

Diagnostic study of a monsoon depression by Geostrophic Baroclinic Model*

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ABSTRACT. Vertical velocities at 850, 700, 500 and 300-mb surfaces, associated with a monsoon depression have been calculated making use of a 4-level geostrophic baroclinic model. The region of upward vertical velocities agrees well with the region of rainfall.

The contributions by the vorticity advection and divergence terms in the vorticity equation have been evaluated, and the importance of the divergence term in formulating a numerical model for the monsoon is brought out.

1. Introduction

The general synoptic features of monsoon depressions have been studied in the past by several workers (Ananthkrishnan and Bhatia 1960, Koteswaram and George 1958 and 1960, Koteswaram and Rao 1963, Malurkar 1944 and 1950, Pisharoty and Asnani 1957 and Rao and Jayaraman 1958). Also, a preliminary attempt to forecast the 500-mb pattern in the case of the monsoon depression by graphical techniques was made by Das and Bose (1958). In this paper, a diagnostic study of the depression has been attempted by numerical methods making use of a 4-level geostrophic baroclinic model.

2. List of symbols

- \mathbf{V} Geostrophic wind velocity
 ζ Relative geostrophic vorticity
 f_0 Standard value of the Coriolis parameter
 ϕ Geopotential
 Z Height of an isobaric surface
 Ψ Stream function for the nondivergent component of velocity

$\omega = \frac{dp}{dt}$ Individual change of pressure

- σ Static stability
 θ Potential temperature
 α Specific volume

∇ Isobaric gradient operator $\left(\mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} \right)$

∇^2 Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$

$J(A, B)$ Jacobian operator

$$\left(\frac{\partial A}{\partial x} \frac{\partial B}{\partial y} - \frac{\partial A}{\partial y} \frac{\partial B}{\partial x} \right)$$

- \mathbf{k} Unit vector in the vertical
 u Wind speed in the X-direction
 v Wind speed in the Y-direction

3. Basic Equations

As the basic equation of motion, we adopted the quasi-geostrophic system proposed by Charney (1948). In this system the Coriolis parameter is regarded as constant except when it is differentiated with respect to latitude, several terms in the vorticity equation (the vertical advection term, the twisting term) are omitted, the divergence equation is replaced by the geostrophic relation, and the measure of static stability in the thermodynamic equation is treated as a function only of pressure. Since the Rossby Number is estimated to be about 0.4 over India, while it is about 0.1 in the middle latitudes, the quasi-geostrophic system of equations is likely to yield less accurate results in low latitudes than in the middle latitudes. Still the quasi-geostrophic system has been used so that it may serve as a first approximation. The equations used in this study are as follows.

The vorticity equation is given by—

$$\frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla (\zeta + f) = f_0 \frac{\partial \omega}{\partial p} \quad (1)$$

The thermodynamic energy equation for adiabatic motion is given by —

$$\frac{\partial}{\partial t} \left(\frac{\partial \phi}{\partial p} \right) + \mathbf{V} \cdot \nabla \left(\frac{\partial \phi}{\partial p} \right) + \sigma \omega = 0 \quad (2)$$

*Based on Sci. Rep. No. 54 of the Institute of Tropical Meteorology, Poona

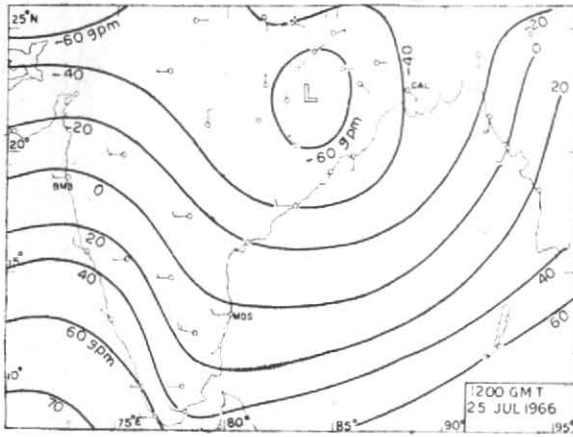


Fig. 1. 1000-mb chart at 12 GMT on 25 July 1966

TABLE 1

Values of static stability σ at different levels

Level (mb)	Static stability $m^2/mb^2/sec^2$
850	1.463×10^{-2}
700	2.453×10^{-2}
500	4.095×10^{-2}
300	6.002×10^{-2}

