551.577.2 : 551.585.3 (54)

Statistical distribution of pentad rainfall over India during monsoon season

D. A. MOOLEY and G. APPA RAO

Institute of Tropical Meteorology, Poona

(*Received 6 July 1968*)

ABSTRACT. Statistical distribution of pentad rainfall during southwest and northeast monsoon seasons, at representative stations in India, has been studied. From the histograms it is seen that these distributions are right (positive) skewed. Gamma distribution function has been fitted to rainfall data and the goodness of fit of the distribution to the rainfall data has been tested by Chi-square tests. These tests show that pentad rainfall may be described by Gamma distribution.

1. Introduction

Barger and Thom (1949), Barger, Shaw and Dale (1959), Friedman and Janes (1957) have applied Gamma distribution for obtaining probabilities of weekly precipitation at stations in U.S.

Suzuki (1964) has fitted hypergamma distribution to 5-day rainfall of Tokyo.

In this paper it is proposed to examine the distribution of pentad rainfall during the southwest and northeast monsoon at Indian stations representing different rainfall regimes over the country.

2. Data

Rainfall distribution is proposed to be studied for 11 stations as shown in Fig. 1. Daily rainfall data for these stations for the period 1901-1960 were obtained from the punched cards and 5-day totals were computed from these. For Port Blair, 5 years' data are not available during this period and the study for this station is based on 55 years data only.

The pentads used are standard pentads, numbered 1 to 73, the first pentad corresponding to 1 to 5 January and seventythird pentad corresponding to 27 to 31 December; 29 February during leap years being ignored.

3. Pentad rainfall distribution during southwest and northeast monsoon seasons

June to September is normally the period of southwest monsoon over India and pentad numbers 31 to 55 cover this period. The time of onset and of withdrawal of southwest monsoon varies from one year to another and from one part of India to another. By the end of June monsoon is normally established over the whole country and by about mid-September it withdraws from northwest India. In view of this, the question arises what are the pentad numbers for the different stations for which rainfall distribution should be considered, if we want these pentads to be representative of monsoon conditions? Obviously, the pentad numbers representative of monsoon conditions would show some variation from one station to another. It is, therefore, necessary to lay down some objective criterion for deciding upon the pentads whose rainfall distribution should be studied.

3.1. *Criteria for selection of pentads*

The two criteria which can be considered are as follows —

(*i*) *Normal dates of onset and withdrawal* — Pentads may be taken from the set of pentads enclosing the normal date of onset of monsoon and its normal date of withdrawal at the stations under consideration. The normal dates of onset and withdrawal of monsoon may be fixed from the normal charts of onset and withdrawal as given in *Climatological Atlas for Airmen* (India met. Dep. 1943).

(*ii*) *Rainfall Probability* — If the pentads are to be representative of monsoon conditions they should satisfy some limit of probability of non-zero rain. All pentads with probability P (non-zero rain) $\geqslant \frac{2}{3}$ may be considered. More stringent limits of this probability may be set if considered necessary for any specific purpose.

Normal dates of onset of monsoon at different stations were computed by sudden jumps in normal pentad rainfall amounts and *not* by the sudden jumps in the probability of rainfall and the normal chart of monsoon onset as given in

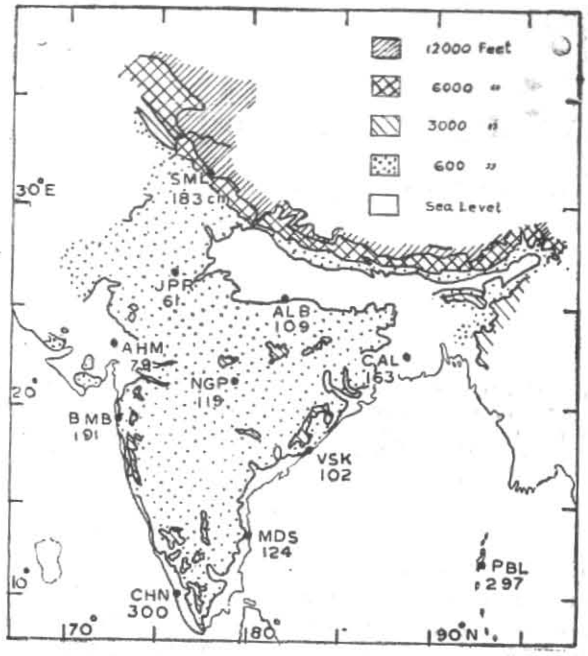

Fig. 1. Relief map of India showing stations with their annual rainfall

Climatological Atlas for Airmen (*loc. cit.*) was prepared on the basis of these individual dates for the different stations. In a similar way, the normal chart of withdrawal was prepared, the criterion in this case being marked decrease in pentad rainfall. Normal dates obtained in this way are likely to be biased as a result of very heavy rainfall during a few years in some pentads. Taking these facts into account it was felt advisable to use criterion (*ii*) given above with $P \geqslant \frac{2}{3}$, in preference to criterion (*i*).

