551·510·62

A simplified method of computing radio refractive index

B. C. SEN and D. K. RAKSHIT

Meteorological Office, Calcutta Airport

(Received 26 December 1969)

ABSTRACT. Dependence of variation in radio refractive index on the anomalous propagation of radio waves in the lower layers of the atmosphere is of interest to the Cloud-physicists and Radio-physicists. For easy and quick evaluation of this parameter at different levels of the atmosphere a set of nomograms have been presented. The values obtained from these nomogrms when compared with those calculated values are found to be within 0·5 to 1 per cent of error.

1. Introduction

The radio refractive index of air (n) for UHF and microwave frequencies can be uniquely defined by three meteorological factors, *viz.*, atmospheric pressure, temperature and humidity. The value of n is of the order of 1·0003. However, in the study of the variability of n it is convenient to handle it in a modified form as shown below.

$$N = (n-1) \times 10^6 = \frac{77\cdot 6}{T}\left(P + \frac{4810e}{T}\right) \quad (1)$$

Where, N is the index of refraction, T the temperature in degrees Kelvin, P the total atmospheric pressure in mb and e the vapour pressure in mb.

It is known to the radio-meteorologists that vertical gradient of radio refractive index in the lower troposphere affects the propagation of radio waves considerably. Study of the vertical distribution of N is important in connection with scatter propagation also. Radio-meteorologists are therefore paying more and more attention to the study of the vertical profile of radio refractive index. Kulshrestha and Chatterjee (1966) studied the radio-climatology of India in which they discussed the distribution of N near the ground surface, at 850-mb level, at 700-mb level and its vertical structure in the lower troposphere. It appears that this type of study will invite attention of meteorologists in future also. These studies require lot of computation with the help of the formula given above. If the results of computation for useful range of meteorological parameters are available in the form of a nomogram, future computation will be easier and time-saving. One form of the nomogram is presented here for the purpose.

2. Construction of the nomogram

Values of N have been calculated for the following ranges of the meteorological parameters which appear to be sufficient for Indian conditions—

Pressure	Surface level to 700 mb
Temperatue	—10°C to 50°C
Vapour Pressure	0 to 80 mb

Equation (1) takes the form $N = 77\cdot 6 \times P'/T$ when the atmospheric pressure P is increased to P' by an amount equal to $P_e = 4810\ e/T$. This formula gives the refractive index of a "dry" atmosphere having temperature $T°$ A and pressure P' mb. In other words if we imagine an atmosphere exerting a pressure equal to P_e mb and extending above the actual atmosphere, its effect on refractive index will be equivalent to that of water vapour present in air. Therefore P_e may very well be called Equivalent Atmospheric Pressure (E.A.P.). The increased pressure P' $(=P+P_e)$ may be called Virtual Atmospheric Pressure (V.A.P.) in analogy with virtual temperature used for calculating moist air density.

Fig. 1(a) is the graph of the equation $P_e = 4810\ e/T$. For a constant temperature this graph is a straight line with a slope of $\tan^{-1} 4810/T$. A family of straight lines will cover the whole range of temperature. Here the lines are drawn at temperature interval of 30°C for lower values of vapour pressure and 10 °C for higher values of vapour pressure. In the actual upper air sounding reports vapour pressure values are not directly available ; instead dew point temperatures are reported. To increase the utility of the graph a scale of dew point temperature has been given by the side of the vapour pressure axis.

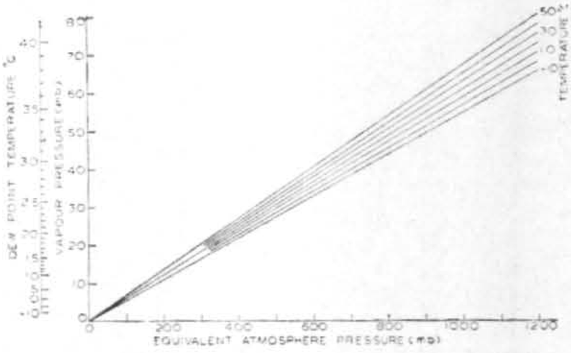

Fig. 1 (a). Graph of the equation $P_e = 4810\ e/T$

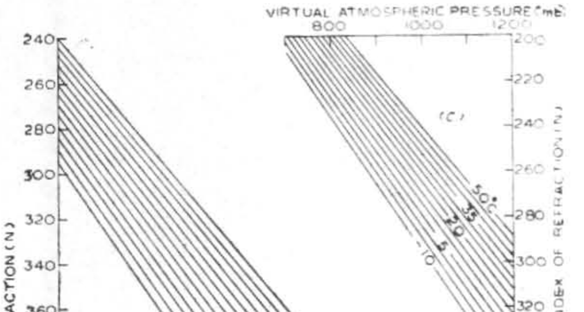

Fig. 1(b and c). Graph of the equation $N = 77 \cdot 6\ P'/T$

Fig. 2

Fig. 1(b) represents the graph of the equation $N = 77 \cdot 6 \times P'/T$. For a constant temperature the graph is a straight line with a slope of $\tan^{-1} 77 \cdot 6/T$. For a given range of temperature the nomogram is a family of straight lines converging to the origin. Fig. 1 (c) is the extension of Fig. 1 (b) with some overlapping region. The lines are drawn at the temperature interval of 5°C.

Since the graph is a straight line it is easy to extrapolate; for instance, if the value of N is required for $P' = 3000$ mb we may pick up the value of N for $P' = 1500$ mb and multiply it by 2 to get the required value of N. The graph for P_e may also be extrapolated similarly, if required.