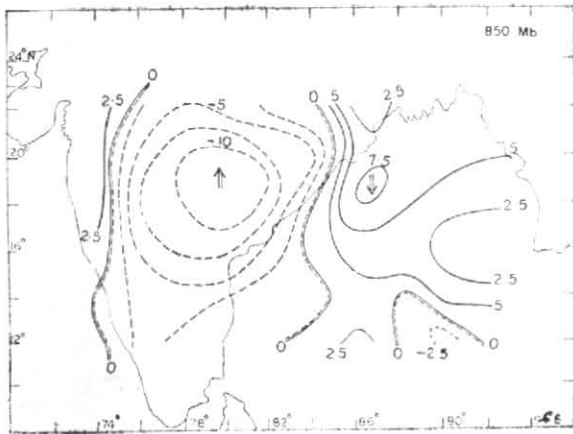


Fig. 2(a). 850-mb surface

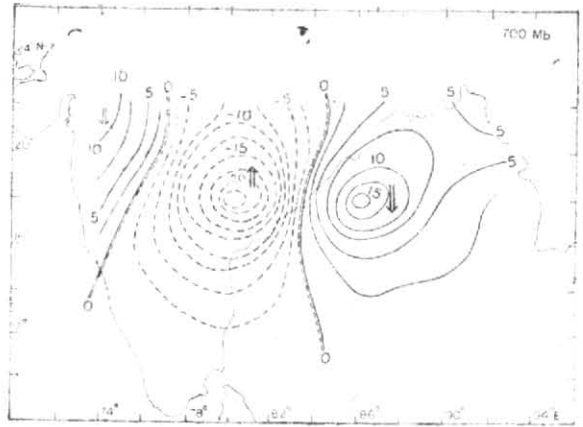


Fig. 2 (b). 700-mb surface

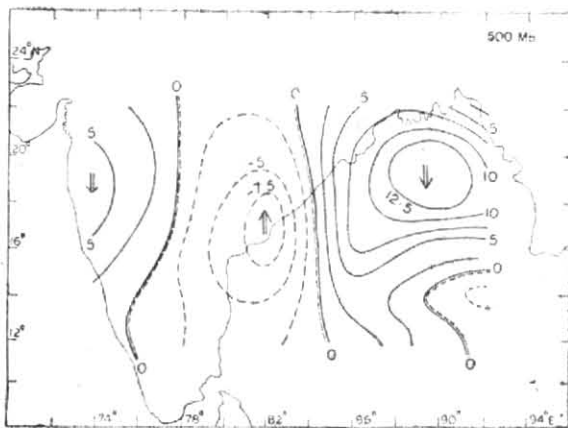


Fig. 2(c). 500-mb surface

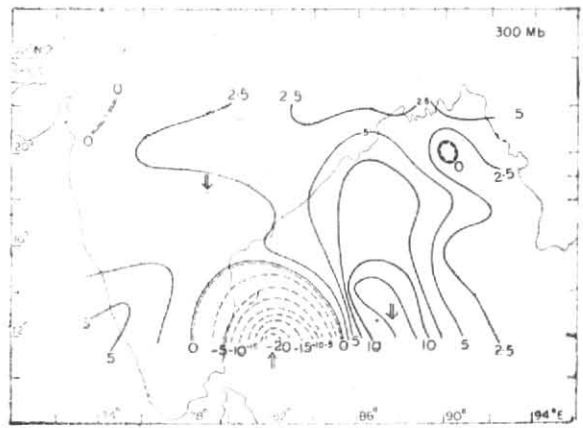


Fig. 2(d). 300-mb surface

Fig. 2. Vertical velocity ω at different surfaces, 1200 GMT, 25 July 1966

$$\left. \begin{aligned} \mathbf{V} &= -\frac{1}{f_0} \mathbf{k} \times \nabla \phi = -\mathbf{k} \times \nabla \psi \\ \zeta &= \frac{1}{f_0} \nabla^2 \phi = \nabla^2 \psi \end{aligned} \right\} \quad (3)$$

$$\alpha = -\frac{\partial \phi}{\partial p}; \quad \sigma = -\alpha \frac{\partial \ln \theta}{\partial p} \quad (4)$$

Static stability σ is considered a constant in any isobaric surface, *i.e.*, $\sigma = \sigma(p)$. As pointed out by Hollmann (*see Ref.*) when the geostrophic approximation is applied in the thermodynamic equation, the horizontal variation of σ has to be omitted, so that the enthalpy does not increase with increasing kinetic energy.