Ramamurty (*see* Ref.) in his study of pentad rainfall at 168 stations of India and Pakistan, has investigated into the dates of onset and withdrawal of rainy season at these stations on the basis of significant changes in pentad rainfall. The criteria for significant changes adopted by him were those laid down by Crowe (1933) or by Savur (1936-37 and 1937). These criteria are based on the significant changes of median pentad rainfall. It may be mentioned that in respect of the stations considered in the present study, the pentads of rainy period arrived at by him, and those arrived at during the present study on the basis of criterion (*ii*) given earlier, are in general agreement.

Utilizing criterion (*ii*), pentads were selected for each station from amongst pentads 31 to 67 covering the period June to November (southwest and northeast monsoon) and rainfall distribution was considered for each of these.

3·2. *Testing for normal distribution*

It is well known that distribution of daily rain is very far from normal and it does not appear that pentad rainfall may be normally distributed. It was, however, decided to carry out tests of significance of skewness and kurtosis of the frequency distribution of pentad rainfall and also Chi-square tests for normality of pentad rainfall at all the stations under consideration to get an idea of the extent of departure from normality.

The estimates of Fishers' (1932) measures of skewness and of Kurtosis g_1, g_2 respectively, of pentad rainfall distribution were computed by using the expressions for consistent and unbiased estimates of the second, third and fourth moments of the distribution as given by Crame'r (1946). Significance of g_1, g_2 was tested by utilising the exact expressions, for mean and variance, first obtained by Fisher (1930) and later by Crame'r (*loc. cit.*).

Significance of skewness, kurtosis and of the frequency distribution of pentad rainfall was tested. The null hypothesis H_0 that the distribution of pentad rainfall was not different from normal distribution was tested against the alternative hypothesis H_1 that it was different from the normal distribution. The results of these tests show clearly that (*i*) for 90 per cent of the pentads skewness and Chi-square values are significant at 1 per cent level and (*ii*) for 70 to 80 per cent of the pentads, kurtosis values are significant at 1 per cent level.

Thus the departure of the pentad rainfall distribution from normality is highly significant.

3.3. *Testing for Gamma distribution*

Next, Gamma distribution function given by

$$\int_o^x \frac{x^{\gamma-1} e^{-x/\beta}}{\beta^{\gamma}\, \Gamma(\gamma)}\, dx$$

where $\gamma > 0$, $\beta > 0$ was fitted to non-zero pentad rainfall data. γ is the shape parameter and β is the scale parameter. For $\gamma \leqslant 1$, the frequency curve is reversed J-shaped and for $\gamma > 1$, it is bell-shaped. Maximum likelihood estimates, $\hat{\gamma}$ and $\hat{\beta}$ which are consistent and efficient were obtained as per method given by Thom (1958). The units of x and β are the same. Thom's method is given below:

$$M, \text{ the likelihood function} = \prod_{i=1}^{n} \frac{x^{\gamma-1} e^{-x/\beta}}{\beta^{\gamma}\, \Gamma(\gamma)}$$

$$\frac{\partial}{\partial y} \log_e M = \sum_{i=1}^{n} \log_e x_i - n \log \hat{\beta} - n\psi(\hat{\gamma})$$

$$= 0 \qquad (1)$$

where, $\psi(\hat{\gamma}) = \dfrac{\partial}{\partial \hat{\gamma}} \log_e \Gamma(\hat{\gamma})$

$$\frac{\partial}{\partial \beta} \log_e M = \frac{1}{\hat{\beta}^2} \sum_{i=1}^{n} x_i - n(\hat{\gamma}/\hat{\beta}) = 0$$

$$i.e., \ \hat{\beta}\hat{\gamma} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$= A, \text{ the arithmetic mean} \qquad (2)$$

Elimination of $\hat{\beta}$ from (2) and (1) gives,

$$\log_e \hat{\gamma} - \psi(\hat{\gamma}) + \log_e (G/A) = 0 \qquad (3)$$

where,

$$\log_e G = \frac{1}{n} \sum_{i=1}^{n} \log_e x_i$$

$$= \log_e \left[(x_1, x_2 \ldots x_n)^{1/n} \right]$$

Thus G is the geometric mean of the n quantities $x_1, x_2, \ldots x_n$. Now $G/A \leqslant 1$, hence $\log_e (G/A) \leqslant 0$. The sign of equality holds when $x_1 = x_2 = \ldots = x_n = A$, which is a trivial case.

Equation (3) can be solved for $\hat{\gamma}$ by using tables of $\log_e \hat{\gamma} - \psi(\hat{\gamma})$ prepared by Masuyama and Kuroiwa (1951).

Thom (*loc. cit.*) has approximated $[\log_e \hat{\gamma} - \psi(\hat{\gamma})]$ to $[1/2\hat{\gamma} + 1/12\hat{\gamma}^2]$

Substituting this in (3) and solving for $\hat{\gamma}$

$$\hat{\gamma} = \frac{-1 - [1 - (4/3) \log_e (G/A)]^{\frac{1}{2}}}{4 \log_e (G/A)}$$

the other root being negative is inadmissible. By substituting this value of $\hat{\gamma}$ in (2), we get $\hat{\beta}$.

P_1, the probability of rain $< x$,

$$= Q + (1-Q) \int_o^x \frac{x^{\hat{\gamma}-1} e^{-x/\hat{\beta}}}{\hat{\beta}^{\hat{\gamma}}\, \Gamma(\hat{\gamma})}\, dx$$

where Q is empirical probability of no rain, being the ratio of occasions of zero rain to the total number of occasions and x includes zero rain also.

