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On objective assessment of convergence and precipitation by dynamic trajectory method

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ABSTRACT. Dynamic trajectories are drawn for intervals of three hours by means of arc-strike technique developed by Good-year and later used by Peterson and others. The wind field required for this purpose is obtained on the basis of streamline analysis at 0·6 km level, while the geostrophic winds have been evaluated from the sea level charts. A fairly accurate idea of areas of low level convergence can be had with the help of these trajectories.

An estimation of moisture inflow into the convergence area is made by the use of an empirical relation involving the computation of precipitable water, and average wind-speed normal to the line of maximum curvature. In order to reduce time on computations, they were done on polar coordinates and a nomogram was also prepared for different latitudes for direct application on the polar diagram.

The results obtained in a few cases are presented here, which show that some improvement is obtainable by using the stream line analysis, particularly in view of the sparse aerological network.

1. Introduction

During computations made by Banerji *et al.* (1967) on vertical velocities for a continuous period of one monsoon, it was noticed that there was marked disagreement in some cases between the computed and observed rainfall. This was further investigated and found that the disagreement was largely due to the position of depressions which caused the rainfall with respect to the grid. It was seen that when the depression was within or nearer the centroid of the grid, the forecast amounts were in close agreement with actuals. Regarding the computed values which were much higher than the actuals, it was realised that higher estimates of vertical velocities were responsible for this.

The vertical velocities were computed for a point occurring at the centroid of a triangular grid from the partial divergence values obtained at the vertices of this grid by Bellamy's method. The inequitable influence of a developing weather situation at any of the three vertices of the Bellamy's (1949) triangle could affect erroneously the resultant convergence and consequently also the computed vertical motion. For instance, if the partial divergence at one of the vertices was largely negative (showing convergence) it could be such that this large negative value outweighed the positive contribution of divergences at the remaining two vertices.

It was, however, found possible to obviate these difficulties, by recourse to dynamic trajectories which is the subject matter of this paper. A second objective was also to see if these dynamic trajectories could be independently used for locating large scale convergences, and for estimating quantitative precipitation. Such studies were conducted by the National Hurricane Research Project (Peterson *et. al* 1960) and the method itself was put to operational use for some time in USA.

2. Principle of dynamic trajectory

A dynamic trajectory may be defined as a trajectory of wind particles whose path with respect to time is governed by the synoptic configuration and in accordance with the equations of motion as applied to a stationary or quasi-stationary pressure system. The basis underlying study of the path of a particle under influence of various forces acting on the particle was provided by the early studies of Bjerknes and later by Pettерssen (1940) and by Haurwitz (1941). The application of these principles was subject of investigations by many later workers, notably Takahashi *et. al* (1954) and Hubert (1957), Peterson and others (1960). Appleby (1954) suggested use of trajectory method for forecasting various meteorological parameters.

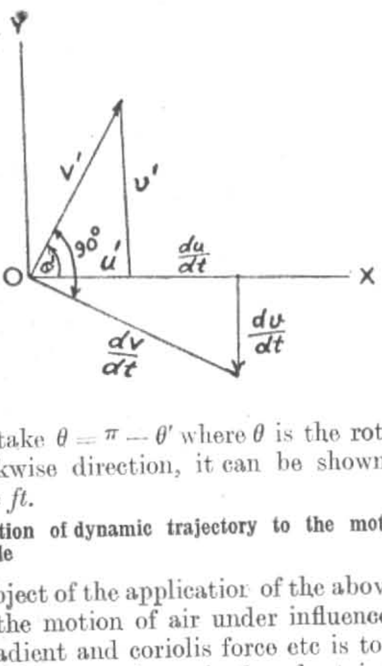
Goodyear (1959), however, stated with the help of simplified equations of motion, three essential characteristics features of dynamic trajectories. They are—(1) The acceleration of geostrophic deviation vector, *i.e.*, ageostrophic wind is zero, or in other words, the ageostrophic component is constant and does not change for small intervals of time; (2) The acceleration of motion is always at right angle to the ageostrophic wind component and (3) The ageostrophic wind (which is constant in magnitude) rotates anticyclonically in northern hemishpere with an angle defined by $\theta = ft$, where f is the coriolis parameter and t is the time interval.

