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A simple method of evaluating vertical velocity over small areas for forecasting heavy rainfall

V. VITTAL SARMA*

Meteorological Office, New Delhi

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ABSTRACT Vertical motion is an important parameter in any study of the atmospheric process. Evaluation of this parameter from derived values of divergence and vorticity involve elaborate calculations. A simple and quick method by adapting the vorticity equation in a simplified form has been attempted in the present study. The method has been applied to a small area formed by four pibal stations in Gujarat. Vertical velocity for six layers between surface and 700 mb were calculated. With these values rainfall over the area was also computed and compared with the actual observed rainfall. The results are compared and discussed.

1. Introduction

Vertical motion on the scale of the synoptic charts is an important parameter in any study of the atmospheric processes by the dynamic meteorologist. Direct measurements of this parameter are however not possible. Computations have to be made to evaluate vertical velocity from other derived parameters like divergence and vorticity. These methods involve elaborate calculations and can be attempted on an operational basis only with the aid of computers.

A simple and quick method of determining vertical velocity from the observed winds will be very helpful for practical purposes of forecasting heavy rainfall over small areas at any meteorological office required to issue warnings to various public services like railways., irrigation etc. Several attempts have been made in this direction and a number of original papers and exhaustive reviews are available in recent meteorological literature (Billa and Nedungadi—*See* Ref.). Some of these methods are fraught with practical difficulties on account of inadequate data or incomplete knowledge of the physical process of the atmosphere. The forecaster faces a special situation when the area for heavy rainfall warning includes a coastal region, with a large expanse of water adjoining it. For instance, if we consider the Gujarat region, upper air data is absent over a large area west of the Gujarat coast. The forecaster has to resort to methods of extrapolation from data of coastal stations separated by thousands of miles. Alternately some simplifying assumptions and approximations can be made in the vorticity equation, so that a simple and a quick method of estimating vertical velocity over a small area adjoining the sea is obtained. With this object, an attempt has been made in the present note to compute vertical velocity over a small area formed by a grid of four pilot balloon stations.

2. A simple form of vorticity equation

Following the usual notations, the equation of vorticity in isobaric co-ordinates can be written as

$$\frac{d}{dt}\left(\zeta+f\right) = \left(\zeta+f\right)\frac{\partial\omega}{\partial p} \tag{1}$$

Assuming that ζ is very much smaller than f, we can write

$$\frac{d}{dt}\left(\zeta+f\right) = f\,\frac{\partial\omega}{\partial p} \tag{2}$$

The left hand side of this equation representing the total rate of change of vorticity can be simplified as follows :

$$\frac{d}{dt}\left(\zeta+f\right) = \frac{d\zeta}{dt} + \frac{df}{dt} \tag{3}$$

*Present affiliation: Central Water & Power Commission, Hydrology Directorate, New Delhi-22

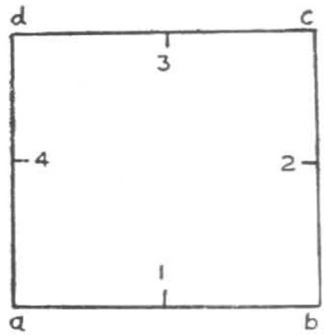

Fig. 1

$$\text{Now} \quad \frac{df}{dt} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} \tag{4}$$

The first two terms $\partial f/\partial t$ and $u(\partial f/\partial x)$ are zero since f is only a function of the angular velocity of the earth and of the latitude. The last term $v(\partial f/\partial y)$ is assumed to be small in comparison with $\partial\zeta/\partial t$ as for predominantly zonal flow. As a first approximation, this may be neglected in comparison with changes in the relative vorticity of the field of motion, *i.e.*, in comparison with changes in the curvature and shear of the isobars or contours.

$$\text{Since} \quad \frac{d\zeta}{dt} = \frac{\partial\zeta}{\partial t} + u\frac{\partial\zeta}{\partial x} + v\frac{\partial\zeta}{dy} \tag{5}$$