P_2, the probability of rain $\geqslant x = 1 - P_1$

$$= (1-Q) \left\{ 1 - \int_o^x \frac{x^{\hat{\gamma}-1} e^{-x/\hat{\beta}}}{\hat{\beta}^{\hat{\gamma}}\, \Gamma(\hat{\gamma})}\, dx \right\}$$

Probabilities of rainfall exceeding specified amounts $x_1, x_2, x_3 \ldots$ were computed on CDC 3600 TIFR, Bombay. x_1, x_2, x_3 etc., were expressed as percentages of mean pentad rainfall, $\bar{X}'$, this mean being mean of rainfall for pentads with non-zero rain. The percentages considered are 20 to 200 in steps of 20, 200 to 300 in steps of 50 and 400. From these probabilities, theoretical frequencies for ranges of rainfall like $< x_1$, x_1 to x_2, x_2 to x_3 etc, were obtained. The corresponding observed or empirical frequencies of the pentad rainfall distribution are also obtained; the Chi-square test of goodness of fit to the distribution is then applied with significance level, $\alpha = 0 \cdot 05$. The results of Chi-square tests for the different pentads for the stations under consideration are tabulated under Table 1 as frequencies of $P(\chi^2 \geqslant \chi_1^2)$, *i.e.*, probability of getting χ^2 exceeding or equalling χ_1^2, the value actually obtained, over di erent ranges of probability. This table shows that $P(\chi^2 \geqslant \chi_1^2)$ is less than $0 \cdot 05$ in only 2 per cent of the cases. In other words, $\chi_1^2 > \chi_{.05}^2$ for appropriate degrees of

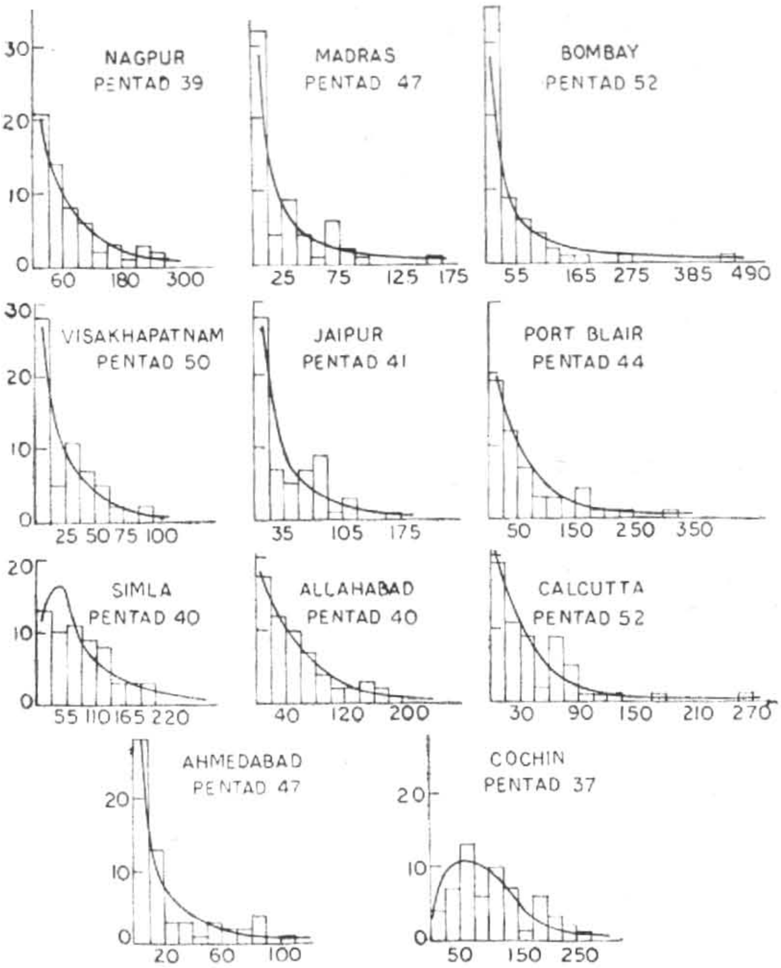

Fig. 2 Histograms of pentad rainfall

Abscissa is rainfall (mm) and ordinates is frequency

freedom in 2 per cent of the cases only. $\chi_{.05}^2$ is the value critical at 5 per cent level. Thus the null hypothesis is not contradicted and pentad rainfall distribution is not expected to be different from Gamma distribution. This distribution can be used to describe the pentad rainfall distribution.

Histograms of rainfall of randomly selected pentads, one from each of the stations, are shown in Fig. 2. Fitted smooth curves are superimposed over these histograms.

Table 2 gives N', the number of occasions of non-zero rain, $\bar{x}'$, the mean rainfall (in cm) on occasions of non-zero rain, s, the standard deviation (in cm) of pentad rainfall, $\hat{\gamma}$ and $\hat{\beta}$, the M.L. estimates of the parameters of rainfall distribution. $\hat{\beta}$ is in cm and $\hat{\gamma}$ is dimensionless parameter.