It will be seen from the above easily that if $\vec{V'}$ denotes the ageostrophic wind, then according to (1) above, we have —

$$\frac{d\vec{V'}}{dt} = 0$$

Let θ' be the angle made by the geostrophic deviation vector $\vec{V'}$ and the east-west component of the geostrophic deviation vector (u') as shown in diagram. Then from the simplified equation of motion, it will be seen that the following solution would result.

$$(i) \quad \frac{dV'}{dt} = 0 \qquad (ii) \quad \frac{d\theta'}{dt} = -f$$



If we take $\theta = \pi - \theta'$ where θ is the rotation in the clockwise direction, it can be shown simply that $\theta = ft$.

3. Application of dynamic trajectory to the motion of air particle

The object of the application of the above principles to the motion of air under influence of pressure gradient and coriolis force etc is to find the expected wind at the end of a short interval of time say, 3 hours. If V_i is the initial wind, the object is to find the final wind V_f at the end of 3 hours. In vector notation the initial and final winds are related by the following—

$$\vec{V_f} = \vec{V_i} + ft\,(\vec{V'} \times \hat{k})$$

where f is the coriolis parameter, t is the time, $\vec{V'}$ is the ageostrophic component, $\hat{k}$ is the unit vertical vector. Goodyear (1959) has shown graphically how the final wind could be evaluated. However, in the present paper, the authors preferred the use of polar diagram for entire computations. The authors had preferred to use the polar diagram in conjunction with the arc strike diagram for simplicity and with a view to reduce computational time, for operational purposes.

To find out $\theta = ft$, a nomogram was prepared in polar coordinates for direct application on the polar diagram (scale used 1 cm = 1 kt). Since f is dependent on the latitude, the nomogram was prepared for different latitude intervals, keeping the time constant at 3 hours (*see* Fig. 3).

4. Details of computing a dynamic trajectory

The detailed steps that are involved in obtaining an average trajectory for 3 hours are as follows in the order given—

(1) The reported winds at 0·6 km are plotted on a weather chart and streamline analysis is made. (The analysis in two cases is shown as sample in Figs. 11 and 12). The 0·6 km was selected to avoid frictional losses at surface.

(2) A careful analysis of the pressure map is made at the start of the trajectories to find the value of geostrophic wind as accurately as possible.

(3) The area which is subjected to convergence is determined by a visual examination, and winds are picked at a place about 18 to 24 hr upstream of this convergence area. This is then known as inflow area. About five to six winds (speed and direction) were noted each spaced equally at intervals of one degree latitude say at A, B, C, D, E etc. The actual wind V_a and geostrophic wind V_g at A were then plotted on the polar diagram.

(4) To find out the final wind V_f, the nomogram for 'striking the arc' was kept below the polar diagram. It may be mentioned here that in practice these above two devices are prepared in a transparent paper. The following details are mentioned in a sample case, as these operations are repeated in every other case for computing final wind. The actual wind V_a is denoted by a point M on the polar diagram, and the geostrophic wind V_g is denoted by N on the polar diagram for the sake of convenience.

$$V_g = \frac{1}{\rho f} \cdot \frac{\partial p}{\partial n} \quad \cdots\cdots (1)$$