Using equations (1) and (2) we obtain the ω -equation—

$$\sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} = f_0 \frac{\partial}{\partial p} \left[\mathbf{V} \cdot \nabla (\zeta + f) \right] - \nabla^2 \left[\mathbf{V} \cdot \nabla \left(\frac{\partial \phi}{\partial p} \right) \right] \quad (5)$$

Krishnamurti and Baumhefner (1966) in their study of a tropical disturbance based on a solution of multilevel baroclinic model have shown that the differential vorticity advection and the Laplacian of the thermal advection (that is, the two terms on the right hand side of Eq. 5) contribute about equally and have the same sign, and contributions by the other terms in the more elaborate ω -equation, such as the twisting term and the vertical advection term, are negligible. Further, as shown by Eliassen (1956), these two terms may be combined into one, and the ω -equation used in the paper is—

$$\sigma \nabla^2 \omega + f_0^2 \frac{\partial^2 \omega}{\partial p^2} \approx f_0 \frac{\partial \mathbf{V}}{\partial p} \cdot \nabla (2\zeta + f) \quad (6)$$

where terms like,

$$2 \left[\frac{\partial \mathbf{V}}{\partial x} \cdot \nabla \left(\frac{\partial^2 \phi}{\partial x \partial p} \right) + \frac{\partial \mathbf{V}}{\partial y} \cdot \nabla \left(\frac{\partial^2 \phi}{\partial y \partial p} \right) \right]$$

on the right hand side of Eq. (6) have been neglected. According to this equation, ascending motion occurs in regions where the thermal wind blows from high to low vorticity, descending motion where the thermal wind blows from low to high vorticity. In the monsoon case, the thermal wind is easterly throughout, though the monsoon current is westerly. So in the region between the ridge line and the trough line, the easterly thermal wind blows from high values of vorticity at the trough region

to the low values at the ridge line, and so upward motion is to be expected in this region and downward motion in the region between the trough and the ridge lines.

The right hand side of Eq. (6) can be shown to be identical with Sutcliffe's expression for the divergence of the thermal wind when the same is evaluated as a mean value in an isobaric layer (Eliassen 1956). If \mathbf{V}_1, ζ_1 and \mathbf{V}_2, ζ_2 denote the velocities and vorticities at two pressure levels p_1 and p_2 ($p_1 > p_2$), and if we write $\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{V}_T$, $\zeta_2 - \zeta_1 = \zeta_T$, then the mean value in the layer between p_1 and p_2 of the right hand side of Eq. (6) may be approximated by—

$$\begin{aligned} & -\frac{f_0}{p_1 - p_2} \mathbf{V}_T \cdot \nabla (\zeta_1 + \zeta_2 + f) \\ & = -\frac{f_0}{p_1 - p_2} \mathbf{V}_T \frac{\partial}{\partial s} (2\zeta_1 + \zeta_T + f) \end{aligned}$$

where $\partial/\partial s$ denotes differentiation in the direction of \mathbf{V}_T . Thus except for the constant factor, this expression is identical with Sutcliffe's expression for the divergence of the thermal wind. But Eq. (6) is somewhat more general than Sutcliffe's though the physical content is the same.

4. Computations

A weak depression formed over the northwest Bay of Bengal on the morning of 25 July 1966 and its location on 1000-mb surface at 1200 GMT of that evening is shown in Fig. 1. Charts for 850, 700, 500, 300 and 150-mb surfaces were analysed only for the isobaric heights.

The area for which height values were read off at 2-degree grid intervals is marked by the inner boundary in Fig. 1. The area north of 26°N was not included partly due to paucity of data in the Himalayan region and also partly to exclude the boundary effects of the Himalayas.

The values for static stability σ for the various constant pressure surfaces were computed from climatological mean values (*see Table 1*). Computations of vorticity and advection of vorticity, and relaxation were carried out with the IBM 1620 Computer at the Institute.

(a) *Vertical velocity*—The boundary conditions for ω , are $\omega = 0$ at 1000 and 150-mb and at the lateral boundary of the region concerned on each pressure surface. Three dimensional relaxation of the ω -equation was carried out and the computed values of ω at 850, 700, 500 and 300-mb surfaces are given in Fig. 2. The rainfall amounts reported for 24 hrs ending at 03 GMT of 26 July 1966 are shown in Fig. 3.