Sajnani (1964) has shown that correlation between pentad rainfall of Bombay station and pentad rainfall of Colaba district is about 0·9 in June, August and September and 0·7 in July. From his study it may be inferred that during southwest monsoon season pentad rainfall at a station could be easily considered as fully representative of an area covered by 40 km (or 25 mile) radius around the station, which area is about one fourth the area of an average-sized district. As such when the interest lies in an area of this size, the parameters of pentad rainfall at the station could be taken to be applicable to this area. Unless the area considered is several times larger than the area represented by a station, the areal pentad rainfall is expected to be Gamma-distributed. Hence it is plausible to expect pentad district rainfall to be Gamma distributed.

3.4. *Application of the additive property of Gamma distribution*

If rainfall for different pentads are statistically independent it is possible to utilize the additive property of Gamma distribution for obtaining the parameters of Gamma distribution for the combined rainfall of 2 or more pentads. From the Gamma parameters of the combined rainfall distribution and Pearson's (1934) *Tables of Incomplete Gamma Function* or the nomograms prepared by Barger and by Thom, both of which are included in a paper by Barger, Shaw and Dale (1959 a), probabilities of rainfall exceeding specified amounts during the combined 5-day periods can be obtained. In combining the rainfall distribution for 2 or more pentads it is necessary that the parameter $\hat{\beta}$ for the pentads is not different. The observed differences may be due to random sampling fluctuations. We must, therefore, check for significance the difference in the parameters for two pentads. Now Fisher (1941) has shown that M.L. estimates of parameters are normally distributed for large samples and Thom (*loc. cit.*) has given the following expressions for variance for M.L. estimates from large samples,

$$\text{Var}\ (\hat{\gamma}) = \hat{\gamma}/N[\hat{\gamma}\psi'(\hat{\gamma})-1]$$
$$\text{Var}\ (\hat{\beta}) = \hat{\beta}^2\psi'\hat{\gamma}/N[\hat{\gamma}\psi'(\hat{\gamma})-1]$$

Digamma, $\psi(\hat{\gamma})$ and trigamma $\psi'(\hat{\gamma})$ functions have been tabulated by Davis (1933). By utilising these, Var ($\hat{\gamma}$) and Var ($\hat{\beta}$) can be computed. Making an assumption that estimates or M.L. parameters obtained from samples of size 60 are normally distributed we can obtain 90 per cent confidence limits for $\hat{\beta}$. If the $\hat{\beta}$ values for the pentads to be combined lie within the 90 per cent confidence limits of $\hat{\beta}$ for one of the pentads, we can consider that these $\hat{\beta}$ values are not different and that these pentads can be combined. In this case, the $\hat{\beta}$ values for the pentads to be combined can be averaged as suggested by Shaw, Barger and Dale (*loc. cit.*) and Friedman and Janes (*loc. cit.*). For the combined period, the Gamma parameter is the sum of $\hat{\gamma}$ values for the pentads and the beta parameter is the average of the $\hat{\beta}$ parameters for the pentads. Masuyamma and Kuroiwa (1951) have tabulated the functions $\hat{\gamma}/[\hat{\gamma}\psi'(\hat{\gamma})-1]$ and $\psi'(\hat{\gamma})/[\hat{\gamma}\psi'(\hat{\gamma})-1]$ for different values of $\hat{\gamma}$. Utilising these values of $\psi'(\hat{\gamma})/[\hat{\gamma}\psi'(\hat{\gamma}))-1]$, 90 per cent or any other desired confidence limits on $\hat{\beta}$ can be obtained.

A separate study has been undertaken to find out if pentad rainfall amounts during monsoon are statistically independent.

3.5. *Reduction to exponential distribution*

When $\hat{\gamma} = 1$, Gamma distribution, reduces to the special case of exponential distribution. Considering sample of size 60 as large and utilising the table of $\hat{\gamma}/[\hat{\gamma}\psi(\hat{\gamma})-1]$ given by Masuyamma and Kuroiwa (1951), the standard error of $\hat{\gamma}$ and 95 per cent confidence limits on $\hat{\gamma}$ have been computed and are given under Table 3 for $\hat{\gamma} = 0{\cdot}70$ to $1{\cdot}70$. If it is desired to know whether rainfall distribution for any particular pentad is not significantly different from the exponential distribution, $\hat{\gamma}$ value for the pentad is referred to in Table 3. If the corresponding 95 per cent confidence limits on $\hat{\gamma}$ include 1 , it may be inferred that the rainfall distribution for the pentad is not significantly different from the exponential distribution. Table 3 suggests that when $0{\cdot}80 \leqslant \hat{\gamma} \geqslant 1{\cdot}45$, the distribution is not significantly different from the exponential distribution. Table 2 shows that some of the values of $\hat{\gamma}$ lie within this range and as such for these pentads rainfall distribution is not likely to be significantly different from exponential distribution.

