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Mean Meridional air-motion and associated transport processes in the atmosphere

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ABSTRACT. Eliassen's equation of quasi-static balance has been re-derived in a more suitable frame of co-ordinates and, by making certain simplifying assumptions, rendered into a form which lends itself easily to solution by relaxation. The differential equation thus obtained is solved for extended sources of heat and momentum similar to those actually encountered in the troposphere. The results show a mean meridional circulation consisting of a strong direct cell in low latitudes and a weak indirect cell in middle latitudes.

In Section 3 of the paper, starting from the idea that the mean meridional circulation is split up into cells and making use of the values of the strength of the circulation deduced by Tucker and Mintz, the sources and sinks of heat and momentum necessary to maintain such mean motion are evaluated. Further, an attempt is made to infer what the character of eddy motion in the meridional plane has to be in order to satisfy balance requirements.

1. Introduction

In the atmosphere we have a self-maintaining system which is accomplishing transport of momentum and heat in a rather interesting manner. Westerly angular momentum is continually picked up from the surface of the earth in low latitudes and transferred to the earth in middle latitudes, while the momentum at any particular place (considering the temporal mean over long periods) remains sensibly constant. Similarly, heat is transported from low levels and latitudes to high levels and latitudes in such a manner that heat balance is maintained everywhere, *i.e.*, the loss of heat due to radiation is exactly made good by the release of latent heat and transport of sensible heat due to air motion. (Here, air motion includes any mean circulation as well as eddy motion, and any conversion of potential energy into thermal energy or *vice versa* by vertical motion is taken into account). The general circulation of the atmosphere has, thus, to satisfy certain integral requirements which may be formulated in terms of the balances not only of angular momentum and energy, but also of mass, water content and other properties.

A simplified explanation of the working of the atmospheric system which finds more or less general acceptance at the present time is as follows. A consideration of the radiation field of the earth and atmosphere as a system leads us to suppose that air tends to move upwards and polewards from regions near the equator. It may further be inferred that such motion in the meridional plane tends to cease when the thermal wind equation is satisfied. But steady baroclinic flow in circumpolar vortices is unstable, and results, as has been shown by Eady (1949), in turbulent motions in planes having roughly half the slope of the temporarily-existing broad-scale isentropic surfaces. This large-scale turbulence which has a horizontal wave-length of about 2000 km and corresponds to the weather systems of middle latitudes, transfers potentially warm air upwards and polewards and potentially cold air downwards and equatorwards. Further, as the axes of the troughs and ridges are bow-shaped, trailing in middle latitudes in a NE-SW direction (as can be inferred from the theoretical condition for persisting structure and maximum amplitude as well as from actual observation), there arises a positive correlation between east and north components of velocity, resulting in a poleward transfer of angular momentum.

The amplitude of these disturbances increases with height up to the tropopause, above which there is heavy damping due to increased static stability and high value of Richardson's number—a deduction which is borne out by observations such as those of Brooks (1950) and Starr (1951). Therefore, the bulk of transfer of angular momentum takes place at the high levels close to the tropopause. The angular momentum thus accumulating in the upper levels in the middle and high latitudes has to come down to the surface where it is destroyed by skin-friction. The momentum is so brought down either wholly by convectional eddies or partly by meridional circulation and partly by convectional eddies.

In low latitudes, it is fairly well established that there is a direct cell of meridional circulation. The pronounced cross-isobaric flow towards low pressure which can be observed near the surface (together with the compensating flow at high levels in the opposite direction necessary because of continuity) is enough evidence for such a circulation. It is not so easy to draw any similar inference in respect of the middle and high latitudes. While one school of meteorological thought (Mintz and others) holds that there is an indirect (Ferrel) cell in middle latitudes, another (Sheppard and others) maintains that there is no such indirect circulation.

The processes of transfer of heat and momentum described above provide perhaps an over-simplified picture of the general way in which the atmospheric system works. A closer examination reveals that there are a number of feed-back mechanisms involved, and that many details are as yet very poorly understood indeed.

The entire problem of the general circulation makes a really fascinating subject of study. In the following two sections, the simple mechanism outlined above is taken to be generally correct, and further investigation of the problem undertaken—the investigation being limited to a study of the problem from two different angles.

2. Modification and solution of Eliassen's equation

No satisfactory dynamical theory of the general circulation has been developed up to the present time. However, in recent years the problem has received considerable attention from a number of workers, chief among whom are Eliassen (1952), Rogers (1954), Phillips (1956) and Fleagle (1957). Each of these authors examines the question from some particular point of view, making a number of assumptions.

Up to the point where he arrives at an equation of quasi-static balance, Eliassen's treatment of the problem is general and involves few assumptions. As the investigation in this section is based upon an adapted form of Eliassen's equation, a very brief review of Eliassen's approach is given in the next paragraph.

Eliassen begins by considering symmetrical circular vortex motion in a gravity field.

He then proceeds to determine the effect of introducing sources of heat and momentum, assuming that the sources are so weak that the vortex will remain close to the state of balance at all times. With this quasi-static approximation, Eliassen arrives at an expression of the form :

$$\frac{\partial}{\partial R}\left\{ A\left(\frac{\partial \eta}{\partial R}\right)_p + B\left(\frac{\partial \eta}{\partial p}\right)_R \right\} + \frac{\partial}{\partial p}\left\{ B\left(\frac{\partial \eta}{\partial R}\right)_p + C\left(\frac{\partial \eta}{\partial p}\right)_R \right\} = \frac{\partial H}{\partial R} + \frac{\partial M}{\partial p}$$

where R is the distance from the earth's axis, η is a stream function defined by

$$\dot{R} = g\,\frac{\sin\phi}{R}\,\frac{\partial \eta}{\partial p} \text{ and}$$

$$\dot{p} = -g\,\frac{\sin\phi}{R}\,\frac{\partial \eta}{\partial R}$$

H and M are quantities proportional respectively to rates of supply of heat and momentum to unit mass of fluid, and the coefficients A, B and C have values

$$A = \frac{g\sin\phi}{\rho C_p R}\left(\frac{\partial S}{\partial p}\right)_R$$

$$B = \frac{-g\sin\phi}{R^4}\left(\frac{\partial c^2}{\partial p}\right)_R$$

$$\text{and } C = \frac{g\sin\phi}{R^4}\left(\frac{\partial c^2}{\partial R}\right)_p$$

Further details may be seen in the original paper. Eliassen does not go on to general solutions of this equation, but merely discusses the character of the resulting meridional motion in the vicinity of point sources of heat and momentum and deduces that meridional motions occur in the direction of weakest stability.

It appears likely that Eliassen's differential equation would yield useful and interesting results if solved by some method such as relaxation, not for just point sources as Eliassen has done but for extended sources of heat and momentum as are actually encountered in the troposphere. But this equation is expressed in R-p co-ordinates—a system in which the axes are not merely orthogonal but are at a variable angle to each other—and as such, is hard to solve by relaxation.

It is, however, possible to re-derive the quasi-static relationship in a more suitable frame of co-ordinates and to simplify it considerably by ignoring the geometry of the earth, in the following manner.

Choosing x-axis towards the east, y-axis to the north and z-axis vertical, let us investigate the mean meridional motion (*i.e.*, motion in the yz-plane).

The zonal component of the equation of motion when averaged over all longitudes yields

$$\frac{Du}{Dt} = \frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = fv + \dot{M}_1$$

where $\dot{M}_1$ is the rate at which momentum is being supplied to unit mass from any source. It is assumed that $\dot{M}_1$ includes the turbulence terms $\widetilde{v'_1\partial u'/\partial y}$ and $\widetilde{w'\partial u'/\partial z}$, and near the surface the residual value of $\widetilde{-\frac{1}{\rho}\frac{\partial p}{\partial x}}$ where it adds to skin-friction.* (Even when the curvature of the earth's surface is taken into account substantially the same equation is arrived at. In Appendix I, the relative order of magnitude of the various terms in the equation of motion in spherical polar co-ordinates is briefly discussed).

Therefore, we have,

$$\frac{\partial u}{\partial t} = \dot{M}_1 + fv - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} \qquad (1)$$

It is convenient to consider the effect of heat on meridional motion in terms of the quantity ϕ defined by $\phi = S/C_p = \log\theta$ (where S is entropy and θ potential temperature). If $\dot{Q}_1$ be the rate at which heat is

*Here and everywhere else in this paper, the upper wavy bar denotes longitude mean. A straight upper bar means time average. The bars are not used when the meaning is otherwise clear.

supplied at some source (per unit mass), we have,

$$\frac{\dot{Q}_1}{TC_p} = \frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + v\frac{\partial\phi}{\partial y} + w\frac{\partial\phi}{\partial z}$$

(T being absolute temperature)
Therefore,

$$\frac{\partial\phi}{\partial t} = \frac{\dot{Q}_1}{TC_p} - v\frac{\partial\phi}{\partial y} - w\frac{\partial\phi}{\partial z} \qquad (2)$$

We shall now assume that the sources are so weak that the thermal wind equation still holds—this corresponds to Eliassen's quasi-static approximation. We may write the thermal wind equation as (see Appendix II):

$$\frac{\partial u}{\partial z} = -\frac{g}{f}\frac{\partial\phi}{\partial y} \qquad (3)$$

Differentiating (1) with respect to z and (2) with respect to y, we obtain

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial z}\right) = \frac{\partial\dot{M}_1}{\partial z} + \frac{\partial(fv)}{\partial z} - \frac{\partial}{\partial z}\left(v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\dot{Q}_1}{TC_p}\right) - \frac{\partial}{\partial y}\left(v\frac{\partial\phi}{\partial y} + w\frac{\partial\phi}{\partial z}\right)$$