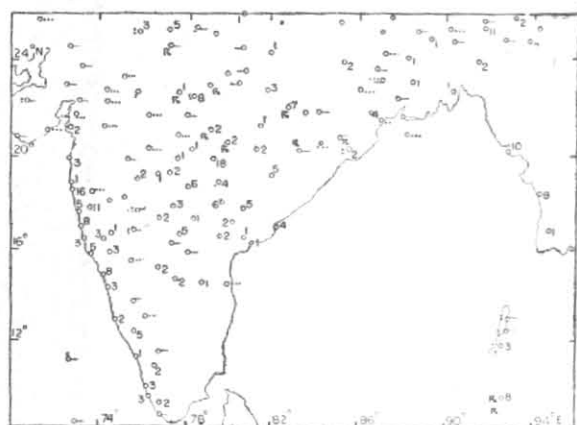


Fig. 3. Rainfall amounts (cm) for 24-hr ending 03 GMT of 26 July 1966

The region of upward vertical velocities at 850 and 700-mb surfaces lies to the southwest of the low pressure area at 1000-mb level and agrees well with the region of precipitation, *i.e.*, the region between 75° and 82°E and 15° and 23°N. It may be pointed out, however, that the rainfall along the west coast where the orography plays a dominant role cannot be explained by this model, since ω is assumed to be zero at 1000-mb level.

Above 500-mb, the wave pattern in the westerlies has disappeared, and at 300 mb and above, the flow pattern south of 25°N is easterly. The pattern of vertical velocity of 300 mb is also different from those at 850, 700 and 500-mb surfaces.

The value of 1×10^{-4} mb/sec for ω corresponds roughly to a vertical velocity of 1 mm/sec and maximum value of about 24 mm/sec occurs for upward motion at 700-mb surface. This is in good agreement with the value of 2 cm/sec obtained by Krishnamurti (1966) in his study of the Indian monsoon circulation on 24 August 1961 by the balanced model.

As may be seen in Fig. 3, large amounts of rainfall reported in some of the stations in the region under consideration were associated with thunderstorms and convective activity but the values of vertical velocity computed do not bring out this feature, because as pointed out by Charney (*loc. cit.*) the vertical velocity calculated by the geostrophic models cannot exceed the value of 10 cm/sec. But it is interesting to observe that convective activity has occurred in the region of large scale vertical motion indicated by the geostrophic model. According to Krishnamurti (1966) also, orographic convective activity becomes more intense and widespread when dynamically produced rising

centres due to large scale motion overlie these regions. It is also relevant to point out that we have assumed adiabatic motion and hence have not taken into account the effect of release of latent heat due to condensation on the development of vertical velocities. The inclusion of latent heat in the ω -equation results in an increase in the magnitude of the computed vertical motion as has been shown by Danard (1966).

The computed distributions of ω can be understood qualitatively by examining a linearised perturbation equation. If the actual motion is thought of as zonally averaged motion on which the disturbance is superimposed we can separate the observation into two components, the zonal mean component denoted by a bar on the quantity, and the perturbation component by the primed quantities

$$\left. \begin{aligned} \left(\bar{\quad} \right) &= \frac{1}{2\pi} \int_0^{2\pi} (\quad) d\lambda \\ \mathbf{V} &= \bar{\mathbf{V}} + \mathbf{V}' \\ u &= \bar{u} + u' \\ v &= v' \\ \omega &= \bar{\omega} + \omega' \\ \phi &= \bar{\phi} + \phi' \\ \Psi &= \bar{\Psi} + \Psi' \end{aligned} \right\} \quad (7)$$

Substituting Eq. (7) in Eq. (6) gives —

$$\begin{aligned} \sigma \nabla^2 (\bar{\omega} + \omega') + f_0^2 \frac{\partial^2}{\partial p^2} (\bar{\omega} + \omega') \\ = f_0 \frac{\partial}{\partial p} (\bar{\mathbf{V}} + \mathbf{V}') \cdot \nabla [2(\bar{\zeta} + \zeta') + f] \end{aligned} \quad (8)$$

On picking out the linear perturbation terms, the differential equation of ω' is obtained as,

$$\begin{aligned} \sigma \nabla^2 \omega' + f_0^2 \frac{\partial^2 \omega'}{\partial p^2} = f_0 \left[2 \frac{\partial \bar{\mathbf{V}}}{\partial p} \cdot \nabla \zeta' + \frac{\partial \mathbf{V}'}{\partial p} \cdot \nabla \bar{\zeta} + \frac{\partial \mathbf{V}'}{\partial p} \cdot \nabla f \right] \end{aligned} \quad (8a)$$

$$= f_0 \left[2 \frac{\partial \bar{u}}{\partial p} \frac{\partial \zeta'}{\partial x} + \beta \frac{\partial v'}{\partial p} \right] \quad (8b)$$

as the second term in Eq. (8a) vanishes, when we assume $\bar{\zeta} = \partial \bar{u} / \partial y = \text{constant}$. If we assume the solution for ω' to be of the form,