3.6. *Utilisation of Gamma parameters*

The values of Gamma parameters for pentad rainfall distribution can be used for computation of probabilities of rainfall exceeding specified amounts as required for planning agricultural activities in the various stages of the crop. The probabilities can be computed by using the Tables of *Incomplete Gamma Function* (*op. cit.*) or nomograms of Barger (*op. cit.*) and of Thom (*op. cit.*). It is necessary to obtain beforehand the requirements of minimum or maximum rainfall for carrying out the requisite agricultural operations. If any activity covers more than 5 days, requisite number of 5-day periods may be combined, Gamma parameters may be obtained for the combined periods and then requisite rainfall probabilities may be worked out. If agricultural operations are being planned for the district as a whole, Gamma parameters for pentad district rainfall may be worked out and from these, requisite rainfall probabilities may be computed.

4. Conclusion

Gamma distribution can be used to describe pentad rainfall at an Indian station during the southwest and northeast monsoon season.

5. Acknowledgement

The authors would like to thank Shri P. Jagannathan, Assistant Director, for going through the manuscript and making valuable suggestions.

TABLE 1

Frequencies for different ranges of probabilities of getting $\chi^2 \geqslant \chi_1^2$ for appropriate degrees of freedom

Chi-square test for null hypothesis of five-day rainfall distribution being not different from Gamma distribution

Station	Probability Ranges							
	<·01	·01—<0·05	0·05—0.09	0·10—0·25	0·26—0·50	0·51—0·75	>0·75	Total
Ahmedabad	1	0	0	6	8	1	1	17
Allahabad	0	0	0	3	3	7	6	19
Bombay (Colaba)	0	1	7	5	6	4	1	24
Calcutta (Alipore)	0	0	2	6	9	6	2	25
Cochin	1	0	3	7	12	8	4	35
Jaipur	0	0	3	4	4	3	0	14
Madras	0	1	3	10	6	9	6	35
Nagpur	0	0	3	6	7	2	5	23
Port Blair	0	2	3	7	10	7	8	37
Simla	0	0	4	3	6	8	2	23
Visakhapatnam	0	0	4	2	10	6	5	27
Total	2	4	32	59	81	61	40	279

TABLE 2

Parameters of pentad rainfall during monsoon

Pentad No.	N'	$\overline{X}'$ (cm)	s (cm)	$\hat{\gamma}$	$\hat{\beta}$ (cm)
			Ahmedabad		
34	40	1·88	1·88	0·718	2·769
35	[illegible]7	2·74	3·79	0·633	4·321
36	44	3·40	3·53	0·957	3·561
37	46	6·35	6·63	0·764	8·319
38	56	5·41	6·02	0·885	6·124
39	53	5·77	7·09	0·702	8·219
40	54	4·70	5·31	0·647	7·244
41	54	5·41	9·88	0·592	9·121
42	53	8·94	16·28	0·656	13·637
43	56	4·80	4·57	0·797	6·022
			Allahabad		
35	44	3·30	5·72	·609	5·441
36	49	3·00	3·94	·738	4·064
37	56	4·24	5·51	·759	5·575
38	55	4·95	5·74	·716	6·[illegible]09
39	56	4·95	4·78	·833	5·[illegible]33
40	54	6·15	4·95	1·419	4·326
41	57	4·27	4·04	·950	4·501
42	55	6·15	5·41	1·200	5·121
43	59	5·66	4·80	1·105	5·118
			Bombay (Colaba)		
32	45	5·51	7·75	·602	9·139
33	57	8·10	8·59	·872	9·301
34	58	9·86	11·79	·78	12·634
35	59	12·68	11·33	·786	16·104
36	60	12·01	16·38	·708	16·957
37	60	11·46	12·29	·957	11·976
38	60	12·17	11·46	·976	12·482
39	60	10·21	10·57	·848	12·037
40	60	9·47	10·36	1·041	9·098
41	60	9·60	10·95	1·022	9·390
42	60	9·09	8·43	1·193	7·623
43	60	9·53	10·69	·995	9.568

TABLE 2 (*contd*)

Parameters of pentad rainfall during monsoon

Pentad No.	N'	$\bar{X}'$ (cm)	s (cm)	$\hat{\gamma}$	$\hat{\beta}$ (cm)
			Calcutta (Alipore)		
31	45	3·48	5·89	·665	5·222
32	51	4·70	6·43	·834	5·629
33	53	5·13	5·05	·834	6·162
34	57	6·32	7·59	1·040	6·081
35	60	5·10	5·46	1·044	4·900
36	59	5·54	5·16	1·293	4·285
37	59	4·37	4·39	1·024	4·290
38	60	5·97	5·77	1·374	4·346
39	60	4·70	3·23	1·785	2·637
40	59	5·03	5·05	1·132	4·445
41	59	5·82	5·11	1·767	3·297
42	60	6·17	6·10	1·269	4·867
43	60	5·03	4·47	1·635	3·068
			Ahmedabad		
44	53	4·12	4·55	·611	6·734
45	50	5·08	5·46	·647	7·892
46	53	3·40	4·95	·656	5·182
47	54	2·57	2·95	·699	3·683
48	46	4·52	4·78	·723	6·243
49	49	4·19	5·56	·517	8·090
50	45	4·32	5·99	·553	7·780
			Allahabad		
44	59	6·99	5·79	1·097	6·375
45	58	5·41	4·65	1·072	5·050
46	57	4·39	4·62	·817	5·364
47	57	5·64	8·99	·901	6·256
48	54	4·65	5·05	·924	5·022
49	55	4·72	4·44	1·089	4·328
50	52	5·74	6·76	·932	6·162
51	53	3·99	5·00	·703	5·657
52	44	4·37	4·55	·947	4·600
53	40	3·05	3·79	·7[illegible]6	4·039
			Bombay (Colaba)		
44	60	6·43	8·36	·891	7·224
45	59	5·03	5·99	·931	5·408
46	60	5·00	7·09	·892	5·624
47	59	5·06	5·49	1·083	4·676
48	60	6·43	9·30	·795	8·092
49	60	5·99	7·32	·764	7·833
50	59	5·18	6·20	·840	6·166
51	57	5·33	11·33	·668	7·971
52	57	4·75	7·62	·593	7·898
53	55	5·59	8·03	·624	8·946
54	53	4·04	9·27	·498	8·125
55	46	4·19	5·97	·600	7·026
			Calcutta (Alipore)		
44	59	6·05	6·10	1·592	3·387
45	60	5·64	4·52	1·430	3·940
46	59	4·70	4·67	1·469	3·200
47	59	5·00	3·61	1·740	2·827
48	60	4·75	4·22	1·297	3·660
49	59	5·33	4·24	1·680	3·183
50	60	4·17	5·23	1·153	3·622
51	60	5·64	5·16	1·377	4·097
52	59	4·42	4·72	·789	5·618
53	56	3·83	4·34	·835	4·600
54	59	4·32	5·46	·991	4·369
55	53	3·73	5·16	·765	4·990