Making use of relation (3) we see that

$$\frac{\partial\dot{M}_1}{\partial z} + \frac{\partial}{\partial z}\left\{\left(f - \frac{\partial u}{\partial y}\right)v\right\} - \frac{\partial}{\partial z}\left(w\frac{\partial u}{\partial z}\right) + \frac{g}{f}\frac{\partial}{\partial y}\left(\frac{\dot{Q}_1}{TC_p}\right) - \frac{g}{f}\frac{\partial}{\partial y}\left(v\frac{\partial\phi}{\partial y}\right) - \frac{g}{f}\frac{\partial}{\partial y}\left(w\frac{\partial\phi}{\partial z}\right) = 0$$

We may now bring in the equation of continuity, assuming as a first approximation that the fluid is incompressible. This enables us to replace v and w by $-\partial\psi/\partial z$ and $+\partial\psi/\partial y$ and rewrite the quasi-static balance equation in terms of the stream function ψ approximately as

$$\frac{\partial\dot{M}_1}{\partial z} + \frac{g}{f}\frac{\partial}{\partial y}\left(\frac{\dot{Q}_1}{TC_p}\right) - \left(f - \frac{\partial u}{\partial y}\right)\frac{\partial^2\psi}{\partial z^2} - \left(\frac{\partial u}{\partial z} - \frac{g}{f}\frac{\partial\phi}{\partial y}\right)\frac{\partial^2\psi}{\partial y\partial z} - \frac{g}{f}\frac{\partial\phi}{\partial z}\frac{\partial^2\psi}{\partial y^2} = 0$$

$$i.e., \quad \frac{\partial\dot{M}_1}{\partial z} + \frac{g}{f}\frac{\partial\dot{\phi}_1}{\partial y} = A\frac{\partial^2\psi}{\partial y^2} + B\frac{\partial^2\psi}{\partial y\partial z} + C\frac{\partial^2\psi}{\partial z^2}$$

Where $\dot{\phi}_1 = \dfrac{\dot{Q}_1}{TC_p}$, $A = \dfrac{g}{f}\dfrac{\partial\phi}{\partial z}$,

$$B = \frac{\partial u}{\partial z} - \frac{g}{f}\frac{\partial\phi}{\partial y}$$

and $C = f - \dfrac{\partial u}{\partial y}$

This relationship is now in a form more convenient for solution by relaxation methods. It is, however, desirable to modify the equation by suitable changes in axes so as (a) to make the coefficients of $\partial^2\psi/\partial y^2$ and $\partial^2\psi/\partial z^2$ nearly equal and (b) to make the coefficient of $\partial^2\psi/\partial y\partial z$ vanish, with a view to minimising the labour in the relaxation process.

We may first stretch the z-axis in the ratio

$$\sqrt{\frac{g}{f}\frac{\partial\phi}{\partial z}\Big/\left(f - \frac{\partial u}{\partial y}\right)}$$

by means of the transformation $z = Z/\sqrt{A/C}$

The approximate magnitudes of the coefficients are as below

$$A = \frac{g}{f} \frac{\partial\phi}{\partial z} \left\{ \begin{array}{l} \text{Under ICAN conditions at sea level (1013 mb)} \\ T = 288°\text{A and} \\ \theta = 287°\text{A} \\ \text{At 15 km (120 mb)} \\ T = 216 \cdot 5°\text{A and} \\ \theta = 399°\text{A; so that} \\ \frac{\partial\phi}{\partial z} = \frac{\log_e (399/287)}{15 \times 10^5} \end{array} \right\}$$

$$= \frac{981}{1 \cdot 45 \times 10^{-4} \sin \text{Lat.}} \times 2 \cdot 2 \times 10^{-7}$$

$$\sim 2 \text{ per sec}$$

$$B = 2 \frac{\partial u}{\partial z} \sim 2 \frac{17 \cdot 25 \text{ m/sec}}{11 \cdot 5 \text{ km}}$$

$$= 3 \times 10^{-3} \text{ per sec}$$

$$C \simeq f = 1 \cdot 45 \times 10^{-4} \sin \text{Lat.}$$

$$\sim 10^{-4} \text{ per sec}$$

We may, therefore, use the transformation $z = Z/100\sqrt{2}$ which changes the equation to

$$\frac{\partial \dot{M}_1}{\partial z} + \frac{g}{f} \frac{\partial \dot{\phi}_1}{\partial y} = 2 \frac{\partial^2\psi}{\partial y^2} + 3\sqrt{2} \times \times 10^{-1} \frac{\partial^2\psi}{\partial y \partial Z} + 2 \frac{\partial^2\psi}{\partial Z^2}$$

(It may be noted that no transformation has been effected on the left hand side, $\partial \dot{M}_1/\partial z$ remaining as before).

Next, we could get rid of the cross-differential term by a suitable rotation of the axes. When the axes are rotated by an angle θ, an expression like

$$a \frac{\partial^2\psi}{\partial y^2} + b \frac{\partial^2\psi}{\partial y \partial Z} + c \frac{\partial^2\psi}{\partial Z^2} \text{ becomes}$$

$$\frac{\partial^2\psi}{\partial y'^2} (a \cos^2\theta + c \sin^2\theta + b \sin\theta \cos\theta) + \frac{\partial^2\psi}{\partial y' \partial Z'} (-2a \sin\theta \cos\theta + 2c \sin\theta \cos\theta - b \cos^2\theta - \sin^2\theta) + \frac{\partial^2\psi}{\partial Z'^2} (a \sin^2\theta + c \cos^2\theta - b \sin\theta \cos\theta)$$

For the vanishing of the cross-differential we require

$$b \cos 2\theta = (a - c) \sin 2\theta$$

$$\text{or } \theta = \tfrac{1}{2} \tan^{-1} [b/(a-c)]$$

In the present case we need to rotate the axes by $\pi/4$. This transforms the expression

$$\frac{\partial^2\psi}{\partial y^2} + 0 \cdot 21 \frac{\partial^2\psi}{\partial y \partial Z} + \frac{\partial^2\psi}{\partial Z^2} \text{ into}$$

$$\left(1 + \frac{0 \cdot 21}{2}\right) \frac{\partial^2\psi}{\partial y'^2} + \left(1 - \frac{0 \cdot 21}{2}\right) \frac{\partial^2\psi}{\partial Z'^2}$$

Thus the equation of quasi-static balance becomes

$$1 \cdot 105 \frac{\partial^2\psi}{\partial y'^2} + 0 \cdot 895 \frac{\partial^2\psi}{\partial Z'^2} = \tfrac{1}{2} \left(\frac{\partial \dot{M}_1}{\partial z} + \frac{g}{f} \frac{\partial \dot{\phi}_1}{\partial y} \right)$$

This equation can now be relaxed with the finite difference formula

$$1 \cdot 105 \frac{\psi_1 + \psi_3 - 2\psi_0}{h^2} + 0 \cdot 895 \frac{\psi_2 + \psi_4 - 2\psi_0}{h^2} = \tfrac{1}{2} \left(\frac{\partial \dot{M}_1}{\partial z} + \frac{g}{f} \frac{\partial \dot{\phi}_1}{\partial y} \right) = R$$

i.e., $1 \cdot 105(\psi_1 + \psi_3) + 0 \cdot 895(\psi_2 + \psi_4) - 4\psi_0 = h^2 R$

2.1. *Boundary conditions*—The boundary conditions that may be assumed are the following. There is no motion at the ground perpendicular to the surface, so that ψ= constant, which may without restriction be set=0, at the ground. Similarly it may be assumed that ψ=0 at the vertical through the pole, the vertical plane through the

TABLE 1

Components of the heat source

Latitude	Rate of rise of temp. (°C/day)			$\frac{g}{f}\frac{\partial\dot{\phi}_1}{\partial y}$ 10^{-10} sec^{-2}
	Latent heat	Radiation cooling	Conv. of eddy flux	
0	1·2	—1·0	0	
5				—93
10	0·9	—1·0	—0·1	
15				—31
20	0·5	—1·0	—0·1	
25				—18
30	0·5	—0·9	—0·5	
35				+10
40	0·6	—0·9	—0·4	
45				+12
50	0·5	—0·9	0·1	
55				+07
60	0·3	—0·9	0·6	
65				+08
70	0·2	—0·9	1·1	
75				—06
80	0·1	—0·9	0·9	
85				—05
90	0	—0·8	0·6	

equator (inter-hemispheric air-motion being supposed to be 0 when averaged over a long period) and also at some high level which corresponds to the tropopause or some higher level.

2.2. *Heat source*—Although the radiation field of the earth and atmosphere as a unit leads us to suppose a source of heat at low levels near the equator and a sink at high levels in the polar regions, the proper way of determining the sources and sinks of heat responsible for air motion in the troposphere would be to consider the release of latent heat at every point and subtract from it the net radiational loss at that point together with the loss due to any evaporation, making further allowance for conduction where it may be significant, *i.e.*, near the earth's surface. For estimating mean meridional circulations, we have also to add the eddy supply of sensible heat. Reliable data of the spatial distribution of the quantities involved are not available. However, an attempt may be made to estimate the rough values of $\frac{g}{f}\frac{\partial\dot{\phi}_1}{\partial y}$ needed for the solution of our equation, by making various assumptions and approximations in the following manner.

From the zonal distribution of precipitation given by Haurwitz and Austin (1944) after the data of Meinardus and by Conrad (1942) it is possible to estimate the mean rate of heating in various zones of latitudes. If R_ϕ be the precipitation per day in cm at latitude ϕ the mean heating rate is nearly $\frac{R_\phi L g}{1013 \times 1000\, C_p}$ per day, L being latent heat at the temperature of clouds (Table 1. Col.1).

The distribution of net radiational cooling in the troposphere under conditions of average cloudiness has been presented by London (1952). Averaging London's figures in the vertical, with respect to pressure, the mean radiational cooling in different zones of latitude could be obtained (Table 1, Col. 2).