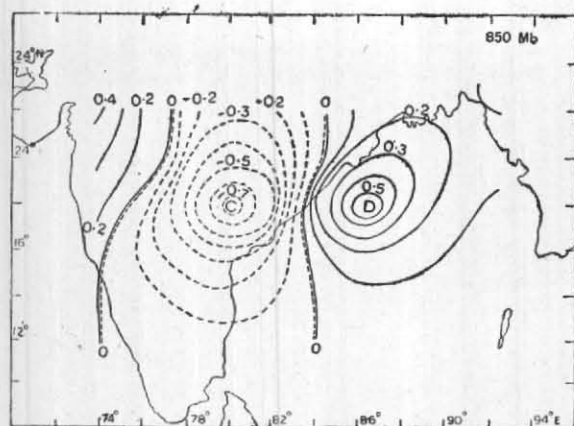


Fig. 4 (a). 850-mb surface

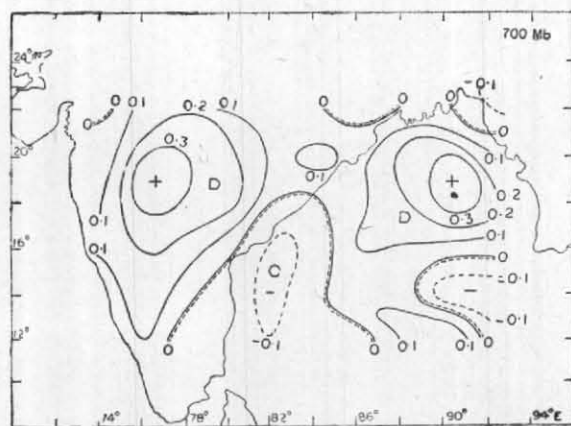


Fig. 4(b). 700-mb surface

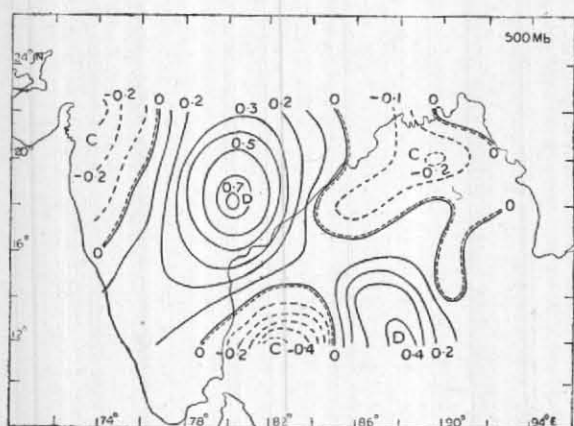


Fig. 4 (c). 500-mb surface

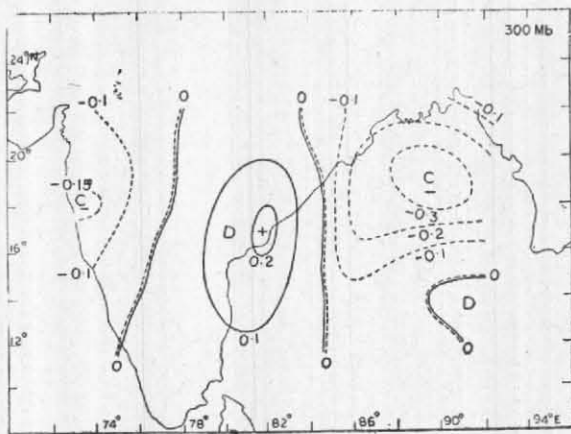


Fig. 4 (d). 300-mb surface

Fig. 4. Divergence at different surfaces at 12 GMT on 25 July 1966

$$\omega' = A \sin kx \sin (n\pi p / p_s)$$

and if L and P are characteristic values for length and pressure,

$$\nabla^2 = -1/L^2, \quad \partial^2 / \partial p^2 = -1/P^2,$$

then,

$$\omega' = \frac{f_0 P^2}{\sigma P^2 + f_0^2} \left[2 \frac{\partial \bar{u}}{\partial p} v' - \beta L^2 \frac{\partial v'}{\partial p} \right] \quad (9)$$

(Murakami—see Ref.)