TABLE 2 (*contd*)

Parameters of pentad rainfall during monsoon

Pentad No.	N'	$\bar{X}'$ (cm)	s (cm)	$\hat{\gamma}$	$\hat{\beta}$ (cm)
			Jaipur		
37	43	2·52	2·72	·909	2·756
38	52	2·62	2·57	·831	3·147
39	49	3·66	3·43	·978	3·749
40	50	3·94	4·27	·902	4·371
41	51	4·34	3·71	·920	4·707
42	51	4·62	4·57	·746	6·185
43	55	3·48	3·89	·803	4·351
			Nagpur		
32	44	1·98	3·07	·640	3·112
33	51	3·38	6·05	·739	4·580
34	52	3·63	4·17	·814	4·468
35	55	5·57	5·56	·945	5·883
36	56	7·11	7·39	·954	7·447
37	60	6·81	6·02	1·377	4·935
38	60	5·82	4·90	1·168	4·971
39	57	7·47	7·14	1·050	7·115
40	58	5·69	4·52	1·121	5·088
41	57	4·90	4·62	·973	5·034
42	56	6·71	5·16	1·521	4·402
43	59	5·26	4·47	·980	5·352
			Simla		
31	44	1·74	1·80	·927	1·887
32	47	1·85	2·08	·830	2·223
33	46	2·29	2·67	·870	2·619
34	51	3·00	3·61	1·076	2·779
35	57	3·33	3·07	1·052	3·162
36	55	4·47	5·28	·735	6·083
37	55	5·13	4·42	1·304	3·924
38	58	6·43	4·57	1·707	3·767
39	60	6·78	5·00	1·744	3·894
40	59	8·26	5·39	1·643	5·029
41	60	6·83	4·67	2·078	3·284
42	60	7·26	5·23	1·964	3·693
43	60	7·49	4·95	2·954	2·535
			Jaipur		
44	51	3·93	4·82	·742	5·022
45	43	5·13	5·46	·860	5·964
46	48	3·94	5·31	·614	6·408
47	52	3·61	4·19	·893	4·034
48	46	4·78	4·62	·700	6·828
49	50	3·40	4·50	·599	5·690
50	41	2·17	2·00	·925	2·337
			Nagpur		
44	59	4·98	5·46	·863	5·781
45	57	3·94	4·50	·732	5·392
46	54	4·34	4·32	1·012	4·290
47	59	5·06	4·90	·780	6·469
48	60	4·83	4·52	·910	5·293
49	56	4·95	4·98	·817	6·060
50	60	4·22	3·63	1·047	4·011
51	55	4·04	4·14	·971	4·166
52	56	3·51	4·65	·800	4·382
53	52	2·74	2·24	1·148	2·380
54	46	3·71	3·51	·756	4·895