Eq. (2) can be rewritten as

$$\frac{\partial\zeta}{\partial t} + u\frac{\partial\zeta}{\partial x} + v\frac{\partial\zeta}{\partial y} = f\frac{\partial\omega}{\partial p} \tag{6}$$

Integrating over a small area A, we have

$$\frac{\partial}{\partial t}\iint_A \zeta\,dx\,dy + \iint_A\left(u\frac{\partial\zeta}{\partial x} + v\frac{\partial\zeta}{\partial y}\right)dxdy = f\frac{\partial}{\partial p}\iint_A \omega\,dx\,dy \tag{7}$$

3. Expression for vertical velocity in terms of u, v components

Now let us consider a rectangular area A formed by four pilot balloon stations a, b, c and d as shown in Fig. 1. The u and v components at these points are suffixed a,b,c and d and at the midpoints between two stations 1, 2, 3 and 4; u_1, v_1 etc are taken as mean of the values at stations a and b and so on.

The vorticity ζ at any point is

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{8}$$

Since we are considering an area A we have

$$\bar{\bar{\zeta}} = \iint_A \zeta\,dx\,dy \tag{9}$$

Substituting suitably, Eqns. (9) and (7) can be written as

$$\bar{\bar{\zeta}} = \int(v_2-v_4)\,dy - \int(u_3-u_1)\,dx \tag{10}$$

$$\text{and,} \quad \frac{\partial\bar{\bar{\zeta}}}{\partial t} + \int\Big[(u\zeta)_2 - (u\zeta)_4\Big]dy + \int\Big[(v\zeta)_3 - (v\zeta)_4\Big]dx = f\frac{\partial\omega}{\partial p} \tag{11}$$

The last two terms in the left hand side of Eq. (11) can be simplified by substituting for ζ and rearranging the terms as shown below:

$$\int\Big[\Big(u\zeta\Big)_2 - \Big(u\zeta\Big)_4\Big]dy = \int\Big[u_2\Big(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\Big)_2 - u_4\Big(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\Big)_4\Big]dy$$

$$= \int\Big[u_2\Big(\frac{\partial v}{\partial x}\Big)_2 - u_4\Big(\frac{\partial v}{dx}\Big)_4\Big]dy - \int\Big[u_2\Big(\frac{\partial u}{\partial y}\Big)_2 - u_4\Big(\frac{\partial u}{\partial y}\Big)_4\Big]dy$$

$$= \int\Big[u_2\Big(\frac{v_2-v_4}{\triangle x}\Big) - u_4\Big(\frac{v_2-v_4}{\triangle x}\Big)\Big]dy - \tfrac{1}{2}\Big(u_c^2 - u_b^2\Big) + \tfrac{1}{2}\Big(u_d^2 - u_a^2\Big)$$

$$= \int\frac{u_2-u_4}{\triangle x}\Big(v_2 - v_4\Big)dy - \Big(u^2{}_c - u_b^2\Big) + \tfrac{1}{2}\Big(u_d^2 - u_a^2\Big) \tag{12}$$

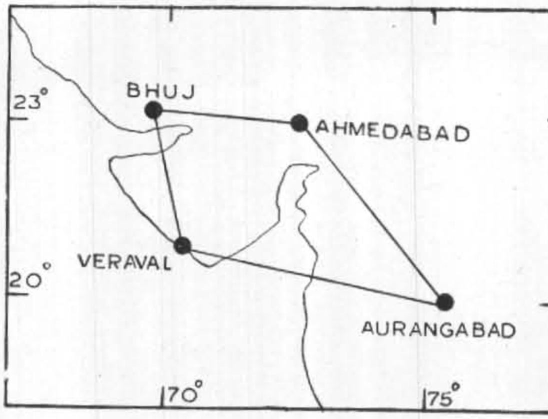

Fig. 2

Similarly, the other term car also be expressed in terms of u, v components as

$$\int \left[\left(v\zeta \right)_3 - \left(v\zeta \right)_4 \right] dx = \tfrac{1}{2} \left(v_c^2 - v_d^2 \right) + + \tfrac{1}{2} \left(v_b^2 - v_a^2 \right) - \int \frac{v_3 - v_1}{\triangle y} \left(u_3 - u_1 \right) dx \quad (13)$$