The second term due to β -effect is not important in our case, as the wave length of the disturbance is of the order 2000-3000 km. The sign of ω' depends only on the first term $2(\partial \bar{u} / \partial p) v'$ and in the monsoon season, $\partial \bar{u} / \partial p$ is positive since the westerlies decrease with height. Therefore ω' will be positive or negative as v' is positive or negative.

Accordingly, ω' is negative in the region from the ridge line to the trough line, where v' is negative;

and ω' is positive from the trough to the ridge line where v' is positive. In other words, upward vertical velocities occur where there is warm advection (i.e., where $-v'(\partial T / \partial y)$ is positive), and downward vertical velocities occur in regions of cold advection. In the monsoon season, $\partial T / \partial y$ is generally positive from the surface level upward, and warm advection can take place only in regions of winds with a northerly component, and hence upward vertical motion occurs just to the west and southwest of a low or depression. It may, therefore, be possible to regard that the monsoon low or depression is a disturbance embedded in a large scale phenomenon of a wave in the lower tropospheric westerlies of the monsoon season.

In order to bring out the correspondence between the direction of the thermal wind and areas of ascent or descent, the thickness pattern for the layer between 850 and 500 mb is presented in Fig. 5(a). The geostrophic vorticity pattern at 700-mb is

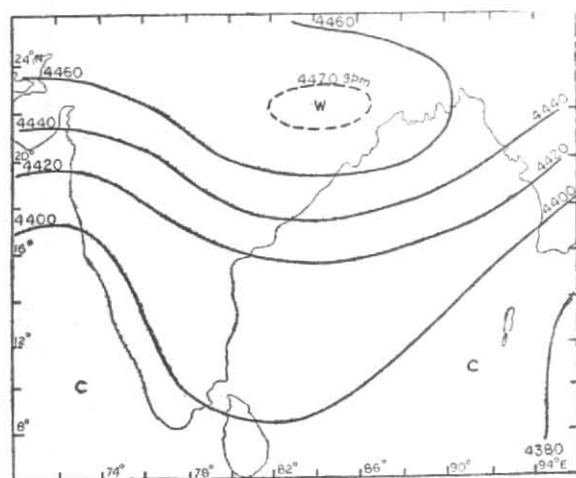


Fig. 5 (a). Thickness pattern for 850-500 mb layer at 12 GMT on 25 July 1966

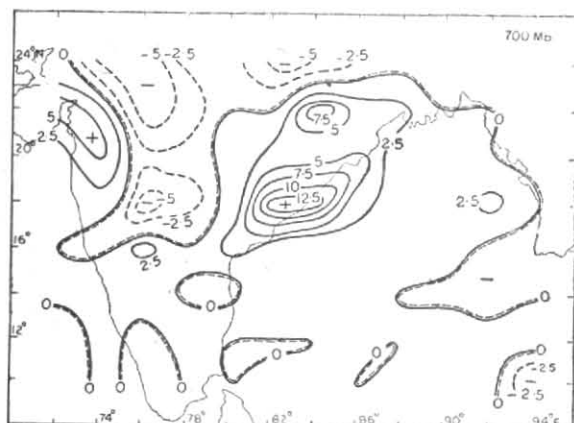


Fig. 5(b). Geostrophic relative vorticity at 700-mb surface at 12 GMT on 25 July 1966

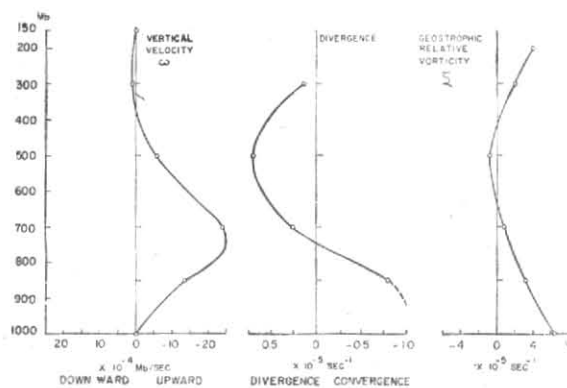


Fig. 6. Vertical velocity, divergence, and geostrophic relative vorticity at 18°N, 80°E

illustrated in Fig. 5(b). It is readily seen from these two charts, and also from Fig. 2(b) that upward vertical velocities occur in regions where the thermal wind blows in the direction of vorticity gradient and downward vertical velocities in regions where the thermal wind and the vorticity gradient are oppositely directed.