TABLE 2 *(contd)*

Parameters of pentad rainfall during monsoon

Pentad No.	N'	$\overline{X'}$ (cm)	s (cm)	$\hat{\gamma}$	$\hat{\beta}$ (cm)
			Simla		
44	60	6·83	5·06	1·772	3·861
45	60	7·62	5·46	1·776	4·293
46	58	7·72	5·79	1·684	4·580
47	59	7·10	6·60	1·536	4·567
48	58	5·56	4·90	1·392	3·995
49	58	5·18	4·27	1·185	4·369
50	56	3·76	3·35	1·081	3·465
51	55	4·21	4·01	·886	4·745
52	47	4·67	4·32	·925	4·999
53	44	3·10	3·38	·761	4·084
			Cochin		
31	60	10·13	8·10	1·135	8·938
32	60	13·56	8·28	1·895	7·160
33	60	14·20	9·07	2·355	6·033
34	60	11·12	7·24	2·365	4·709
35	60	11·48	6·96	2·022	5·677
36	60	12·45	8·36	1·954	6·365
37	60	10·72	6·30	2·501	4·280
38	60	10·21	6·78	1·602	6·368
39	60	9·75	7·26	1·764	5·537
40	60	9·63	7·34	1·382	6·465
41	59	10·34	8·66	1·426	7·254
42	60	8·31	5·64	2·024	4·097
43	60	7·49	5·87	1·289	5·812
			Madras		
33	40	·94	1·07	·777	1·194
34	42	1·04	1·17	·782	1·331
35	50	1·30	1·40	·933	1·392
36	51	1·22	1·52	·735	1·674
37	58	1·09	1·42	·910	1·196
38	53	1·47	1·68	·816	1·791
39	53	1·27	1·65	·828	1·834
40	52	1·75	2·21	·862	2·032
41	56	1·73	1·88	·940	1·836
42	54	2·24	2·97	·834	2·672
43	54	1·55	1·72	·873	1·760
			Port Blair*		
31	53	10·36	8·74	1·208	8·580
32	54	8·76	7·72	1·197	7·361
33	54	8·97	6·15	1·708	5·255
34	54	8·69	6·88	1·553	5·584
35	54	9·30	7·37	1·201	7·739
36	54	8·71	8·23	1·258	6·924
37	54	6·15	5·49	1·305	4·714
38	54	6·20	6·12	·927	6·703
39	53	6·93	5·84	1·169	5·926
40	54	6·22	5·21	1·070	5·850
41	54	6·93	7·54	0·888	7·803
42	54	6·15	5·21	1·480	4·155
43	54	7·34	7·04	1·487	4·938

*The study for this station is based on 55 years' data

TABLE 2 (*contd*)

Parameters of pentad rainfall during monsoon

Pentad No.	N'	$\overline{X}'$ (cm)	s (cm)	$\hat{\gamma}$	$\hat{\beta}$ (cm)
			Visakhapatnam		
32	46	1·80	2·39	·590	3·056
33	41	1·68	1·83	·619	2·723
34	48	3·23	4·24	·651	4·938
35	47	2·03	3·43	·749	2·695
36	52	2·36	2·24	·944	2·499
37	54	2·71	3·20	·756	3·599
38	50	2·11	2·13	·999	2·118
39	53	2·16	2·74	·761	2·845
40	50	1·75	2·46	·914	1·918
41	52	2·39	4·32	·775	3·086
42	55	1·93	1·96	1·023	1·890
43	54	2·77	3·48	·718	3·863
			Cochin		
44	59	8·23	6·10	1·328	6·200
45	60	6·10	5·87	1·464	4·158
46	58	6·55	6·93	1·108	5·913
47	59	5·59	3·94	1·734	3·226
48	60	4·55	5·00	·828	5·494
49	59	3·20	3·23	·987	3·244
50	56	3·96	4·65	·830	4·785
51	56	4·19	4·52	·694	6·025
52	55	3·25	3·27,	·861	3·787
53	56	4·06	3·43	1·190	3·414
54	56	5·51	5·51	1·057	5·217
55	57	6·25	6·10	·758	8·260
			Madras		
44	52	2·54	2·34	1·118	2·271
45	52	1·88	1·96	1·051	1·778
46	50	2·13	2·74	·750	2·832
47	52	2·97	3·12	·803	3·703
48	54	1·68	1·70	1·128	1·496
49	56	2·46	2·92	·633	3·894
50	50	2·08	1·91	1·027	2·040
51	51	2·56	2·67	·926	2·764
52	48	2·51	3·22	·646	3·884
53	51	2·51	2·82	·765	3·294
54	48	1·73	2·92	·658	2·637
55	47	2·69	2·41	1·156	2·329
			Port Blair		
44	54	6·91	7·06	·924	10·013
45	53	6·88	6·53	1·351	5·100
46	54	5·49	4·90	1·249	4·387
47	54	6·58	5·00	1·395	4·714
48	54	7·44	5·92	1·466	5·083
49	53	7·90	6·35	1·185	6·662
50	54	8·20	6·81	1·228	6·668
51	52	7·95	6·22	1·323	6·012
52	54	7·26	5·44	1·563	4·656
53	53	8·67	7·62	1·058	8·212
54	53	7·70	5·87	1·929	5·949
55	54	6·33	5·28	1·063	5·959

TABLE 2 (*contd*)

