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# A New Theory for Cup Anemometers

S. RAMACHANDRAN

*Institute of Tropical Meteorology, Poona*

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**ABSTRACT.** The summary of a transient analysis of cup anemometer is presented. It is shown that the steady state calibration curve and the transient characteristics of anemometers can be calculated purely from the theoretical consideration using the raw data on the inertia, static aerodynamic forces on the cups, minimum wind speed at which the practical anemometer begins to rotate and any particular value of observed speed of rotation in the middle of the range of operation. The calculated characteristics are shown to agree with the published experimental data for the British cup generator anemometer.

## 1. Introduction

The wind speed in the natural atmosphere, near the ground level, is a random parameter, its instantaneous value being always subject to continuous transient variations. The anemometers, on the other hand, are designed and calibrated inside a wind tunnel, where particular care and attention are paid to keep the wind speed constant for a 'sufficiently long time' before taking a reading. A basic question now arises, as to how far are these two facts mutually compatible, whether the anemometers are faithful during their exposure in natural wind and how far are the calibration readings in the wind tunnel representative ? The question has been answered by the writer in a thesis (Ramachandran 1966), containing a detailed application of the transient theory formulated for various anemometers. In this article, a summary of the theory for the cup anemometers is presented.

## 2. Discussion

When wind blows on a cup of an anemometer at an angle $\theta_i$ to its normal and if the cups are rotating at a speed $n_i$ revolutions per second, air blows on the cup at a relative speed given by —

$$v_r = v_i - 2\pi n_i R \cos\theta_i \qquad (1)$$

where $v_i$ is the wind speed, $R$ is the distance of the centre of the cup from the axis of the rotor. Here we have assumed that one need not take the vector difference between the velocities. The rate at which the air blows on the cup is $\rho A v_r$ grams per second where $\rho$ is the density of air. Since its own speed is $v_i$ , the force due to this air on the cup may be written as $\rho A [C_N(\theta_i)] v_i v_r$ , where $C_N(\theta_i)$ is the normal force coefficient. The values of normal force coefficient for different shapes and Reynold numbers have been published by Brevoort and Joyner (1934). Now, one may imagine an 'effective mass' of air given by $\rho A [C_N(\theta_i)]$ blowing on the cup imparting this force on it. Then we may define an effective area $A\theta_i$ for the cup which would be given by the product $A [C_N(\theta_i)]$ of the actual area and the normal force coefficient. For a three cup anemometer the other two cups would have effective areas $A_{\theta i+120^\circ}$ and $A_{\theta i+240^\circ}$. When we consider all the three cups together we may write this quantity as $\rho [a_i v_i - 2\pi n_i b_i R]$ where $a_i$ and $b_i$ are given by —

$$a_i = A_{\theta i} + A_{\overline{\theta i+120^\circ}} + A_{\overline{\theta i+240^\circ}} \qquad (2a)$$

$$\text{and } b_i = A_{\theta i}\cos\theta_i + A_{\overline{\theta_i+120^\circ}}\cos(\theta_i + 120^\circ) + A_{\overline{\theta_i+240^\circ}}\cos(\theta_i + 240^\circ) \qquad (2b)$$

and $i$ is a suffix to denote a particular orientation of the anemometer as indicated by the value of $\theta_i$. Values of $a_i/A$ and $b_i/A$ ($A$ being the area of the face of the cup) have been computed from the data of Brevoort and Joyner (1934) for the shape of the cup used in the British cup generator anemometer. These values are presented in Fig. 1. When the cups are rotating the configuration with respect to the wind direction is repeated for every 120°, so that the average rate at which the mass of air is intercepted and the average torque acting on the anemometer may be evaluated. In an interval of time in which the cups rotate through 120° at any speed the change in the speed is within about 5 per cent of $n$ in all practical cases. Since $n$ does not change much in this interval there is no need to take into account the detailed variations of wind accelerations in the interval. However, the wind speed remains a continuous variable even in this interval. We may assume that the mean acceleration of the wind is constant during a rotation of the cups through 120°. With this assumption, it may