The poleward transport by eddies of sensible heat has been computed by Mintz (1955a) and by Starr and White (1955); from the convergence of this flux the rate of heating in any latitude zone can be derived (Table 1, Col. 3).

We shall now consider the heating of the latitude belts due to the convergence of upward eddy flux of heat. Since the eddy flux at the top of the atmosphere is zero, the heat gained due to eddy conduction at the surface only need be considered. The latitudinal distribution of the quantity of heat exchanged between the sea and the atmosphere is given by Jacobs (1951). From an examination of this data, it will be realised that the difference between the maximum heat exchange at about 40° N (of 40 gm cal cm²/day equivalent to 0·16° C/day) and the minimum at 0° (of 10 gm cal cm²/day or 0·04° C/day) is so small that the contribution to $\frac{g}{f}\frac{\partial\dot{\phi}_1}{\partial y}$ can be neglected.

Apart from the mean meridional circulation itself, there are no other factors that can make a material contribution to the change of temperature of each zonal ring. The heat for surface evaporation is assumed to be extracted mainly from the sea, and molecular conduction is ignored.

As a first step we shall investigate only the mean motion that results from introducing the mean $(g/f)\ (\partial\dot{\phi}_1/\partial y)$ at just one level at the middle of the troposphere, between 500 and 600 mb. The net rate of heating or cooling per day is easily converted to $\dot{\phi}_1$. As $\phi = \log\ \theta,\ \dot{\phi}_1 = \dot{\theta}/\theta = \dot{T}/T$. Therefore ΔT per day$=T\dot{\phi}_1 \times 8{\cdot}64\times10^4$. Fig. 1 shows the smoothed values of $\frac{g}{f}\ \frac{\partial\dot{\phi}_1}{\partial y}$.

2.3. *Momentum source*—In low latitudes, while westerly momentum is being injected into the atmosphere at low levels, large scale eddies are removing such momentum in the upper levels of the troposphere. Thus we can form a general picture of a source beneath a sink in the region of surface easterlies, and similarly a sink beneath a source in the region of surface westerlies. The latitudinal variation of the source at low levels can be obtained from the values of τ_j, the surface friction stress plus mountain stress at different latitudes from estimates made by Priestly (1951), Mintz (1955 a) and White (1949).

For the solution of our equation we need to estimate $\partial\dot{M}_1/\partial z$. What concerns us primarily is the latitudinal variation of $\partial M_1/\partial z$, and, therefore, we can afford to make some crude assumption in arriving at $\dot{M}_1/\partial z$ in each zonal belt, the same sort of assumption applying to the various belts. It will be assumed that in the vertical the distribution of sources and sinks are roughly as indicated schematically in Fig. 2, for low latitudes—the inflow of momentum at the surface giving rise to a more or less uniform source up to 700 mb (3 km), and eddies creating a uniform sink above 450 mb (6·3 km). In the region of surface westerlies, the vertical distribution assumed is similar with the signs reversed. With such assumption, making use of the known values of τ_j extrapolated upto 90° N in conformity with the surface wind, (Fig. 3), the corresponding values of $\partial\ \dot{M}_1/\partial z$ at different latitudes at about 4·7-km level are computed and presented in Fig. 4.

2.4. *Relaxation Tables*—The grid chosen for relaxation has an interval of 3° latitude (333 km) between successive grid points. This automatically fixes the interval of vertical height—remembering that the z-axis has been stretched $100\sqrt{2}$ times—at 2.35 km. We thus obtain a net-work of 105 nodal points, inside a rectangular boundary on which ψ is set at 0.

Relaxation has been carried out separately for heat source alone, momentum source alone and combined heat and momentum sources, and the results are presented in Figs. 5, 6 and 7 respectively. In these figures, the numbers in brackets represent $R\times10^{10}$, while the other numbers represent values of ψ' (where $\psi'=\psi\times10^{10}/h^2$ cm²/sec, h being $333\times10^5\sqrt{2}$ cm).

Isopleths of ψ' are drawn at intervals of $\psi'=10$. These represent stream lines, the direction and speed of flow at any point which could be determined from the signs and magnitudes of the velocity components

$$v = -\ \frac{\partial\psi}{\partial z}\ \text{and}\ w = +\ \frac{\partial\psi}{\partial y}$$

2.5. *Discussion of results*—The results show that meridional circulation is split up into cells—a strong direct circulation cell in low latitudes and a weak indirect cell in the middle latitudes. The very weak direct cell in high latitudes will be left out of this discussion mainly because it is the result of extrapolated values of τ_j, and secondly because the accuracy of our equation, which ignores the geometry of the earth, is questionable in very high latitudes.

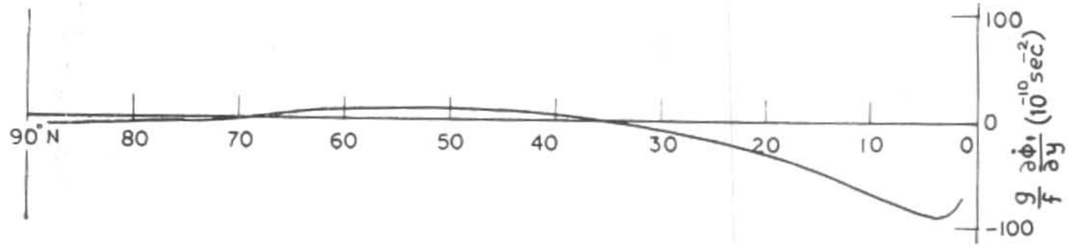

Fig. 1

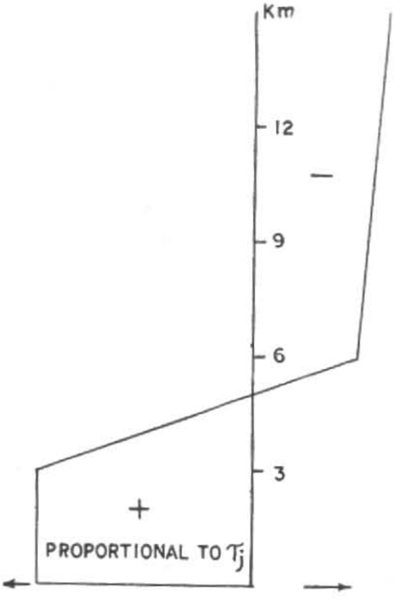

Fig. 2. Assumed schematic distribution in the vertical of momentum sources

(The above figure is for low latitudes. In the region of surface westerlies the signs are reversed)

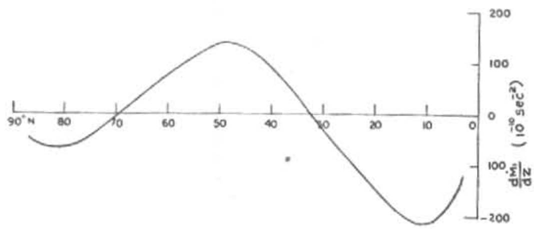

Fig. 4. Corresponding $\frac{\partial \dot{M}_1}{\partial z}$ (at 4·7 km level)