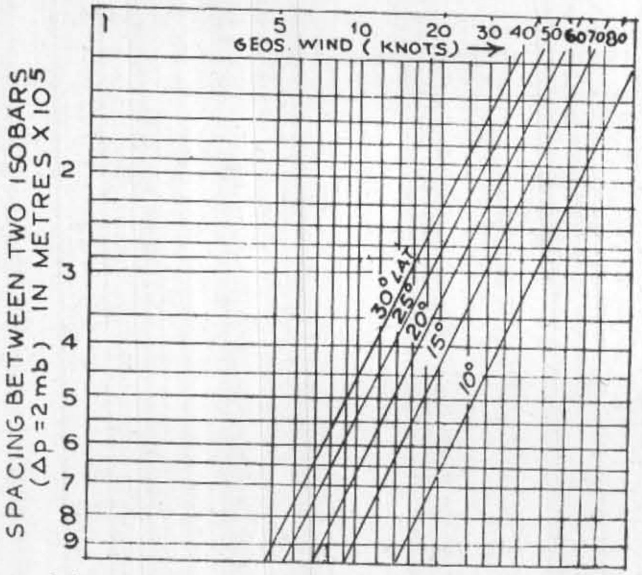


Fig. 1. Nomogram used for computing geostrophic wind for 2 mb interval

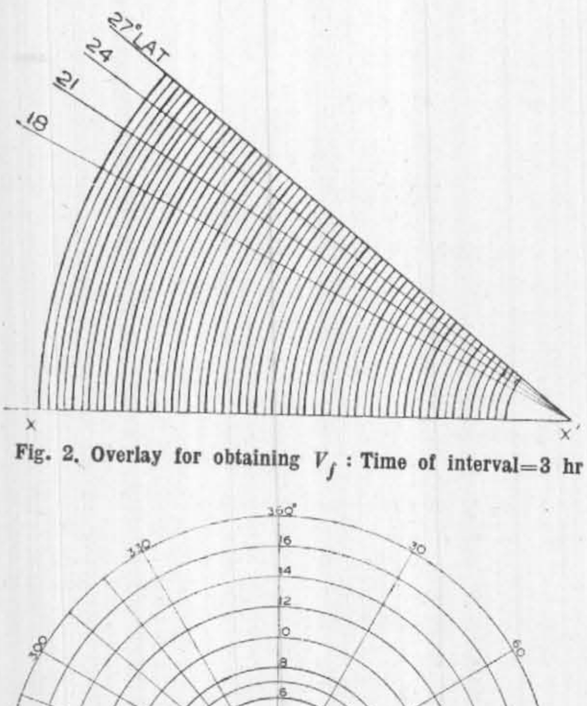


Fig. 2. Overlay for obtaining V_f : Time of interval=3 hr

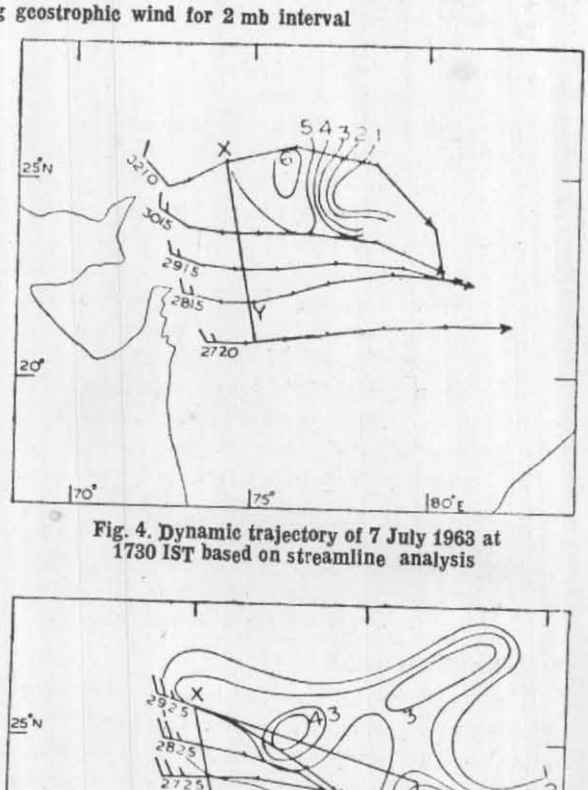


Fig. 4. Dynamic trajectory of 7 July 1963 at 1730 IST based on streamline analysis

Fig. 3. Polar diagram

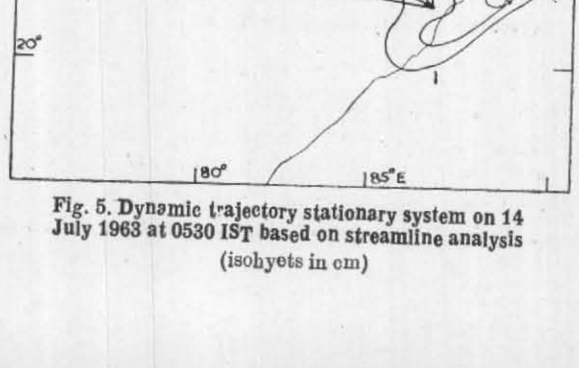


Fig. 5. Dynamic trajectory stationary system on 14 July 1963 at 0530 IST based on streamline analysis