Thus by means of Eqns. (10), (12) and (13), the left hand side of equation can be expressed in terms of u, v components of winds at the stations. If we denote this value by $G_{(p)}$ then

$$f \frac{\partial}{\partial p} \int \overline{\overline{w}} = G_{(p)}$$

$$\text{Or,} \qquad \frac{\partial}{\partial p} \int \overline{\overline{w}} = \frac{1}{f} G_{(p)} \quad (14)$$

Integrating between two layers p_o and p and assuming the vertical velocity at the ground level p_o to be zero, we have the vertical velocity between layers p_o and p

$$\overline{\overline{w}}_p = - \frac{1}{f} \int_p^{p_0} G_{(p)} \, dp \quad (15)$$

The vertical velocities can be calculated for any number of layers by substituting successive values of $\overline{\overline{w}}_p$ for the lower layer. Since $G_{(p)}$ has been expressed in terms of u,v components, the calculation of vertical velocity reduces to a small number of operations involving constants, sums and differences of u, v components or their squares. This can be done within a few minutes manually.

4. Application of the method to Gujarat area

For a practical application of finding vertical velocity by this method, a small area in Gujarat bounded by the four pilot balloons stations, *viz*, Veraval, Aurangabad, Ahmedabad and Bhuj was considered (Fig. 2). It will be seen that this actual 4-station grid deviates to some extent from a regular rectangle considered for deriving the expression for vertical velocity above. For the sake of simplicity, the values were not interpolated at regularly spaced grid points. This no doubt introduces some limitation in this method of computation.

This grid is located on the Gujarat coast in one of those areas where the forecaster has to depend on extrapolation for want of upper air data in the Arabian Sea to the west of the grid. Unlike the coastal belt further south of this grid where the rainfall is more influenced by the orographic lift due to the Western Ghats, the area under consideration is mostly in the plains. Large scale convergence associated with marked increase in vertical velocity can result in heavy rainfall over this region

Daily values of vertical velocity were calculated for two continuous periods of 30 days each during the monsoon season of 1964 and 1965 These two periods consisted of alternate dry and wet spells, some of them recording heavy rainfall. Values of $\overline{\zeta}$ were calculated with the wind data observed at 00 and 12 GMT and their difference used for $\partial\overline{\zeta}/\partial t$, the first term on the left hand side of equation (11). This is then added to the other two terms to obtain $G_{(p)}$ for this particular layer. The upper limit of the layer for the computation of vertical velocity was limited to 700 mb (3 km) since on many days during the monsoon season,