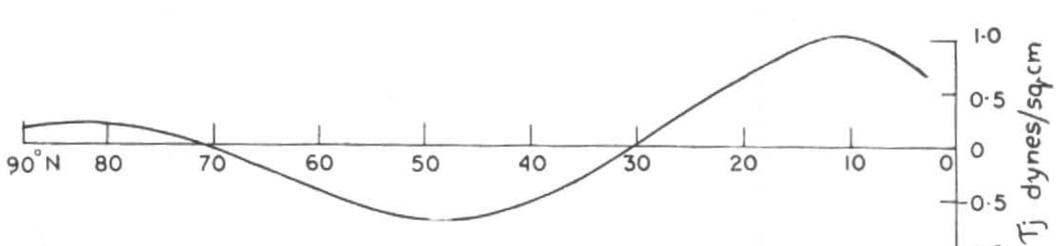

Fig. 3. Annual mean values of τ_j

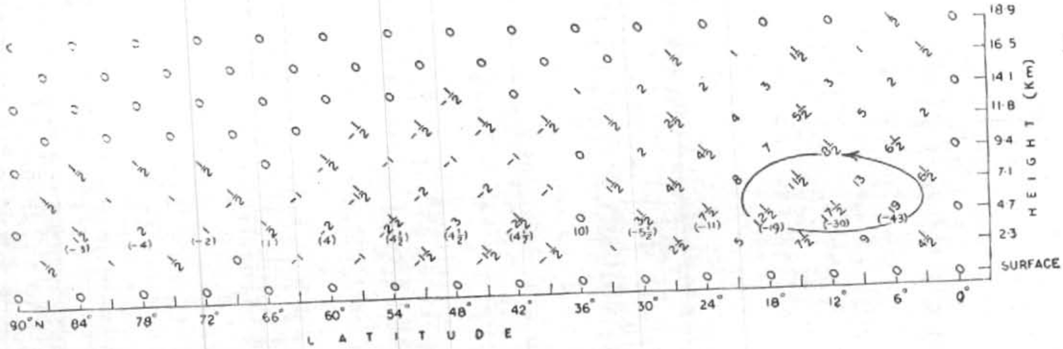

Fig. 5. Heat source alone

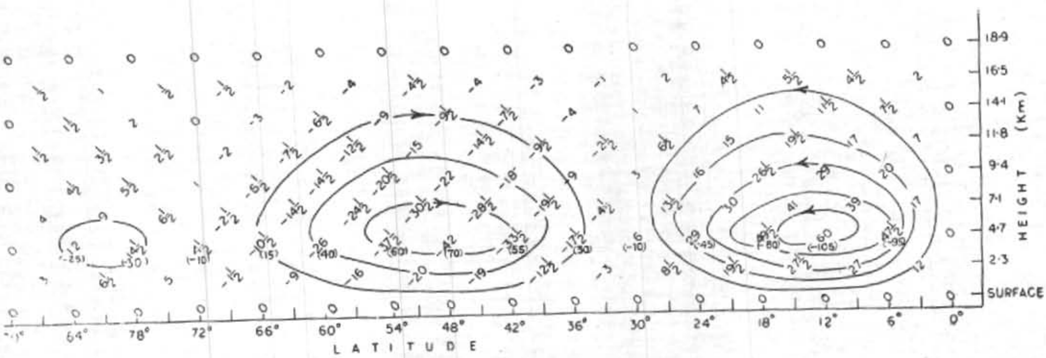

Fig. 6. Momentum source alone

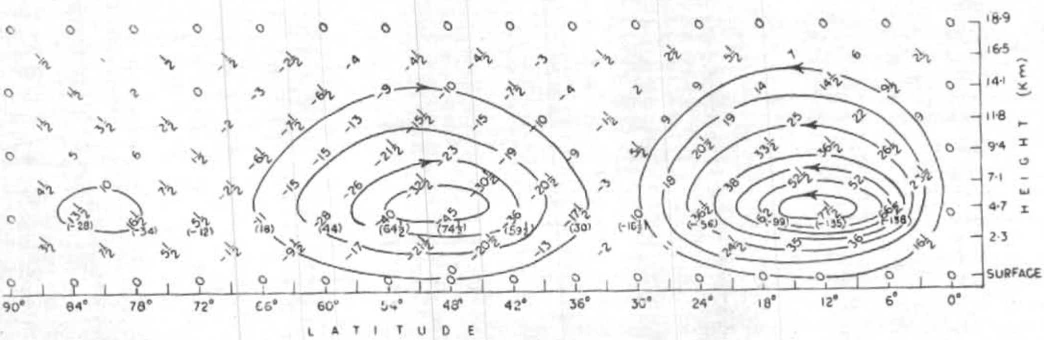

Fig. 7. Heat and momentum sources

Figs. 5—7. Figures in brackets are values of $R \times 10^{10}$. Other figures are values of $\psi' = \psi \times 10^{10}/h^2$ cm²/sec. The directed lines are isopleths of ψ' at intervals of $\psi'=10$)

The low latitude direct cell fits in rather well with the generally accepted ideas about mean motion in the meridional plane in present-day meteorology. The strength of circulation in this cell, too, is in fair agreement with all other investigations, as will be shown later.

The mid-latitude indirect cell indicated by our results is in conflict with the conclusions of Sheppard (1952, 1954) according to whom the convergence of flux due to large scale eddies in middle latitudes is more than off-set by the divergence of vertical flux (due to small scale eddies), so that where we have assumed a source, there would be a weak sink which if presupposed during solution of our equation would give rise to a weak direct cell. Sheppard's deductions are supported by his observations that over the sea-surface in the region of surface westerlies, the cross-isobaric flow is slightly towards "High". On the other hand, climatological rainfall distribution data (with a subsidiary maximum in latitudinal variation) indicate the possibility of a limb of mean ascent in the middle latitudes. Further, Tucker's (1954) careful analysis of actual wind data yields an indirect cell in middle latitudes. Mintz (1955 b) also deduces from various considerations an indirect (Ferrel) cell in middle latitudes. Although it is not possible to make any definite assertion, we might say that the balance of evidence is in favour of our picture of a source over a sink in middle latitudes, so that our conclusions of a weak indirect circulation in these latitudes is probably correct.

A rough estimate of the values of the velocity components, $v\left(=-\frac{\partial\psi}{\partial z}=-\frac{\partial\psi'}{\partial z}\times 10^{-10}\,h^2\right)$ and $w\left(=\frac{\partial\psi}{\partial y}=\frac{\partial\psi'}{\partial y}\times 10^{-10}\,h^2\right)$ may be made from the results of relaxation. In the low latitude (Hadley) cell, the mean value of v at 15° N between the heights of 8 to 15 km works out to be roughly 20 cm/sec, and the mean value of w at 7 km height between 2° and 12° N, nearly 0·15 cm/sec. In the Ferrel cell, the mean v at 48°N between 8 and 15 km is nearly −10 cm/sec, and the mean w at 7 km between 35° and 45°N nearly −0·06 cm/sec. As these values do not disagree violently with the results of Tucker and Mintz, we are led to believe that the sources and sinks assumed by us are at least of the right order of magnitude.

Another interesting fact that emerges from our analysis is that the momentum sources and sinks are the main factor in driving the circulation. We might conclude that the middle latitude circulation is frictionally driven, and that even the low latitude (Hadley) cell is influenced much more by momentum sources than by heat sources (as the values of $\partial\dot{M}_1/\partial z$ are higher than those of $\frac{g}{f}\frac{\partial\dot{\phi}_1}{\partial y}$) and so is in a sense mainly frictionally driven.

3. Evaluation of sources and sinks

In Section 2 it was inferred, by solving a simplified form of Eliassen's equation with certain assumptions about the sources of heat and momentum, that the mean meridional motion consists of a strong direct circulation cell in low latitudes and a weak indirect cell in middle latitudes. Not many more useful conclusions could be drawn in view of the approximations involved in the simplified form of the equation and in view of the uncertainties concerning the sources and sinks. It is little use attempting to refine our equation (by taking into account the earth's curvature, including smaller terms that have been ignored, etc) unless we have a better knowledge of the sources and sinks. The real problem appears to be to determine the location and magnitude of sources of heat and momentum.

In this section, we shall assume that our conclusion of a meridional circulation split up into cells is correct, and making use of the values of the strength of the circulation deduced by Tucker and Mintz, we shall proceed to evaluate precisely the sources and sinks necessary to maintain such mean circulation.

Considering mean motion in the meridional plane, we may write, as in Section 2

$$\frac{\partial u}{\partial t} = \dot{M}_1 + fv - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z}$$

$$\frac{\partial \phi}{\partial t} = \frac{\dot{Q}_1}{TC_p} - v\frac{\partial \phi}{\partial y} - w\frac{\partial \phi}{\partial z}$$

When averages over long periods of time only are considered, in the meridional plane the field of u and the field of ϕ remain constant (without any change locally with time), so that we have

$$\dot{M}_1 = v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv$$

$$\dot{\phi}_1 = v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z}$$

Therefore, if the mean value of u and ϕ in the meridional plane are known, and the velocity components v and w of the mean meridional circulation in the troposphere are assumed to be known, then one can determine the sources $\dot{M}_1$ and $\dot{\phi}_1$ at each point in the troposphere.

There is little difficulty in obtaining the mean fields of u and ϕ. Mintz (1954) has worked out the meridional cross-section of the mean zonal wind. Petterssen (1950) has presented in graphical form the mean meridional distribution of temperature. The data in tabular form (for the year 1949 only) is also available from Mintz (1955b). From these the mean field of ϕ can readily be worked out.

To obtain the data of v and w is not such an easy matter. However, two reasonably good sources from which the values could be extracted are Tucker's *Mean Meridional Circulations in the Atmosphere* (1954) and Mintz's *A Model of the Mean Meridional Circulation* (1955b.)

Tucker's method of evaluating v is direct. Using data from about 53 upper air ascent stations for a period extending over two years, he has worked out the values of v. The great advantage of Tucker's method lies in that no reference is made in his computations to the pressure-field (barring a few exceptional cases in which, when observations even to such low levels as 850 and 700 mb were wanting, geostrophic winds had to be used). Tucker evaluates w from continuity considerations from the simple equation:—

$$w_h \rho_h = \int_0^h \rho_z \operatorname{div}_H \mathbf{V}_z \, dZ$$

where $\operatorname{div}_H \mathbf{V}_z$ is the horizontal divergence of $\mathbf{V}$ at height z (*i. e.*, $\frac{\partial v}{a\partial \phi} - \frac{v}{a}\tan\phi$), ρ refers to density and w to vertical velocity at heights indicated by subscript.

Mintz's computations are based on far more extensive data (nearly three quarter million grid-points and basic data cards in all being used), but his method is complicated and involves various assumptions. He starts by assuming that above 700 mb the vertical eddy flux of momentum vanishes. The poleward eddy flux of momentum is obtained from geostrophic winds (*i.e.*, assuming that $\widetilde{u'v'} = \widetilde{u_g'v_g'} = \widetilde{u_g v_g}$). In brief, the reasoning by which Mintz is able to compute v and w is as follows— Considering the meridional plane to be divided into blocks of unit pressure increment and unit lateral width, with sides N to the north, S to the south, T at top and B at bottom, if R represents absolute angular momentum ($R = ur + \Omega r^2$) and M the flux of mass, conservation of momentum requires

$$R_N M_N - R_S M_S + R_T M_T - R_B M_B = -\frac{\partial}{a\partial\phi}\left(\frac{2\pi r^2}{g}\widetilde{u_g v_g}\right),$$

the vertical eddy flux being neglected. Also from the equation of mass continuity we have, $M_N - M_S + M_T - M_B = 0$. For the top northernmost block $M_N = M_T = 0$. So the two equations can be solved to obtain M_S and M_B. But M_B for this block becomes M_T for the next lower northernmost block, and M_S becomes M_N for the adjacent top block to the south and so for these two blocks the remaining values of M can be determined. Thus proceeding from the northern top corner, it is possible to solve for all values of M. And the M's are simply related to v and w by the equations

$$M_y = \frac{2\pi r}{g} \overline{\overline{v}} \Delta p \text{ and } M_z = 2\pi ra\, d\phi \rho w$$

so that v and w can be computed.

In the computations of $\dot{\phi}_1$ and $\dot{M}_1$, which follow, the values of v ad w used are the weighted means from the results of Tucker and Mintz. It appears, at first sight, that to make use of Mintz's values to evaluate the momentum sources would be begging the question, considering that Mintz's results are based upon certain momentum sources. But, although Mintz has utilised the data of poleward geostrophic eddy flux, his method involves various other assumptions, and in the end the results are smoothed out and present a very consistent picture indeed of the mean meridional circulation. By working backwards from these results, one may hope to obtain by reiteration a better distribution of the momentum sources than the one with which Mintz started. We need have no hesitation at all in using Mintz's data for the evaluation of heat sources, since heat sources do not come in, in his calculations.