(isohyets in cm)

TABLE 1

Inflow volume and other computed parameters for a few cases

Date	Time* (IST)	L (km)	V_n (kmph)	w (cm)	T	I_v (inflow vol.) $\times 10^{-4}$	Objective forecast pptn (I_v/A)	Observed isohyetal average (cm)	Area (sq. km) $\times 10^4$
7 Jul 1963	1730	400	19	7·1	12	64·7	3·4	3·6	18·8
14 Jul 1963	0530	415	36	5·3	15	118·8	5·7	2·3	20·7
15 Jul 1963	0530	350	38	5·8	15	115·7	7·7	2·5	15·0
13 Sep 1963	0530	470	30	6·5	12	110·0	7·1	5·4	15·5
11 Aug 1964	1730	600	27	6·1	15	148·2	5·2	5·4	28·6
19 Aug 1964	1730	460	16	6·5	15	71·8	3·3	3·0	22·0
25 Aug 1964	1730	290	28	6·2	15	75·5	4·0	2·7	19·4

*Commencement of trajectory

The centre of curvature of the arc, *viz.*, X is made to coincide with the point N, such that the base line XX′ coincides with the line MN. Now the point M is moved along that particular arc which touches the point M till it intersects the relevant latitude line. This point on the polar diagram defines the final wind denoted by P. The mid-point of the line joining P and M of the polar diagram then gives the average wind during the three-hour period. This then is the average trajectory, a particle will make during the period of three hours.

The steps from (1) to (4) are repeated to obtain the trajectories at successive three-hour intervals. Similarly the trajectories are obtained for points B, C, D, etc and are shown from Figs. 4 to 10.

4. Estimation of rainfall

The trajectories (which are nearly four to five in each case) have indicated the convergence areas, and thereby resulting rising of air. If sufficient moisture is also taken up along with this, then precipitation would result. An expression for estimation of rainfall on the basis of dynamic trajectories has been given by Peterson *et. al* (1960) as follows—

$$I_v = LV_n wT$$

where I_v is the inflow volume, L is the distance between points X and Y (*see* Fig. 4 for instance), V_n is the wind speed normal to XY, w is the precipitable water and T is the forecast period.

The computed parameters are shown in Table 1. The observed isohyetal average is also given in the last column of this table.

5. Discussion of Results

The dynamic trajectories have been drawn in a number of cases (a few of them are shown in Figs. 4 to 10). In a majority of cases trajectories have indicated that the heavy rainfall centres are located in the region in between the point of maximum curvature of trajectories and the point where all these appear to converge. An interesting feature of the study is that the maximum rainfall zone was usually not at the point of convergence of trajectories but was located upstream. Study of efficiency of rainfall was also made in a few cases in the following.

The precipitation efficiency E may be defined as $E = P_v/I_v$ where P_v is the precipitation volume and I_v is the inflow volume. The efficiency is based on the supposition that the moisture (which flows into the region) which does not precipitate within the inflow areas is either transported out, or causes an increase in the precipitable water within this area. This efficiency, however, will be required to be determined from a very large number of cases. The precipitation efficiency was found to depend upon two factors, *viz.*, (1) geostrophic vorticity or the shear of geostrophic wind and (2) isobaric gradient. Computations of precipitation efficiency in a few cases reported by Banerji and Rao (*see* Ref.) show that 80-85 per cent of the total possible inflow volume of moisture could be precipitated in the region of study.