From the figure for vertical velocity⁷ (Fig. 2), it is seen that the values of ω at 700-mb surface are larger in magnitude than those at 850 or 500-mb surface. Thus the level of non-divergence (where $\partial\omega/\partial p = 0$ and where ω will have a maximum or minimum value) may be considered to be close to 700 mb. This seems to be in agreement with the theoretical result obtained by Kuo (1953) who has stated that for disturbances with wavelength < 3000 km the level of maximum ω will be lower than 500 mb.

(b) *Divergence* — We have computed divergence with the values of ω by using the continuity equation $\nabla \cdot \mathbf{V} = -\partial\omega/\partial p$. The patterns of divergence and convergence (negative divergence) are given in Fig. 4. The maximum lower level convergence occurs at about 850-mb surface, with corresponding compensating divergence around 500-mb surface, with a level of non-divergence close to but below 700-mb surface. At 850-mb surface, the region of convergence agrees well with the region of rainfall and the maximum value of convergence is $0.79 \times 10^{-5}/\text{sec}$ at the grid point of 18°N, 80°E. At 700-mb surface this region has positive divergence and from this it can be inferred that the level of non-divergence over this region must lie below 700-mb surface. At 500-mb surface these values of divergence have increased over this region. A value of $0.70 \times 10^{-5}/\text{sec}$ occurs at the same grid point 18°N, 80°E. At 300-mb surface the values of divergence have decreased. Profiles of vertical velocity ω , divergence, and geostrophic relative vorticity at the grid point 18°N, 80°E, have been illustrated in Fig. 6.

(c) *Vorticity* — A comparison of the values of divergence obtained by this method, with the values of geostrophic relative vorticity—Figs. 4(b) and 5(b)—shows that the values of large scale divergence are one order of magnitude less than those of the geostrophic vorticity, as is to be expected.

(d) *Direction of movement* — The vorticity equation (1) with the geostrophic assumption can be written in the form —

$$\frac{g}{f_0} \nabla^2 \frac{\partial Z}{\partial t} = -\mathbf{V} \cdot \nabla (\zeta + f) + f_0 \frac{\partial \omega}{\partial p} \quad (10)$$

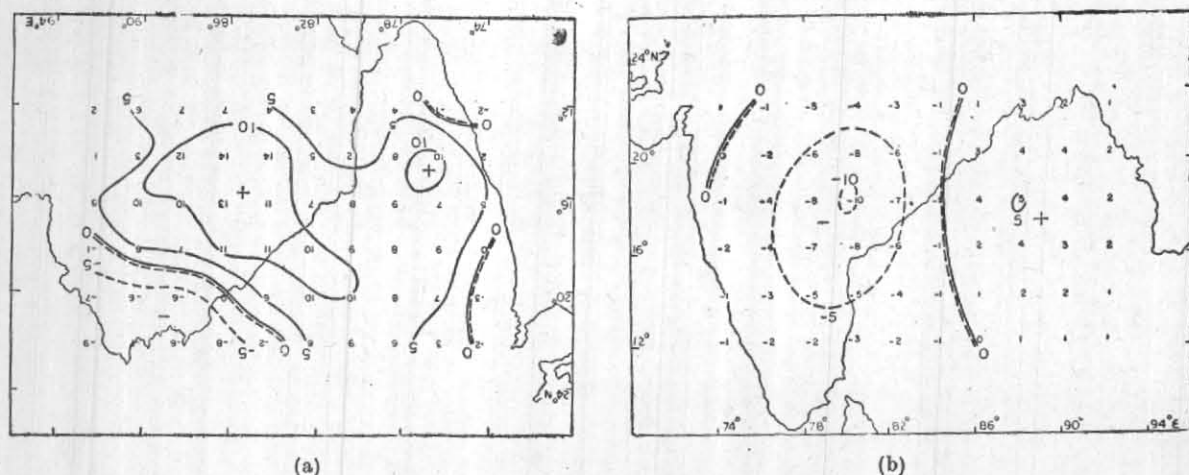


Fig. 7. 24-hr height change (gpm) of 850-mb surface due to (a) Vorticity advection and (b) Divergence terms

$$\frac{\partial Z}{\partial t} = \nabla^{-2} J(\eta, Z) + \frac{f_0^2}{g} \nabla^{-2} \frac{\partial \omega}{\partial p} \quad (11)$$

where $\eta = \zeta + f$, and the symbol ∇^{-2} represents relaxation of the quantity following it. The first term on the right side of Eq. (11) is obtained by computing the geostrophic advection of η from the initial data of the height field, and then relaxing it. The second term is evaluated with the values of ω computed for the various constant pressure surfaces. The height change for 24 hrs is calculated at one step.