In Table 2 are shown the values of the velocity components u, v, w and temperature T that have been assumed (as well as the values of ϕ derived from T) at the various grid-points which are at intervals of 5° latitude and 100 mb pressure. At each grid-point the values of $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial z}$, $\frac{\partial \phi}{\partial y}$ and $\frac{\partial \phi}{\partial z}$ were evaluated and the results entered in Table 3.

3.1. *Results and Discussions*—The computed values of $\dot{M}_1 \left(= v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - v f \right)$ at various grid-points are tabulated in Fig. 8. It is perhaps more useful to see the picture in terms of the rate at which angular momentum has to be supplied to blocks of 100-mb depth and 5° width representing entire zonal rings. Taking each of our figures to represent the mean value of $\dot{M}_1$, for a ring of lateral width $2\frac{1}{2}°$ on either side (a width of $R\pi/36$ in all) and a vertical depth of 50 mb on either side $\left(i.e., \frac{100 \times 1000}{\rho g} \text{ dynes/cm}^2 \right)$, the mass in the entire ring would be

$$\rho\, 2\pi R \cos\phi \times R \frac{\pi}{36} \times \frac{100 \times 1000}{\rho g}$$

and the value of angular momentum source corresponding to the entire ring would be

$$\dot{\Omega} = \dot{M}_1 R \cos\phi \times 2\pi^2 R^2 \cos\phi \times \frac{10^5}{36g}$$

These values have been calculated and plotted in Fig. 9. At the bottom of Fig. 9 are figures representing the angular momentum imparted per sec, by the zonal belt of the earth's surface of lateral width $2\frac{1}{2}°$ on either side. These are computed from data of τ_j (= surface friction stress +mountain stress), from the expression

$$\Gamma_\phi = \tau_j \times R\cos\phi \times 2\pi R \cos\phi \times R \frac{\pi}{36}$$

Similarly, the values of $\dot{\phi}_1 \left(= v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z} \right)$ calculated at all the grid-points are presented in Fig. 10. Fig. 11 represents the corresponding rate of cooling or heating of the atmosphere in °C per day ($\triangle T$ per day $= T\dot{\phi}_1\, 8{\cdot}64 \times 10^4$ because $\dot{\phi}_1 = \frac{\dot{Q}_1}{TC_p} = \dot{T}/T$).

	75° N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	1	1	23	25	25	40	38	28	23	0	-25	-28	-38	-11	-3	0
300	-1	0	22	28	27	38	32	21	12	-4	-24	-23	-14	-12	-3	0
400	-1	2	17	27	26	26	24	15	3	-9	-19	-17	-12	-11	-3	-1
500	0	-2	12	21	20	16	14	11	0	-7	-11	-12	-8	-3	-1	-1
600	2	-2	7	15	15	3	9	8	0	-7	4	5	-1	0	0	0
700	1	0	0	1	1	-3	-1	-1	-3	-4	0	1	3	4	3	0
800	0	2	-11	-28	-20	-18	-20	-9	-1	0	4	9	11	9	6	0
900	-1	4	-29	-49	-45	-42	-48	-34	-12	4	17	25	30	21	11	1
1000	-1	7	-48	-83	-75	-66	-75	-54	-23	22	32	44	51	38	19	1

Fig. 8. $\dot{M}_1$ in 10^{-4} cm sec^{-2}

	75° N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	0	0	6	9	12	23	27	23	22	0	-29	-35	-50	-15	-4	0
300	0	0	5	10	13	21	23	17	12	-4	-28	-29	-18	-16	-4	0
400	0	0	4	10	12	15	17	12	3	-9	-22	-21	-16	-15	-4	-1
500	0	0	3	7	9	9	10	9	0	-7	-13	-15	-11	-4	-1	-1
600	0	0	2	5	7	2	6	7	0	-7	5	6	-1	0	0	0
700	0	0	0	0	0	-2	-1	-1	-3	-4	-1	1	4	5	4	0
800	0	0	-3	-7	-9	-11	-14	-7	-1	0	5	11	12	12	8	0
900	0	1	-7	-17	-21	-25	-34	-29	-12	4	20	31	40	29	15	1
1000	0	1	-12	-29	-35	-39	-54	-46	-22	23	37	55	67	52	27	1
SURFACE TORQUE in 10^{24} gm cm^2 sec^{-2}		0	-20 ↓	-50 ↓	-70 ↓	-100 ↓	-100 ↓	-100 ↓	-40 ↓	10 ↑	50 ↑	90 ↑	120 ↑	120 ↑	60 ↑	30 ↑

Fig. 9. $\dot{\Omega}$ in 10^{24} gm cm^2 sec^{-2}

Eddy motion on scales of all magnitudes must be such as to produce the sources and sinks of momentum shown in Fig. 9. A simplified scheme of momentum transfer consistent with these sources and sinks is drawn up in Fig. 12. This figure shows in the low latitudes and low levels a convergence of upward eddy flux. At higher levels in low latitudes there is a marked divergence of horizontal eddy flux brought about by large-scale eddies. There may also be a slight divergence of downward eddy flux due to small-scale eddies, considering that the high level sink at low latitudes appears to be slightly stronger than the high level source at middle latitudes. In middle latitudes, at high levels there is convergence due to horizontal eddy flux (large-scale eddies), and at a divergence of vertical downward eddy flux (small-scale eddies).

Very near the equator, it seems possible (from an examination of Fig. 9) that there is a small upward eddy flux at all levels, as τ_j at the equator is appreciably different from zero and transport of momentum due to meridional circulation is not effective.

TABLE

Mean values of velocity

Scheme *u* *v*
w *T*
(*ϕ*)

Pressure level (mb) Height (km)	75°N		70		65	
100	150	—	200	—	235	—
16·2	—	202	—	211	—	214
		(396)		(409)		(423)
200	85	12	128	—07	163	—175
11·8	0	208	13	211	16	214
		(199)		(217)		(222)
300	57	12	94	09	128	—157
9·2	0	212	37	214	48	216
		(102)		(108)		(119)
400	45	09	70	20	96	—117
7·2	—06	224	54	225	81	227
		(069)		(077)		(092)
500	29	—03	59	20	85	—75
5·6	—09	233	57	234	98	236
		(049)		(053)		(063)
600	21	—22	40	18	57	—43
4·2	—08	240	54	242	109	244
		(022)		(031)		(039)
700	08	—15	33	07	52	07
3·0	—10	246	48	248	102	250
		(006)		(013)		(021)
800	0	03	12	—15	25	96
2·0	—10	247	37	248	78	250
		($\bar{1}$·968)		($\bar{1}$·972)		($\bar{1}$·985)
900	—06	07	12	—34	18	218
1·0	—06	248	23	249	49	251
		($\bar{1}$·939)		($\bar{1}$·947)		($\bar{1}$·952)
1000	—04	11	10	—56	14	362
0·1	—	249	—	250	—	252
		($\bar{1}$·912)		($\bar{1}$·916)		($\bar{1}$·924)

2

components and temperature

u in 10 cm/sec
v in 10^{-1} cm/sec
w in 10^{-3} cm/sec
T in °A
$(\varphi) = \log \theta$ in 10^{-3} + Const.

60		55		50		45		40	
256	—	253	—	228	—	203	—	200	—
—	216	—	218	—	218	—	217	—	215
	(435)		(442)		(442)		(439)		(430)
198	—190	225	—195	228	—356	225	—377	244	—280
06	216	13	217	14	218	—03	218	—08	218
	(231)		(239)		(244)		(244)		(244)
165	—219	197	—205	207	—326	207	—313	224	—218
18	218	37	220	41	223	—07	224	—23	226
	(130)		(138)		(148)		(157)		(163)
130	—210	157	—198	163	—227	195	—242	210	—164
34	230	51	232	52	235	—18	238	—45	240
	(097)		(110)		(118)		(130)		(138)
114	—164	139	—162	148	—86	147	—156	149	—135
46	239	43	242	50	245	—11	248	—59	251
	(071)		(083)		(097)		(109)		(120)
80	—115	100	—116	106	—13	120	—92	140	—109
53	246	28	249	42	253	04	256	—59	259
	(051)		(063)		(074)		(087)		(099)
73	—03	90	—06	93	42	87	13	82	—04
54	253	17	256	36	259	13	263	—53	266
	(030)		(040)		(055)		(070)		(081)
42	169	50	164	60	164	66	202	69	160
43	253	14	257	29	261	10	265	—42	269
	($\bar{1}$·992)		(007)		(022)		(037)		(053)
27	384	40	371	42	369	45	457	27	357
26	254	08	258	18	262	07	267	—25	272
	($\bar{1}$·963)		($\bar{1}$·976)		($\bar{1}$·995)		(012)		(031)
26	641	39	617	32	612	11	761	—04	591
—	255	—	259	—	264	—	269	—	275
	($\bar{1}$·936)		($\bar{1}$·952)		($\bar{1}$·971)		($\bar{1}$·990)		(012)

TABLE

Mean values of velocity

Scheme	u	v
	w	T
		(ϕ)

Pressure level (mb) Height (km)	35		30		25	
100	209	—	207	—	216	—
16·2	—	213	—	208	—	204
		(418)		(396)		(375)
200	294	—246	348	—03	366	431
11·8	—17	218	—41	218	—45	219
		(244)		(244)		(247)
300	262	—136	297	35	298	367
9·2	—50	229	—121	231	—135	234
		(175)		(179)		(200)
400	250	—52	280	49	250	204
7·2	—76	243	—165	245	—193	249
		(152)		(162)		(177)
500	165	—24	175	47	162	32
5·6	88	254	—152	258	—183	262
		(133)		(146)		(160)
600	150	—41	140	38	100	—21
4·2	—96	263	—138	267	—163	270
		(113)		(121)		(141)
700	84	—10	83	04	68	—55
3·0	—95	270	—119	274	—137	277
		(093)		(108)		(122)
800	49	76	48	—37	10	—153
2·0	—75	273	—94	277	—108	281
		(070)		(084)		(098)
900	20	170	12	—85	—10	—345
1·0	—46	277	—57	281	—67	286
		(050)		(064)		(078)
1000	—07	282	—15	—139	—40	—573
0·1	—	281	—	286	—	291
		(033)		(047)		(068)

2 (*contd*)

components and temperature

u in 10 cm/sec
v in 10^{-1} cm/sec
w in 10^{-3} cm/sec
T in °A
(φ) = log θ in 10^{-3} + Const.