The dynamic trajectory technique was also

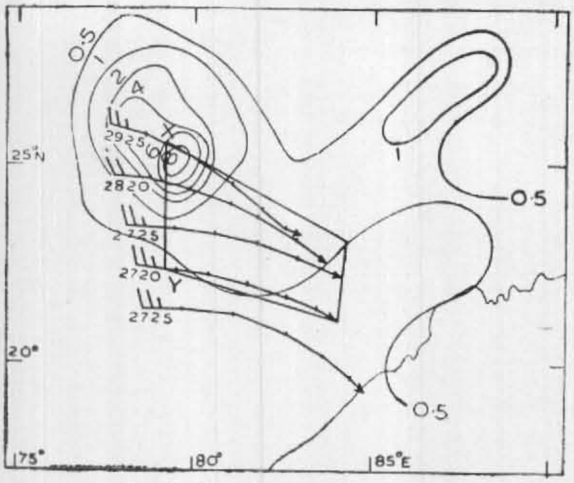


Fig. 6. 15 July 1963 at 0530 IST

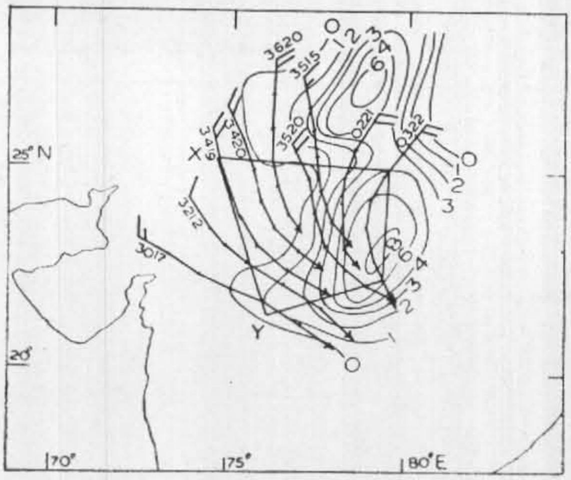


Fig. 7. 13 September at 0330 IST

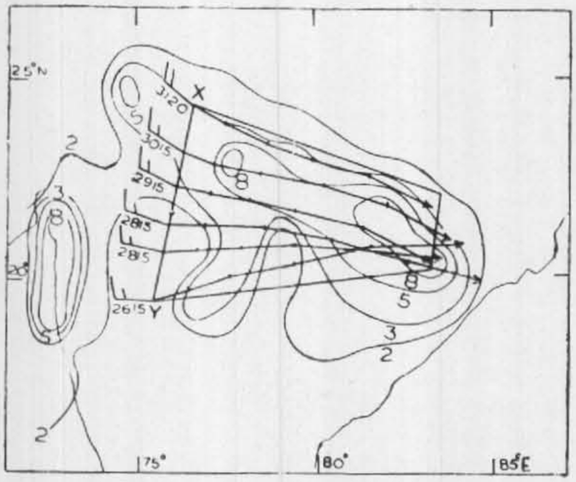


Fig. 8. 11 August 1964 at 1730 IST

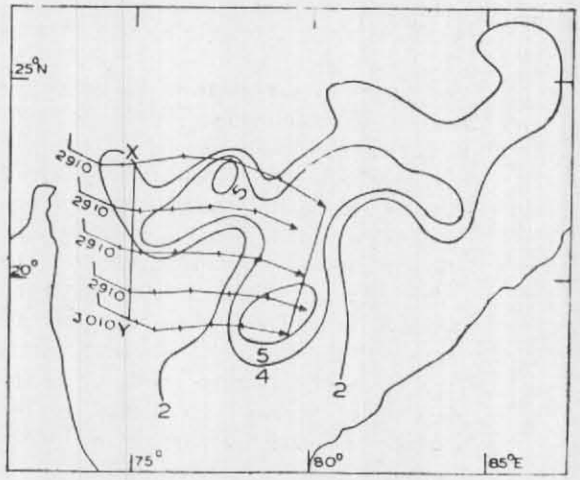


Fig. 9. 19 August 1964 at 1730 IST

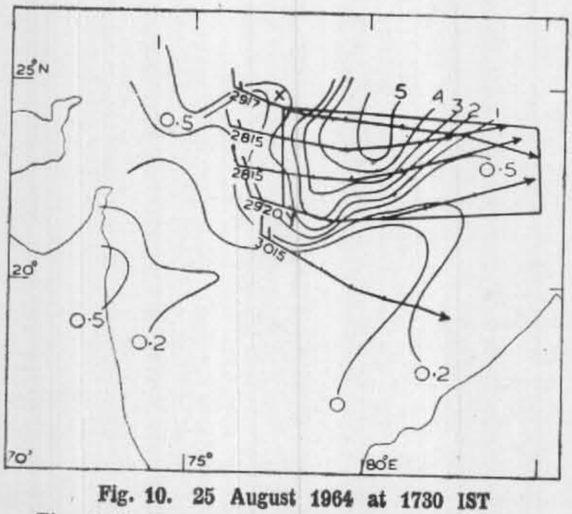


Fig. 10. 25 August 1964 at 1730 IST
Figs. 6-10. Dynamic trajectories based on streamlines (isohyets in cm)