The first term represents the change in height of an isobaric surface due to the advection of absolute vorticity by the geostrophic wind at the isobaric surface. As positive values of absolute vorticity in the trough region will be advected eastward by a westerly current, the trough in the westerlies will move eastward due to this effect of the first term in Eq. (11). Since the monsoon depression is situated in the westerly current in the lower troposphere, the effect of this term at each isobaric level should be to make the depression move eastward. As is well-known, the monsoon depression moves westward and it is therefore to be surmised that the effect of the second term should act in the opposite sense, and predominate over that of the first term at each isobaric level, when the depression moves westward. Hence it is important to estimate the effect of the second term, that is, the term of divergence of horizontal velocity ($\partial \omega / \partial p$) in the monsoon case. Gambo *et al.* (1956) have discussed the effect of this term for the middle latitude systems.

The 24-hr changes in the height of 850-mb surface, caused by these two terms were computed from Eq. (11) and are presented in Fig. 7 as an illustration. It is interesting to find that except

for a small region between 86° to 92° E and 12° to 18° N, the height change due to the divergence term is opposite in sign to that due to vorticity advection. It may be stated that from 1200 GMT of 25 July to 1200 GMT of 26 July 1966 the low had a small northwestward movement and became less marked by the morning of 27 July 1966.

The divergence term thus plays a crucial role in determining the direction of motion of the monsoon depression. As the vertical velocity in the divergence term depends on the thermal wind advecting the absolute vorticity (Eq. 6), we may infer that the thermal wind, or in other words, the baroclinicity of the atmosphere is an important factor to be taken into account.

From an understanding of the physical significance of these two terms it will be possible to make a few inferences.

(i) When the monsoon westerly current is very strong and when the monsoon depression is still stationary the divergence term in Eq. (11) should also have large and opposite values in order to compensate and predominate over the vorticity advection term. This in turn would require larger values of upward vertical motion and hence the precipitation should be relatively heavier than when the monsoon westerly current is not so strong.

(ii) When the monsoon current is strong, a fast westward moving depression will give heavier precipitation than in case (i).

(iii) In the case of a weak monsoon current, if a low pressure area or depression were to give widespread precipitation at will be moving faster than when the current is strong.

(iv) If the monsoon low or depression has to move eastward faster than the monsoon westerly current the divergence term in Eq. (11) should act

in the same sense as the vorticity advection term. In such a situation, we should expect convergence in the lower troposphere, upward vertical velocity and rainfall to the east and northeastern sector of the depression. It is known that the rainfall belt shifts from the southwest sector to the northeast sector when the depression begins to recurve and move eastward.

In order that there is upward vertical motion to the east of the depression, it follows from the discussion of Eq. (9) above that the westerlies should increase with height (and the easterlies should decrease with height), v' being positive to the east of the trough. This means that the depression now gets embedded in a region where the thermal gradient is directed from the south to the north instead of being directed from the north to the south as in the monsoon field. Thus recurvature and eastward movement of a depression are possible only when there are extra-tropical westerlies pervading over the region where the depression is situated. This would mean that the more favourable time for recurvature and eastward movement of a depression, will be towards the latter half of the

monsoon season.

From the time the westward moving depression begins to slow down to the time it has started moving eastward, we should expect the effect of the convergence term to decrease slowly and later change its sign, and therefore we should expect a decrease in the upward vertical velocities and consequently the intensity of rainfall should also decrease during this transition period.

5. Acknowledgement

The authors have great pleasure to express their thankfulness to Dr. T. Murakami, Meteorological Research Institute, Japan, for suggesting this study and for his many helpful discussions during the investigation and for useful comments on the manuscript. They are also thankful to Dr. Bh. V. Ramana Murty for his interest in the study and for going through the manuscript and for making useful suggestions. They thank Dr. P. R. Pisharoty for his interest in the study. Their thanks are also due to Mr. A. G. Pillai for his assistance in some of the computations and to the colleagues who prepared the diagrams.

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