20		15		10		5		0 °N	
209	—	157	—	05	—	—30	—	—50	—
—	201	—	199	—	198	—	197	—	197
	(361)		(352)		(346)		(340)		(340)
310	692	130	1075	07	1045	—33	802	—66	434
—30	220	—22	221	15	222	38	224	50	224
	(251)		(258)		(263)		(270)		(270)
235	598	100	803	03	797	—12	602	—75	324
—120	237	—65	239	45	241	113	243	149	244
	(211)		(221)		(225)		(238)		(240)
160	371	60	485	0	476	—33	364	—36	196
—160	252	—76	254	62	256	157	258	208	259
	(191)		(196)		(206)		(214)		(216)
113	140	30	198	—10	194	—27	148	—33	80
—166	265	—80	267	61	269	154	270	204	271
	(175)		(182)		(187)		(194)		(196)
57	26	10	10	—25	10	—40	08	—25	04
—154	272	—69	274	54	276	138	278	183	279
	(151)		(157)		(163)		(170)		(172)
32	—89	—05	—127	—50	—125	—50	—95	—25	—51
—132	279	—57	281	46	283	117	285	154	286
	(131)		(138)		(146)		(152)		(155)
—03	—280	—25	—365	—70	—358	—66	—273	—22	—147
—104	284	—45	286	36	288	91	290	120	291
	(112)		(118)		(124)		(129)		(131)
—27	—649	—49	—874	—79	—857	—72	—654	—21	—353
—66	289	—29	291	22	293	57	295	75	296
	(092)		(099)		(106)		(114)		(116)
—74	—1009	—60	—1347	—70	—1497	—60	—1142	—21	—617
—	294	—	296	—	298	—	300	—	301
	(078)		(085)		(092)		(100)		(102)

TABLE

Computed values of space-

Scheme $\frac{du}{dy}$ $\frac{d\phi}{dy}$

$\frac{du}{dz}$ $\frac{d\phi}{dz}$

Pressure level (mb)	75°N		70		65	
100	—106	—234	—74	—240	—54	—240
200	—88	—207	—68	—207	—59	—126
	133	42	151	43	167	43
300	—74	—153	—61	—153	—59	—198
	87	28	126	30	146	28
400	—69	—180	—50	—207	—54	—180
	78	15	97	15	120	16
500	—63	—126	—47	—126	—47	—162
	80	16	100	15	130	14
600	—58	—153	—36	—153	—39	—180
	81	17	100	15	104	16
700	—54	—135	—34	—135	—34	—153
	95	25	127	27	145	25
800	—50	—153	—25	—153	—25	—180
	70	34	105	33	170	34
900	—45	—135	—17	—117	—16	—144
	21	30	11	30	58	32
1000	—40	—108	—09	—108	—07	—180
	—60	33	120	33	180	32
f in 10^{-4}/sec	1·405		1·367		1·318	

Pressure level (mb)	35		30		25	
100	—18	306	25	387	—58	315
200	—110	036	—85	—063	18	—099
	—76	35	—129	31	—117	25
300	—79	—144	—45	—225	41	—288
	98	19	148	17	251	15
400	—60	—216	—10	—225	48	—261
	270	12	339	09	378	11
500	—34	—234	—03	—243	50	—261
	333	13	467	14	500	12
600	—07	—198	10	—252	50	—270
	312	15	358	15	362	15
700	—07	—243	11	—267	43	—207
	459	20	418	17	409	20
800	—06	—279	18	—252	49	—252
	320	22	355	22	390	22
900	—05	—297	25	—252	55	—252
	295	19	332	19	263	16
1000	—04	—315	33	—315	61	—279
	200	19	120	19	—100	13
f in 10^{-4}/sec	0·834		0·727		0·615	

3

derivatives of u, v and ϕ

$\frac{du}{dy}$ in 10^{-7} sec^{-1} $\frac{d\phi}{dy}$ in 10^{-12} cm^{-1}

$\frac{du}{dz}$ in 10^{-5} sec^{-1} $\frac{d\phi}{dz}$ in 10^{-8} cm^{-1}

60		55		50		45		40	
—20	—162	33	—072	59	018	29	117	—16	189
—68	—153	—27	—198	14	—054	—02	0	—68	—045
128	44	79	43	30	42	—06	40	—33	38
—76	—171	—40	—162	04	—171	—04	—135	—59	—162
148	29	148	28	141	27	65	25	74	23
—60	—162	—32	—189	0	—180	— 03	—180	—40	—198
142	16	161	15	164	14	167	13	208	12
—59	—180	—32	—234	0	—234	09	—207	—14	—216
167	15	190	16	190	15	250	14	233	13
—43	—216	—24	—207	0	—216	0	—225	—12	—234
158	16	189	17	212	16	231	15	258	15
—43	—171	—16	—225	05	—270	20	—234	0	—207
173	27	227	25	209	24	243	23	322	20
—41	—198	—14	—270	—16	—270	—25	—279	05	—297
230	34	250	32	255	30	210	29	275	25
—39	—216	—12	—288	—27	—324	—30	—324	10	—342
84	30	58	29	147	27	279	25	384	22
—36	—252	—11	—315	38	—342	36	—369	16	—387
270	31	400	25	420	27	450	26	270	22
1·260		1·191		1·114		1·028		0·935	

20		15		10		5		0°N	
85	207	184	117	168	108	50	090	15	0
93	—099	273	—108	147	—081	89	—081	20	0
—37	21	81	19	03	18	—26	15	36	14
184	—189	209	—126	101	—144	70	—144	15	—072
326	13	152	13	15	13	0	12	—65	12
170	—171	150	—135	80	—144	40	—144	10	—072
339	09	195	11	36	11	42	12	—117	12
26	—198	111	—108	51	—108	21	—090	0	—090
343	13	167	13	83	14	23	15	—37	15
90	—144	80	—108	45	—108	0	—108	0	—099
312	17	135	17	154	17	87	16	—31	16
86	—144	74	—117	41	—117	—23	—117	—06	—099
273	18	159	18	204	18	118	18	—14	19
77	—180	40	—108	20	—099	—40	—081	—12	—070
295	20	220	20	145	19	110	19	—20	19
68	—189	30	—126	10	—117	—42	—108	—12	—054
374	18	237	17	0	17	—32	16	—05	15
59	—153	—04	—126	0	—126	—44	—108	—10	—054
—270	17	—490	17	—790	16	—720	15	—210	16
0·497		0·376		0·252		0·127		0	

	75°N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	0	6	9	6	9	8	1	0	-7	-13	-15	-14	-15	-6	-3	2
300	0	9	16	10	13	16	2	0	-7	-21	-30	-27	-19	-6	5	14
400	-1	8	15	10	11	11	2	-2	-8	-20	-27	-21	-12	0	15	23
500	-1	12	15	10	10	10	2	-3	-10	-22	-23	-21	-12	7	22	30
600	-1	12	20	11	8	7	0	-6	-14	-21	-24	-27	-11	9	22	30
700	-2	12	25	14	4	6	0	-10	-18	-20	-22	-22	-9	10	22	30
800	-3	12	25	9	3	0	-3	-15	-18	-20	-18	-12	0	11	18	24
900	-2	6	13	0	-7	-8	-10	-17	-17	-12	-7	1	6	14	16	18
1000	0	1	-6	-16	-19	-20	-21	-20	-9	0	15	16	17	18	12	6

Fig. 10. ϕ_1 in 10^{-9} sec^{-1}

	75°N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	0	-0.1	-0.2	-0.1	-0.2	-0.1	0	0	0.2	0.3	0.3	0.3	0.3	0.2	0.1	0
300	0	-0.2	-0.3	-0.2	-0.3	-0.3	-0.1	0	0.2	0.4	0.6	0.d	0.4	0.2	-0.1	-0.3
400	0	-0.2	-0.3	-0.2	-0.2	-0.2	0	0	0.2	0.4	0.5	0.4	0.2	0	-0.3	-0.5
500	0	-0.3	-0.3	-0.2	-0.2	-0.2	-0.1	0	0.2	0.4	0.5	0.4	0.3	-0.2	-0.5	-0.7
600	0	-0.3	-0.5	-0.3	-0.2	-0.2	0	0.2	0.3	0.4	0.5	0.6	0.2	-0.2	-0.5	-0.7
700	0	-0.3	-0.5	-0.3	-0.1	-0.1	0	0.3	0.4	0.4	0.5	0.4	0.2	-0.2	-0.5	-0.7
800	0	-0.3	-0.5	-0.2	-0.1	0	0.1	0.3	0.4	0.4	0.3	0.3	0	-0.2	-0.4	-0.6
900	0	-0.1	-0.2	0	0.2	0.2	0.2	0.4	0.4	0.3	0.2	0	-0.1	-0.3	-0.4	-0.5
1000	0	0	0.1	0.3	0.4	0.4	0.4	0.4	0.2	0	-0.3	-0.3	-0.3	-0.4	-0.3	-0.2