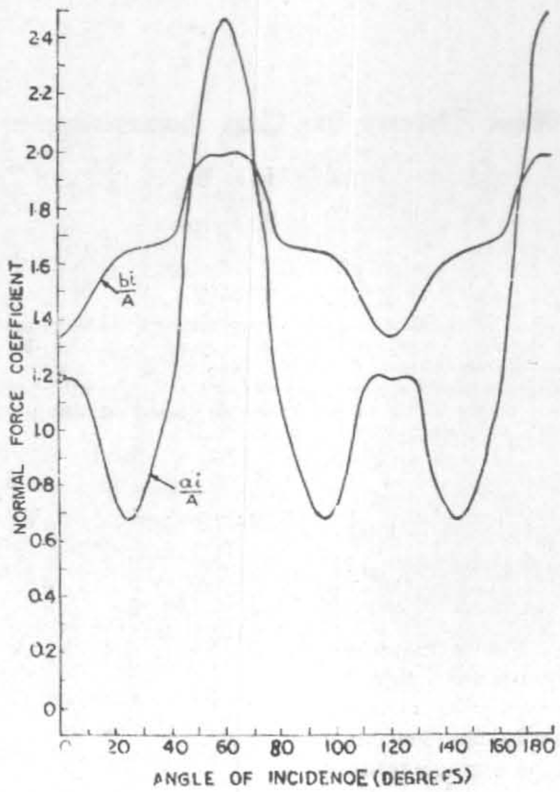


Fig. 1


**Fig. 2. Comparison of calculated and experimental values of speeds of rotation for various steady wind speeds**

I — Theoretical values for a frictionless anemometer

II — Theoretical values for an actual anemometer with friction

III — Experimental values for the actual anemometer be shown that the aerodynamic torque on the cups is given by —

$$\text{Torque} = \rho R \left[ a_m \{v(t)\}^2 + \tfrac{1}{2} a_m \int_0^t f(t-\tau)\, v(\tau)\, d\tau - 2\pi R\, b_m \int_0^t n'(t-\Gamma)\, v(\Gamma)\, d\Gamma \right] \quad (3)$$

where the function $f(t-\tau)$ is given by —

$$f(t-\tau) = v'(t)\, u(t-\tau) \quad (4)$$

where $v'(t)$ = acceleration of wind, $u(t-\tau)$= the delayed unit step function given by—

$$\left. \begin{array}{ll} u(t-\tau) = 0 & \text{for } t < \tau \\ u(t-\tau) = 1 & \text{for } t \geqslant \tau \end{array} \right\} \quad (5)$$

$a_m$ = mean value of $a_i$ ,

$b_m$ = mean value of $b_i$ , $\tau$ = the time variable in the interval for motion through 120°,

$\Gamma$=the time variable in the intervals from the instant when the anemometer started from rest to the instant $t$.

If $I$ be the moment of inertia, $B_0$ the torque due to static friction, $B_1\, n$ is the torque due to dynamic friction (and magnetic drag, if present, as in the case of cup generator anemometer) the equation of balance for the cup anemometer may be written as —

$$2\pi I \frac{d}{dt} [n(t)] + B_0 + B_1[n(t)] = 2D[v(t)]^2 + D \int_0^t f(t-\tau)\, v(\tau)\, d\tau - C \int_0^t n'(t-\Gamma)\, v(\Gamma)\, d\Gamma \quad (6)$$

where $D = \frac{1}{2} \rho R a_m$, $C = 2\pi \rho R^2 b_m$ and $n' = dn/dt$

If a step function change in wind velocity is considered, as given by —

$$\left. \begin{array}{ll} v = v_0 & \text{for } t < 0 \\ v = v_1 & \text{for } t \geqslant 0 \end{array} \right\} \quad (7)$$

The solution for the above equation can be shown to be —

$$n(t) = \frac{Dv_1^2 - B_0}{B_1 + Cv_1} \left[ 1 - \exp(-t/T) \right] - \frac{Dv_0 - Cn_0}{B_1 + Cv_1} v_1 \left[ 1 - \exp(-t/T) \right] + n_0 \exp(-t/T) \quad (8)$$

The exponentials involve a time constant $T$, given by —

$$T = 2\pi I / (B_1 + Cv_1) \quad (9)$$

which is larger for a smaller wind speed.

Taking reciprocals, we may write Eq. (9) as —

$$1/T = (1/T_n) + (1/T_a) \quad (10)$$

where $T_n$ may be called the 'natural time constant' being equal to $2\pi I/B_1$ and $T_a$ may be called 'the aerodynamic time constant' being given by $2\pi I/Cv_1$. The quantity $2\pi I/C$ is the same as the so called 'distance constant'.