In tracing the path of radio waves and locating ducts etc, the parameter which is of more practical value than N is the modified radio refractive index. It is defined as—

$$M = \left(n - 1 + \frac{h}{R} \right) \times 10^6 = N + \frac{h}{R} \times 10^6 \qquad (2)$$

where, M is the modified radio refractive index, h the height in km and R the radius of the earth in km.

If h is taken in metres and R in km, which is the usual practice, the above equation may be written as—

$$M = N + (h/R) \times 10^3 \qquad (3)$$

So M can be calculated easily by adding the value of $(h/R) \times 10^3$ to that of N. Fig. 2 gives the value of $(h/R) \times 10^3$ for any height upto 3200 m, assuming the value of R to be 6370 km. Fig. 3

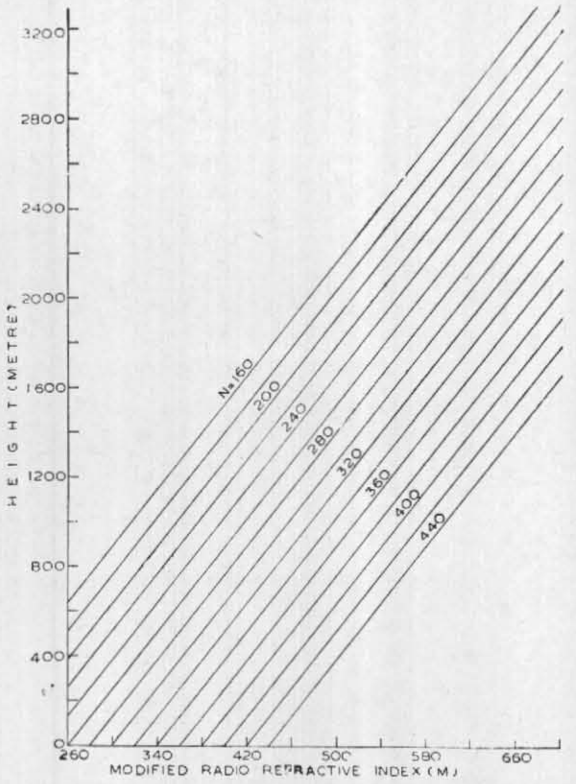

Fig. 3

gives the values of M directly for different values of N in the range of 160 to 440. The figure is self explanatory.

3. Method of computation

The steps to be followed for finding out the values of N and M from the nomograms may now be explained with the help of examples. Let us find out N when—

$$P = 975 \text{ mb} \; ; \; T = 29°\text{C and } T_d = 9°\text{C}$$

From the scale given on the left hand side of Fig. 1 (a) we find saturation vapour pressure at 9°C as 11·5 mb. From the graph in Fig. 1 (a) we find E.A.P. (P_e) corresponding to e =11·5 and T= 29° as 180 mb. Therefore V.A.P. (P') = $P+P_e$ = 975+180 = 1155 mb. From Fig. 1 (b) we find N corresponding to P'=1155 mb and T= 29°C as 297 which is the radio refractive index. A few more worked out examples are given in Table 1.

As an example for calculating M, let us take N= 284 and h=1956 m. Let us first find out the point on Fig. 3 corresponding to M = 280 and h = 1956. Now if we shift this point to the right by two smallest divisions (1 smallest division = 2 N-unit) parallel to the abscissa we get the required point and the value of M is 591. If N is equal to 278 instead of 284 we have to shift the point to the left by one smallest division.

4. Discussion

The values obtained from the nomograms were compared with the values arrived at by computing with the help of a log table. The error was found to be not more than 1 per cent. The vapour pressure can be read from the given scale correct to first place of decimal. An error of 0·1 mb in vapour pressure will give rise to an error of about 2 mb in E.A.P. Maximum error in picking up E.A.P. may be 2·5 mb which is equivalent to half of a smallest division. An error of 5 mb in V.A.P. leads to an error of about 1 N-unit, *i.e.*, an error of about 0·5 per cent.

The parameters given in Cols. 1—4 in Table 1 are available from radiosonde observations. After finding the values of N a graph may be plotted with height as ordinate and N as abscissa. This gives the vertical profile of N.

On the other hand one may depict the vertical profile of N on Figs. 1(a) and 1(b) themselves in the following way. Instead of picking up the values of N from the nomogram if the significant points of the tephigram are transferred on it and

TABLE 1

Height (m)	T (°C)	T_d (°C)	P (mb)	P_e (mb)	$P'=P+P_e$ (mb)	N	M
2	27	25	1000	510	1510	391	391
1956	21	15	800	278	1078	284	591
3091	13	01	700	110	810	220	706

joined by smooth lines it will form what we may call an ENPEEGRAM (N-P gram). Actual height may be written by the side of each point for proper interpretation of the diagram.

Similarly, the vertical M-profile may be depicted on Fig. 3 itself by joining the significant points, plotted on it, by smooth lines.

The nomograms will help to find out the vertical profile of N or M over any station readily, avoiding any elaborate computations, from the available radiosonde observation.

5. Acknowledgement

The authors are extremely grateful to Dr. A. K. Mukherjee, Meteorologist-in-Charge, Calcutta Airport for kindly going through the manuscript and offering valuable advice.

REFERENCE

Kulshrestha, S. M. and Chatterjee, K. P. 1966 *Indian J. Met. Geophys.*, **17**, pp. 367-384 *and* pp. 545-558.