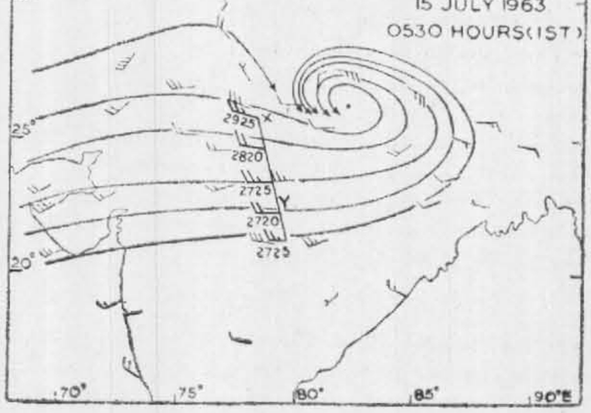


Fig. 11

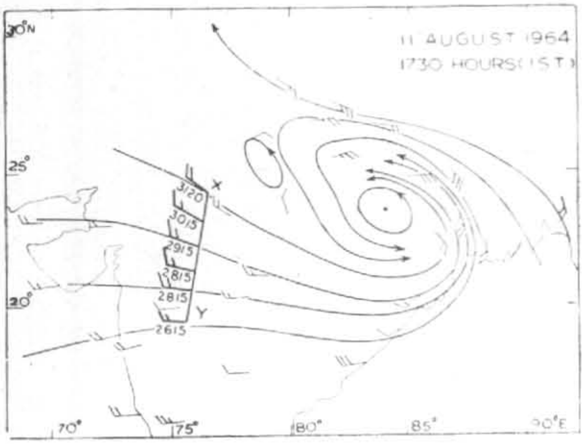


Fig. 12

employed at the National Meteorological Centre at Suitland (U.S.A.) on the basis of National Hurricane Research Project report (Peterson *et. al* 1960) but later computer methods were preferred.

6. Possible sources of error

(*a*) ***Winds and acceleration of air motion*** — The geostrophic deviation or the ageostrophic wind vector plays a very important part, and as such the accuracy of the dynamic trajectory depends on the correct evaluation of geostrophic winds or the pressure analysis. This was explained clearly by Miller (1946). A second point in the source of error is the assumption that the geostrophic wind will be constant over short intervals of time. Although this error can be minimised by selecting a shorter interval of time, in practice, it is known that the changes in wind at a place are not strictly due to ageostrophic component alone. The effectiveness of the trajectory is largely dependent on the accuracy with which the movement and changes of pressure systems are located on the surface map.

(*b*) *Changes in moisture field*—Since precipitable water which is a measure of average water vapour content aloft is used in the formula, it is necessary to forecast this parameter accurately. The changes in water vapour content of the air can take place due to (1) evaporation, (2) condensation and (3) advection. The process involved in evaporation and condensation being rather slow in the atmosphere, the possibility of sudden changes of moisture can be due to advection. One great limitation in this is that aerological charts are not available at intervals less than 12 hr to study the moisture advection.

7. Conclusion

The computation of dynamic trajectories shows that it is possible to locate accurately the areas of large scale convergence. This can also be applied to calculate the expected precipitation within the convergence area. A comparison has been made of computed rainfall depth with the actuals which reveals that the trajectories can be applied to forecasting purposes. The method as applied in this paper, refers to a stationary system in as much as that the geostrophic winds obtained in space are picked and used in the computations. This would involve automatically that the movement of a pressure system in space is such that it does not affect the magnitude of the geostrophic winds used in the calculations. Further investigations are in progress.

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