Fig. 11. Rate of cooling in °C/day

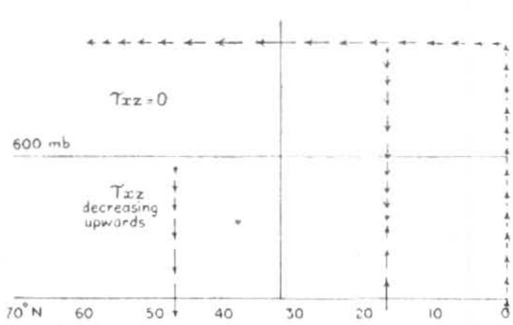

Fig. 12. Transfer of momentum by eddies (case 1)

	75°N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	-0.2	-0.2	-0.2	-0.3	-0.3	-0.4	-0.4	-0.5	-0.5	-0.6	-0.7	-0.8	-1.0	-1.2	-1.2	-1.3
							0.2	0.2	0.2			0.2	0.3	0.5	0.6	0.8
300	-0.2	-0.2	-0.3	-0.3	-0.4	-0.4	-0.5	-0.5	-0.6	-0.7	-0.9	-1.0	-1.2	-1.3	-1.3	-1.3
					0.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.5	1.1	1.3	1.6
400	-0.3	-0.4	-0.4	-0.5	-0.6	-0.6	-0.6	-0.7	-0.8	-1.0	-1.1	-1.2	-1.2	-1.2	-1.3	-1.3
	0.1	0.1	0.2	0.3	0.4	0.6	0.6	0.6	0.6	0.6	0.6	0.7	1.0	1.2	1.5	1.6
500	-0.6	-0.7	-8.7	-0.7	-0.8	-0.8	-1.0	-1.0	-1.2	-1.3	-1.4	-1.4	-1.5	-1.3	-1.2	-1.1
	0.1	0.3	0.5	0.6	0.8	0.8	1.0	1.0	1.0	0.9	0.9	1.0	1.1	1.3	1.5	1.6
600	-1.1	-1.1	-1.1	-1.1	-1.1	-1.1	-1.2	-1.2	-1.4	-1.5	-1.6	-1.7	-1.6	-1.2	-1.1	-1.1
	0.1	0.4	0.5	0.6	0.8	0.9	1.0	1.0	1.0	0.9	0.9	1.0	1.1	1.3	1.5	1.6
700	-1.2	-1.5	-1.5	-1.5	-1.6	-1.6	-1.6	-1.7	-1.7	-1.7	-1.8	-1.6	-1.3	-1.1	-1.0	-1.0
	0.2	0.4	0.5	0.6	0.8	0.9	1.0	1.0	1.0	0.9	0.9	1.0	1.1	1.3	1.5	1.6
800	-1.0	-1.4	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.6	-1.5	-1.3	-1.2	-1.0	-1.0	-1.0	-1.0
	0.2	0.4	0.5	0.6	0.8	0.9	1.0	1.0	1.0	0.9	0.9	1.0	1.1	1.3	1.5	1.6
900	-0.4	-0.5	-0.6	-0.8	-1.0	-1.1	-1.2	-1.0	-0.8	-0.7	-0.7	-0.6	-0.6	-0.6	-0.7	-0.8
	0.2	0.4	0.5	0.6	0.8	0.9	1.0	1.0	1.0	0.9	0.9	1.0	1.1	1.3	1.5	1.6
1000	-0.1	-0.2	-0.3	-0.4	-0.5	-0.5	-0.5	-0.4	-0.3	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	-0.4
RAINFALL cm/Year	15	30	40	50	70	80	90	90	90	85	85	90	110	140	160	180

Fig. 13. Rate of cooling due to radiation in °C/day (after London) and rate of heating due to latent heat °C/day (figures in the second line against each level)

The manner in which this picture will alter if we discredit the Ferrel cell altogether is discussed later.

From Figs. 10 and 11, we cannot proceed direct to draw any inferences about eddy motion, without first eliminating other causes which contribute to the formation of sources and sinks of heat in the atmosphere, such as radiation, release of latent heat and evaporation. These factors are discussed immediately below.

Perhaps the most satisfactory figures yet available of radiational cooling are those produced by London (1952, 1953). It might be well at this stage to recall briefly the procedure by which London has arrived at his distribution. He obtains the figures of heating due to solar radiation by using Mugge and Moller's empirical formula $a = 0{\cdot}172(u^*)^{0\cdot3}$ (where α is the energy absorbed in a column of effective optical depth u^*), from which it follows that the rate of change of temperature per day is

$$\triangle T = 5{\cdot}9 \times 10^3 \ \frac{\triangle a}{\triangle p}$$

Infra-red cooling is computed from the divergence of flux obtained graphically employing Elsasser radiation charts. From these, London works out the net radiational cooling in the troposphere, separately for conditions of clear sky and average cloudiness. His figures (average cloudiness) are entered in Fig. 13.

Rainfall figures represent the excess of water condensed in vertical columns (above the places where rainfall is measured) over the amount which has re-evaporated without coming down to the surface, therefore, assuming that the heat for evaporation at the surface is extracted from the ground or sea, we may evaluate from rainfall data the net heat released in vertical columns of the atmosphere. It is difficult to determine how this heat is distributed in the vertical. It will be assumed—although this is no better than a very crude approximation—that due to convective stirring in clouds, any latent heat released gets distributed uniformly with respect to mass between the average cloud base and average cloud top levels at various latitudes. On this basis, if at a particular latitude ϕ where the rainfall is R_ϕ, P_b and P_t are pressures at average cloud base and top levels, the mean heating rate between cloud base and top becomes $\dfrac{R_\phi \, L \, g}{(P_b - P_t) \ 1000 \ C_p}$,

	75°N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	-0.2	-0.2	-0.2	-0.3	-0.3	-0.4	-0.2	-0.3	-0.3	-0.6	-0.7	-0.6	-0.7	-0.7	-0.6	-0.5
300	-0.2	-0.2	-0.3	-0.3	-0.2	-0.1	-0.2	-0.2	-0.2	-0.4	-0.6	-0.7	-0.7	-0.2	0	0.3
400	-0.2	-0.3	-0.2	-0.2	-0.2	0	0	-0.1	-0.2	-0.4	-0.5	-0.5	-0.2	0	0.2	0.3
500	-0.5	-0.4	-0.2	-0.1	0	0	0	0	-0.2	-0.4	-0.5	-0.4	-0.4	0	0.3	0.5
600	-1.0	-0.7	-0.6	-0.5	-0.3	-0.2	-0.2	-0.2	-0.4	-0.6	-0.7	-0.7	-0.5	0.1	0.4	0.5
700	-1.0	-0.9	-1.0	-0.9	-0.8	-0.7	-0.6	-0.7	-0.7	-0.8	-0.9	-0.6	-0.2	0.2	0.5	0.6
800	-0.8	-1.0	-1.1	-1.0	-0.8	-0.7	-0.6	-0.6	-0.6	-0.6	-0.4	-0.2	0.1	0.3	0.5	0.6
900	-0.2	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	0	0.2	0.2	0.2	0.4	0.5	0.7	0.8	0.8
1000	-0.1	-0.2	-0.3	-0.4	-0.5	-0.5	-0.5	-0.4	-0.3	-0.2	-0.2	-0.2	-0.2	-0.2	-0.3	-0.4

Fig. 14. Net cooling °C/day (or heating °C/day) due to radiation and latent heat

	75°N	70	65	60	55	50	46	40	35	30	25	20	15	10	5	0
200 mb	-0.2	-0.3	-0.4	-0.4	-0.5	-0.5	-0.2	-0.3	-0.1	-0.3	-0.4	-0.3	-0.4	-0.5	-0.5	-0.5
300	-0.2	-0.4	-0.6	-0.5	-0.5	-0.4	-0.3	-0.2	0	0	0	-0.1	-0.3	0	-0.1	0
400	-0.2	-0.5	-0.5	-0.4	-0.4	-0.2	0	-0.1	0	0	0	-0.1	0	0	-0.1	-0.2
500	-0.5	-0.7	-0.5	-0.3	-0.2	-0.2	-0.1	0	0	0	0	0	-0.1	-0.2	-0.2	-0.2
600	-1.0	-1.0	-1.1	-0.8	-0.5	-0.4	-0.2	0	-0.1	-0.2	-0.2	-0.1	-0.3	-0.1	-0.1	-0.2
700	-1.0	-1.2	-1.5	-1.2	-0.9	-0.8	-0.6	-0.4	-0.3	-0.4	-0.4	-0.2	0	0	0	-0.1
800	-0.8	-1.3	-1.6	-1.2	-0.9	-0.7	-0.5	-0.3	-0.2	-0.2	-0.1	0.1	0.1	0.1	0.1	0
900	-0.2	-0.2	-0.3	-0.2	0	0	0	0.4	0.6	0.5	0.5	0.4	0.4	0.4	0.4	0.3
1000	-0.1	-0.2	-0.2	-0.1	-0.1	-0.1	-0.1	0	-0.1	-0.2	-0.5	-0.5	-0.5	-0.6	-0.6	-0.6

Fig. 15. Net heat loss to be made good by turbulence—°C/day

pressures being in mb and other quantities in C.G.S. units. The distribution of latent heat worked out accordingly (from rainfall data extracted from sources mentioned in Section 2) is entered in Fig. 13. At the top of clouds, in order to avoid large discontinuities, some slight smoothing has been resorted to. At the bottom, however, this has not been done in the belief that eddy conduction from the surface should reduce the discontinuity.

Fig. 14 shows the combined effect of radiational cooling and release of latent heat.

In Fig. 15, the resultant effect of the mean meridional circulation, radiative cooling and the distribution of latent heat (obtained by superposing Figs. 11 and 14) is represented. The only factor that has been left out of account is the transport of sensible heat by eddy motion of all scales.