Parameters of pentad rainfall during monsoon

Pentad No.	N'	$\overline{X'}$ (cm)	s (cm)	$\hat{\gamma}$	$\hat{\beta}$ (cm)
		Visakhapatnam			
44	56	1·85	2·95	·726	2·535
45	52	1·52	1·42	1·070	1·433
46	55	1·85	1·83	·901	2·062
47	47	3·18	3·63	·776	4·074
48	57	2·49	2·97	·773	3·216
49	52	2·49	2·39	1·142	2·184
50	53	2·74	2·36	·940	2·916
51	56	1·91	2·49	·748	2·545
52	54	3·40	5·26	·943	3·716
53	56	3·40	3·38	·719	4·735
54	56	4·04	4·85	·753	5·372
55	52	3·84	4·95	·727	5·273
		Cochin			
56	54	5·01	6·60	·950	5·913
57	57	5·69	5·89	·951	5·987
58	53	6·45	5·94	1·209	5·334
59	58	5·08	4·39	·860	5·903
60	57	5·18	4·83	·853	6·073
61	56	5·49	5·03	1·127	4·862
62	57	3·53	2·79	1·219	2·901
63	50	4·57	4·52	·960	4·768
64	47	5·21	4·52	·852	6·109
65	46	3·94	4·62	·658	5·964
		Madras			
56	48	4·52	7·04	·503	8·999
57	46	4·32	5·97	·612	7·084
58	50	4·42	5·66	·605	7·318
59	56	5·87	7·44	·664	8·837
60	48	7·72	8·31	·902	8·573
61	50	9·35	8·28	1·003	9·825
62	44	10·95	9·37	·921	11·890
63	46	9·12	8·99	·925	9·855
64	45	7·31	6·86	·758	9·614
65	42	7·21	6·66	·846	8·542
66	43	4·47	5·00	·836	5·359
67	41	5·59	4·70	·904	6·193
		Port Blair			
56	54	6·88	5·08	1·158	5·946
57	54	5·26	5·18	1·046	5·034
58	52	4·19	3·61	1·360	3·076
59	51	3·84	3·43	1·343	2·860
60	54	4·12	4·37	·866	4·750
61	50	4·22	4·34	·774	5·453
62	52	4·27	4·93	·910	4·679
63	51	4·72	6·88	·704	6·698
64	50	4·50	4·72	·769	5·837
65	48	4·80	4·52	·858	5·596
66	49	4·91	4·32	·689	5·682
67	39	4·80	5·97	·934	5·136
		Visakhapatnam			
56	44	4·32	5·97	·550	7·859
57	47	4·17	4·78	·743	5·606
58	44	5·64	5·77	·851	6·629

TABLE 3

Standard error of $\hat{\gamma}$ ($\sigma_{\hat{\gamma}}$) and 95 per cent confidence limits of $\hat{\gamma}$

$\hat{\gamma}$	$\sigma_{\hat{\gamma}}$	95% confidence limits for $\hat{\gamma}$	$\hat{\gamma}$	$\sigma_{\hat{\gamma}}$	95% confidence limits for $\hat{\gamma}$
0·70	·109	·486—0·914	1·25	·205	·849—1·651
0·75	·117	·520—0·980	1·30	·214	·880—1·720
0·80	·126	·553—1·047	1·35	·223	·914—1·786
0·85	·134	·587—1·113	1·40	·231	·946—1·854
0·90	·143	·620—1·180	1·45	·240	·979—1·921
0·95	·152	·652—1·248	1·50	·249	1·011—1·989
1·00	·161	·685—1·315	1·55	·258	1·044—2·056
1·05	·169	·718—1·382	1·60	·267	1·077—2·123
1·10	·178	·751—1·449	1·65	·276	1·109—2·191
1·15	·187	·783—1·517	1·70	·285	1·149—2·259
1·20	·196	·815—1·584			

REFERENCES

Barger, G. L. and Thom, H. C. S. 1949 *Agron. J.* 41, 519-526.

Barger, G. L., Shaw, R. L. and Dale, R. F. 1959(a) Chances of receiving selected amounts of precipitation in the North Central Region of United States. Agri. and Home Economics Exp. Station. Iowa State Univ., Ames., Iowa.

1959 (b) Gamma distribution parameters from 2 and 3-week precipitation totals in North Central Region of the United States. Agric. and Home Economics Exp. Station, Iowa State Univ., Ames, Iowa.

Crame'r, H. 1946 *Mathematical Methods of Statistics*, Asia Publ. House. pp. 362.

Crowe, P. R. 1933 *Scot. geogr. Mag.*, 49, pp. 73-91.

Davis 1933 *Higher Mathematical Functions*, Vols. I & II, Principia Press, Bloomington.

Fisher, R. A. 1930 *Proc. roy. Soc.*, 130, p. 16.

1932 *Statistical methods for Research Workers*, Oliver & Boyd, Edinburgh.

1941 *Phil. Trans.*, Series A, **222**, pp. 309-368.

Friedman, Don G. and Janes, Byron, E. 1957 Estimation of rainfall probabilities, Storrs Agric. Exp. Station, Bull. 332.

India met. Dep. 1943 *Climatological Atlas for Airmen*

Masuyamma and Kuroiwa 1951 Table for the likelihood solution of Gamma distribution and its medical applications. Statistical Application Res. Vol. I, 1.

Pearson, K. 1934 *Tables of Incomplete Gamma Function*. Cambridge Univ. Press, England, Rev. Ed.

Ramamurty, K. — A study of the rainfall regimes in India. Ph. D. Thesis submitted to the Univ., Madras, 1967.

Sajnani, P. J. 1964 *Indian J. Met. Geophys.*, **15**, 3, p. 483.

Savur, S. R. 1936-37 *Curr. Sci.*, **5**, pp. 419-421.

1937 *Proc. Indian Acad. Sci.*, **5**, pp. 222-234.

Suzuki, E. 1964 *Pap. Met. Geophys., Tokyo*, **15**, 1.

Thom, H. C. S. 1958 *Mon. Weath. Rev.*, **86**, 4, pp. 117-122.