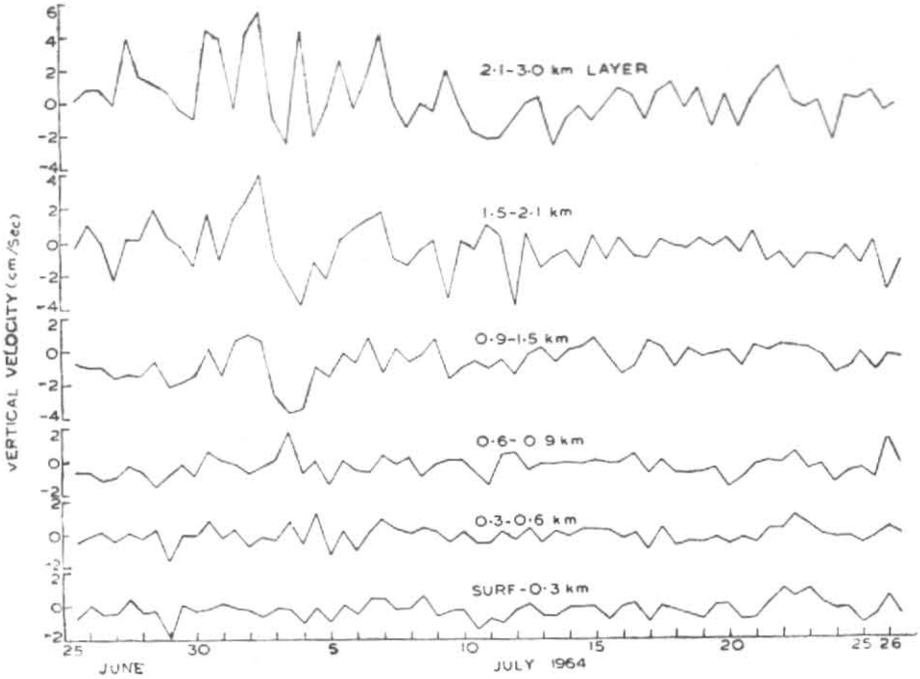

Fig. 3. Daily values of vertical velocity for the six layers (surface to 3 km)

pibal data were not available. However for this layer, all available data for the six levels were used and vertical velocities computed. This would enable one to take into account any shallow circulation in the lower troposphere leading to convergence and vertical motion. It is a familiar experience of forecasters in India that during monsoon months such shallow circulations in the lower troposphere below 3 km cause fairly widespread distribution of rainfall.

Figs. 3 and 4 show the daily values of vertical velocity at 00 and 12 GMT computed for six layers between surface and 3 km. There is a marked variation of the vertical velocity with a number of peaks and a large range from —6 cm/sec to +9 cm/sec. The vertical velocities generally increase with height and significant upward velocities are observed above the 850-mb level. It is seen that the maximum values of positive vertical velocity and also the increasing trend are associated with subsequent rainfall over the area whereas dry spells are preceded by either small or negative values of vertical velocity. Similar characteristics and velocities of this magnitude have been obtained by workers in India (Das 1951, Banerji *et al.* 1967) following more elaborate methods.

5. Computation of rainfall

Having obtained the vertical velocity, the next attempt was to compute the rate of precipitation and made a quantitative estimate of the rainfall over the area by assuming a saturated atmosphere. Different methods have been adopted by various workers for this purpose. In a recent study Sarker (1966) has used the following equation for the rainfall intensity (mm/sec)—

$$I=0\cdot036\,[\rho_0 W_0\,(x_0-x')+\rho_1 W_1\,(x'-x_1)] \quad (16)$$

where

ρ = density of dry air (kg/m^3)

W = vertical velocity (cm/sec)

x = humidity mixing ratio of saturated air, the suffixes representing the bottom and the top of the layer (gm.kg/m)

x' at the middle of the layer is taken as $\frac{1}{2}(x_0+x_1)$.

This derivation is an improvement on the earlier formulae. It considers both the continuity of moisture and the continuity of mass when there is divergence or convergence within a layer. Humidity mixing ratio data obtained from the radiosonde ascents of Ahmedabad and the computed values of vertical velocity were used in the above equation, to find the rate of precipitation and thence the total rainfall for 24 hours. These were compared with the actual rainfall over the area represented by the mean of eleven India Meteorological Department reporting stations inside or very close to the grid in question. Figs. 5 and 6 show the two comparative curves of observed and computed rainfall for the two periods in 1964 and 1965.