An examination of Fig. 15 reveals that on the average there is a general cooling

	75°N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	—	—	—	—	—	—	—	—	—	0	-29	-35	-50	-15	-4	0
300	—	—	—	—	—	—	—	—	—	-4	-28	-29	-18	-16	-4	0
400	—	—	—	—	—	—	—	—	—	-9	-22	-21	-16	-15	-4	-1
500	—	—	—	—	—	—	—	—	—	-7	-13	-15	-11	-4	-1	-1
600	—	—	—	—	—	—	—	—	—	-7	5	6	-1	0	0	0
700	—	—	—	—	—	—	—	—	—	-4	-1	1	4	5	4	0
800	—	—	—	—	—	—	—	—	—	0	5	11	12	12	8	0
900	—	—	—	—	—	—	—	—	—	4	20	31	40	29	15	1
1000	—	↓	↓	↓	↓	↓	↓	↓	↓	23 ↑	37 ↑	55 ↑	67 ↑	52 ↑	27 ↑	1 ↑
SURFACE	TORQUE	0	-20	-50	-70	-100	-100	-100	-60	10	50	90	120	120	60	30

Fig. 16. $\dot{\Omega}$ **in 10^{24} gm cm^2 sec^{-2}**

(Hadley cell alone operative)

	75°N	70	65	60	55	50	45	40	35	30	25	20	15	10	5	0
200 mb	-0.2	-0.2	-0.2	-0.3	-0.3	-0.4	-0.2	-0.3	-0.3	-0.3	-0.4	-0.3	-0.4	-0.5	-0.5	-0.5
300	-0.2	-0.2	-0.3	-0.3	-0.2	-0.1	-0.2	-0.2	-0.2	0	0	-0.1	-0.3	0	-0.1	0
400	-0.2	-0.3	-0.2	-0.2	-0.2	0	0	-0.1	-0.2	0	0	-0.1	0	0	-0.1	-0.2
500	-0.5	-0.4	-0.2	-0.1	0	0	0	0	-0.2	0	0	0	-0.1	-0.2	-0.2	-0.2
600	-1.0	-0.7	-0.6	-0.5	-0.3	-0.2	-0.2	-0.2	-0.4	-0.2	-0.2	-0.1	-0.3	-0.1	-0.1	-0.2
700	-1.0	-0.9	-1.0	-0.9	-0.8	-0.7	-0.6	-0.7	-0.7	-0.4	-0.4	-0.2	0	0	0	-0.1
800	-0.8	-1.0	-1.1	-1.0	-0.8	-0.7	-0.6	-0.6	-0.6	-0.2	-0.1	0.1	0.1	0.1	0.1	0
900	-0.2	-0.1	-0.1	-0.2	-0.2	-0.2	0	0	0.2	0.5	0.5	0.4	0.4	0.4	0.4	0.3
1000	-0.1	-0.2	-0.3	-0.4	-0.5	-0.5	-0.5	-0.4	-0.3	-0.2	-0.5	-0.5	-0.5	-0.6	-0.6	-0.6

Fig. 17. Net heat loss to be made good by turbulence in °C/day

(Hadley cell alone operative)

in the troposphere of the order of 0·2 to 0·4°C/day. This has to be made good by the transport of sensible heat from the ground by eddy conduction.

Jacob's figures for the exchange of sensible heat between the sea and the atmosphere led us in the previous section to infer a mean rate of heating between 0·04 and 0·16° C/day. Our present analysis suggests that Jacob's figures may be somewhat too low.

There appear in Fig. 15, a small heat source and a sink, which may not be entirely spurious. The source is at about 15°N 900-mb level and the sink at about 70°N 700 to 600-mb level. The slope at which the large-scale eddies have to effect transport of heat between source and sink, is not at too much variance with what Eady (1949) has shown from dynamical considerations, *i.e.*, roughly half the slope of the broad-scale isentropic surfaces.

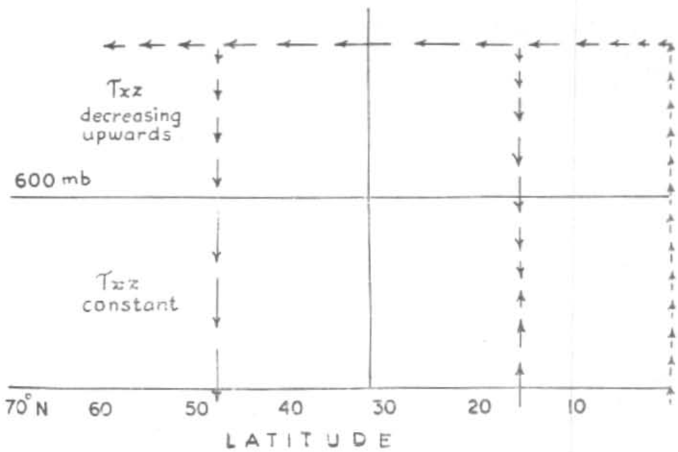

Fig. 18. Transfer of momentum by eddies

Finally, we may just examine what the results would be if the Ferrel cell is supposed not to exist—whether, considered from the points of view of both momentum and heat sources, such a supposition leads us to a more consistent picture of transfers by eddy motion, when only the Hadley cell is operative, we have to make the mean velocity components v and w both 0 (in Figs. 8 and 10) north of $32\frac{1}{2}$°N. The sources of momentum and heat required to be produced by turbulent motion now are represented in Figs. 16 and 17.

A modified scheme of eddy transport of momentum to fit in with Fig. 16 is illustrated in Fig. 18. In this case, the divergence of downward eddy flux at upper levels in middle latitudes neutralises the convergence due to horizontal eddy flux. In the lower levels of mid-latitudes there can be no divergence of downward flux due to small scale eddies (as we have no sink). Thus the essential difference between the case of Fig. 12 and that of Fig. 18 is that while in the former τ_{xz} (shearing stress) decreased in mid-latitudes from the surface upwards becoming 0 above about 600 mb, now τ_{xz} remains constant in lower levels and decreases rapidly in the levels where the weather systems are causing horizontal convergence of momentum.

There is little difference between the pictures of heat sources in Figs. 15 and 17. In the absence of the Ferrel cell the small heat source in low latitudes appears very slightly displaced towards the equator, while the sink at high latitudes is now weaker or really more diffuse.

Our analysis does not help us to decide which, if either, of the alternative schemes of eddy motion considered presents a true picture. This is hardly surprising, considering that all along we have been dealing with quantities smaller than the limits of accuracy with which they are measured. When more precise data are available better results may be expected.

4. Acknowledgement

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Appendix I

The zonal component of the equation of motion in spherical polar co-ordinates may be written,

$$\frac{\partial u}{\partial t}+\frac{u}{r}\frac{\partial u}{\partial \lambda}+\frac{v}{R}\frac{\partial u}{\partial \phi}-$$

$$-\frac{vu\tan\phi}{R}+w\frac{\partial u}{\partial z}+w\frac{u}{R}$$

$$=-\frac{1}{\rho r}\frac{\partial p}{\partial \lambda}+2v\,\Omega\sin\phi-2w\Omega\cos\phi$$

where R = distance from centre of earth and $r = R\cos\phi$.

Averaging over all longitudes and over time, we obtain

$$\frac{v}{R}\frac{\partial u}{\partial \phi}+w\frac{\partial u}{\partial z}-v\frac{u}{R}\tan\phi+w\frac{u}{R}$$

$$=vf-w2\Omega\cos\phi+\dot{M}_1$$

where $\dot{M}_1$ is the source due to the terms ignored such as eddy terms, and below the level of mountain tops, the residual of

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\rho R\cos\phi}\frac{\partial p}{\partial \lambda}\,d\lambda$$

On comparing the order of magnitude of the terms, we find

$\frac{u}{R} << \frac{\partial u}{\partial z}$, $w\,2\,\Omega\cos\phi << v\,f$ and

$\frac{u}{R}\tan\phi << f$ except very near the poles.

Appendix II

Equation (3) in Section 2 is in form slightly different from the usual manner in which the thermal wind equation is expressed. It is, however, readily derived as follows.

From the expression for the geostrophic wind and the equation of state, we may write

$$u_g=-\frac{1}{\rho f}\frac{\partial p}{\partial y}=-\frac{RT}{fp}\frac{\partial p}{\partial y}$$

$$\frac{\partial u_g}{\partial z}=-\frac{R}{f}\frac{\partial}{\partial z}\left(\frac{T}{P}\frac{\partial p}{\partial y}\right)$$

But . $\frac{\partial \log p}{\partial z}=-\frac{\rho g}{p}=-\frac{g}{RT}$

So $$\frac{\partial u_g}{\partial z}=-\frac{R}{f}\left\{\frac{\partial T}{\partial z}\frac{\partial \log p}{\partial y}+\right.$$

$$\left.+T\frac{\partial}{\partial y}\left(-\frac{g}{RT}\right)\right\}$$

$$=-\frac{R}{f}\left\{\frac{\partial T}{\partial z}\frac{\partial \log p}{\partial y}+\frac{1}{T}\frac{g}{R}\frac{\partial T}{\partial y}\right\}$$

$$=-\frac{g}{f}\left\{\frac{\partial \log T}{\partial y}+\frac{R}{g}\frac{\partial T}{\partial z}\frac{\partial \log p}{\partial y}\right\}$$

and when $\frac{\partial T}{\partial z}$ is not appreciably different from $-\frac{g}{C_p}$,

$$\frac{\partial u_g}{\partial z}=-\frac{g}{fC_p}\left\{C_p\frac{\partial \log T}{\partial y}-R\frac{\partial \log p}{\partial y}\right\}$$

$$=-\frac{g}{fC_p}\frac{\partial s}{\partial y}=-\frac{g}{f}\frac{\partial \phi}{\partial y}.$$