger in the Bay regions. In Fig. 3(d) also, the difference is small over the land region increasing to large values in the Bay region. The values computed by the two methods are not strictly comparable as the values of the vertical velocity computed by the ω-equation are smoothed values obtained by three-dimensional relaxation, while values computed from Kuo's analytical expression are unsmoothed spot values. The reason for large values of difference at 300 mb surface may be due to the fact that, in this particular snyoptic situation, the wave pattern in the westerlies at 500-mb level and below, did not extend to 300-mb level

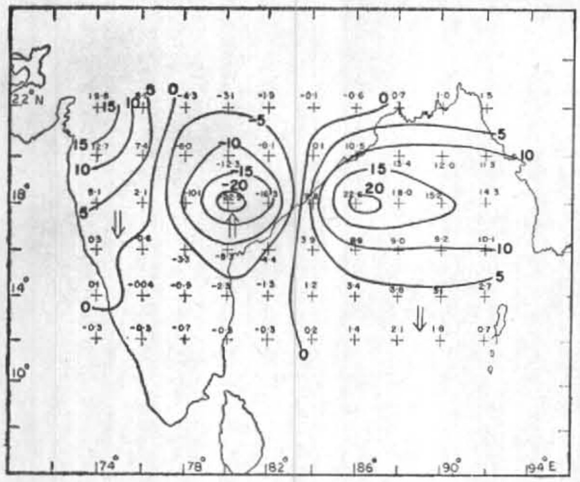

Fig. 3(a). 700 mb

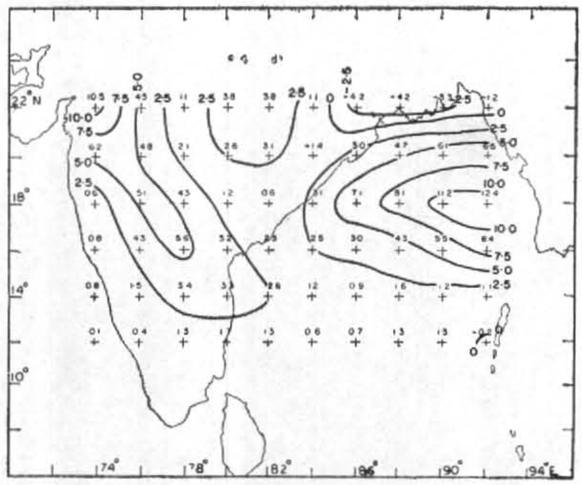

Fig. 3(b). 700 mb

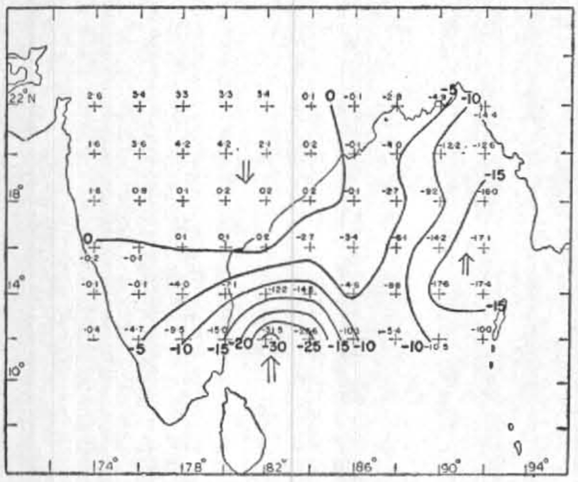

Fig. 3(c). 300 mb

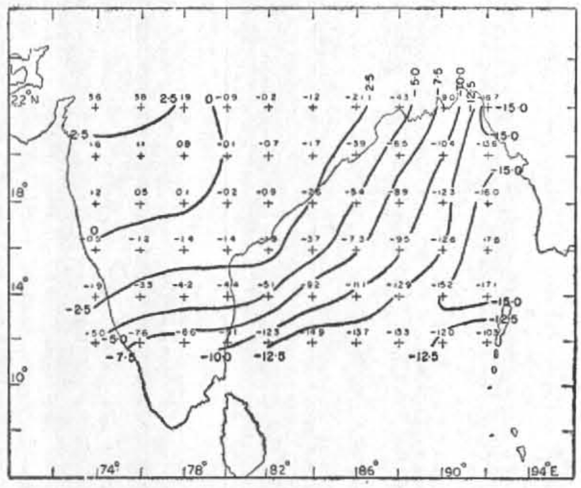

Fig. 3(d). 300 mb

Figs. 3(a) and 3(c). ω_k vertical velocity at 700 mb and 300 mb levels at 1200 GMT on 25 July 1966 — Kuo's method (units: 10^{-4} mb/sec)

Figs. 3(b) and 3(d). $\omega_k - \omega_l$ Difference in vertical velocities computed from Kuo's expression and from simplified ω-equation at 700 and 300 mb (units : 10^{-4} mb/sec)

and the assumption of the existence of wave pattern and also a wavelength of 2000 km appropriate at 1000-mb level cannot, therefore, be valid for 300 mb.

4. Conclusion

(1) This study has shown that the use of the simplified equation will suffice for diagnostic purposes.

(2) Since Kuo's expression for ω gives estimates of ω comparable with the values obtained by relaxation of the differential equation for ω and since it does not involve much time and labour to calculate spot values, it can be used to evaluate vertical velocities at 850 mb or 700 mb for qualitative purposes.

(3) Since Kuo's model gives fairly reliable values of ω, the extent to which the other aspect of Kuo's theory will be applicable to discuss the stability properties and structure of monsoon depressions requires examination.

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