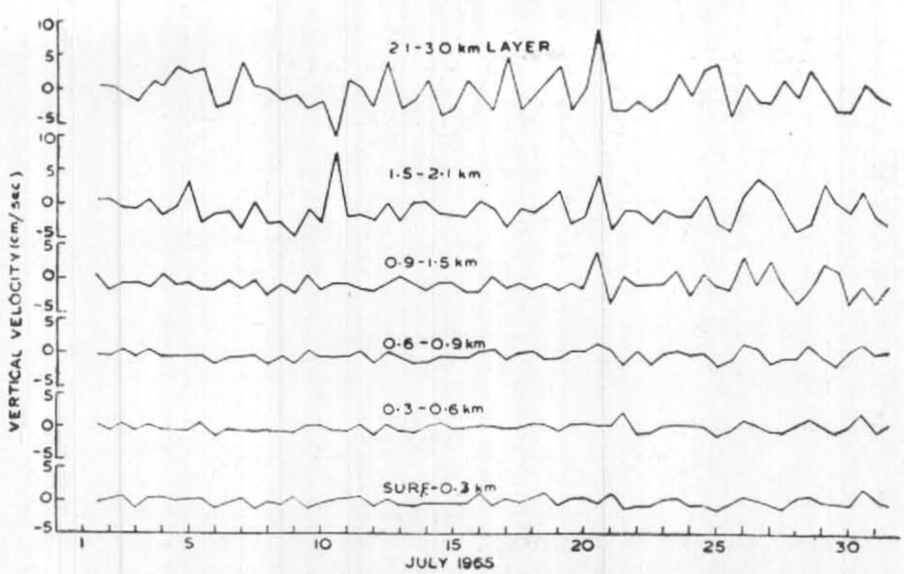

Fig. 4. Daily values of vertical velocity for the six layers (surface to 3 km)

6. Discussion

In both the cases, particularly for the period 27 June to 27 July 1964, there are a number of occasions when the peaks of the two curves show a good agreement. It will also be seen that the trend of actual rainfall variation is also indicated by the computed rainfall curve.

Although there is a general agreement, it will be noticed that there are some exceptions when neither the heavy rainfall nor the increasing trend was shown by the computed rainfall curve. It is also evident that the computed values are always much less than the actual rainfall recorded.

Obviously there are some limitations in this method of computing both the vertical velocity and rainfall. It may be reiterated that the derivation involves certain simplifying assumptions. A plausible explanation for the lower values of computed rainfall is that the vertical velocity values were calculated for only the lower troposphere extending to 700 mb. It is well known that the moisture in the westerlies during the monsoon extends to higher levels in this region and hence it is possible that there is a significant contribution to the precipitation from the higher layers. Banerji *et al.* (1967) have shown that only 35 per cent of the rainfall recorded is contributed by the layer between 850 and 700 mb. In view of the paucity of data for higher levels, this limitation will not be overcome until more rawin stations are established n place of the existing pilot balloon stations.

Secondly, the humidity mixing ratio values of Ahmedabad may not be adequately representative of moisture conditions inside the grid. Since Ahmedabad is situated on the eastern boundary of the four station grid and the prevailing winds in the lower troposphere are westerly during the monsoon season, the humidity values at Ahmedabad are expected to be lower than the true mean for the area. Consequently the rainfall computed will also be lower than the observed values. Some test calculations were made to substitute the humidity data of Bombay situated on the west coast instead of Ahmedabad. Although the rainfall amounts were comparable, this could not also be satisfactory since Bombay is located too far away from the area under discussion. It might be pointed out that without proceeding to the quantitative estimate of the rainfall, the forecaster can utilise this method to compute vertical velocity and study the trend of vertical motion in the lower troposphere. These indications together with the analysis of the synoptic charts might be of use to issue a heavy rainfall warning without elaborate calculations at least in the case of a coastal region where there is an additional handicap owing to lack of data from the sea area.

With the proposed net work of more rawin stations and additional radiosonde stations in this region, it might be possible to improve the

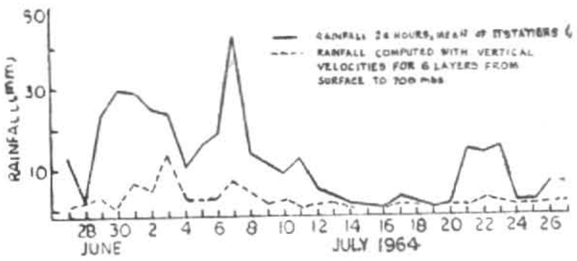

Fig. 5

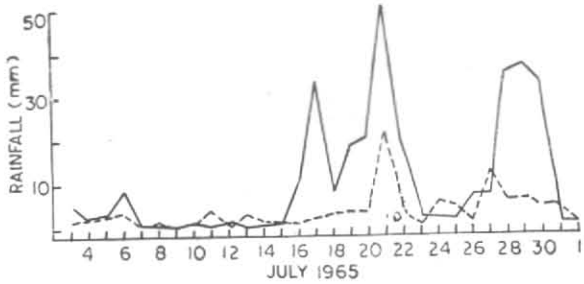

Fig. 6

estimate of vertical velocity and rainfall by this simple method.

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