For a steady state condition, the speed of rotation of an ideally frictionless anemometer would be given by —

$$n_1 = (D/C)\, v_1 \text{ and } n_0 = (D/C)\, v_0 \quad (11)$$

However, when both static and dynamic frictions are present the relation is given by —

$$n = (Dv^2 - B_0) / (B_1 + Cv) \quad (12)$$

where $v = v_1$ or $v_0$ for $n = n_1$ or $n_0$

The minimum value of wind speed at which the anemometer will start to rotate from rest is given by —

$$v_{\min} = [B_0/D]^{\frac{1}{2}} \quad (13)$$

The constants $D$ and $C$ in Eq. (11) are calculated empirically from the knowledge of inertia and static forces on cups held rigidly in the path of a steady wind. Thus the calibration curve of a frictionless anemometer may be calculated from a knowledge of $D$ and $C$ without the need of calibrating it with different wind speeds. However, the practical anemometers are not ideal, but have a considerable amount of static and dynamic friction, in addition to the magnetic drag as in the case of cup generator anemometer. The constant $B_1$ for a practical anemometer is calculated from Eq. (12) using the observed value of a particular speed of rotation at a wind speed in the middle of the range of operation. The constant $B_0$ is calculated from Eq. (13) using the observed wind speed at which the anemometer begins to rotate. Fig. 2 gives the curves of speeds of rotation for (*i*) a cup generator anemometer which would be completely frictionless, (*ii*) for a practical anemometer which would start to rotate at a wind speed of 40 knots, and (*iii*) the published calibration data for the same instrument (Air Ministry 1956). The effect of friction and the magnetic drag is to shift the

TABLE 1

Calculated values of time constants and distance constant and the corresponding values obtained from experimental data

| Velocity | | Time constant | | Exptl. $D$ neglecting friction and $T_n$ | Corrected $D$ from exptl. data for $T_n$ =10 sec | Theoretical value for distance constant |
|---|---|---|---|---|---|---|
| Final | Initial | Experimental | Calculated from Eq. (9) | | | |
| (kt) | (kt) | (sec) | (sec) | (m) | (m) | (m) |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 35 | 17·5 | 0·7 | 0·9 | 12·9 | 13·5 | 18·2 |
| 35 | 26 | 0·7 | 0·9 | 12·9 | 13·5 | 18·2 |
| 26 | 13 | 1·1 | 1·36 | 14·6 | 16·6 | 18·2 |
| 26 | 17·5 | 1·3 | 1·36 | 17·4 | 19·9 | 18·2 |
| 17·5 | 9 | 1·7 | 1·66 | 15.3 | 18·5 | 18·2 |
| 17·5 | 13 | 1·85 | 1·66 | 17·5 | 20·4 | 18·2 |
| 13 | 9 | 2·6 | 2·26 | 17·4 | 22·4 | 18·2 |

Note : 1. Col. 6 shows the distance constant from experimental data after neglecting the effect of natural time constant and col. 7 those corrected for natural time constant

2. $D$ = Distance constant

curve to the right. There is a fair agreement between the curves calculated with the values of $B_0$, $B_1$, $D$ and $C$ and the experimental curve. The natural time constant is determined by the ratio of inertia $2\pi I$ to the constant of dynamic friction $B_1$. This has been calculated as 10 sec for the anemometer under discussion. However, the aerodynamic time constant is in reality not a constant but is the ratio of the so called 'distance constant' to the final value of wind speed in a step function. The value of the distance constant for the anemometer under discussion is calculated as 18·2 metres. The values of the time constants computed from Eq. (9) along with the published experimental values are presented in Table 1. The same table also gives the values of distance constant derived from the experimental data as compared to the value computed from $I$ and $C$. The discrepancies are due to the assumption that $B_1$ is a constant even in transient conditions.

### 3. Conclusions

The agreement between the calculated characteristics and the experimental data proves the validity of the application of the principles of transient theory to the response of the anemometers. The experimental fact that the speed of response of the anemometer is fast for rising wind speed and slow for falling wind speed is explained in this paper on theoretical grounds.

### 4. Acknowledgement

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