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# An Iterative Scheme for diagnostic studies : A proposal

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(*Received 26 June 1968*)

ABSTRACT. An iterative scheme is suggested to compute the vertical velocity. The scheme is such that vertical velocity due to diabatic effects may be computed in terms of the known and measurable quantities and hence does not require any parameterization for diabatic heating. The diabatic thermodynamic energy equation and one of the components, involving the diabatic heating, of Balanced Baroclinic Omega Equation form a pair of equations from which the term involving the diabatic heating is eliminated and an expression for the vertical velocity due to diabatic heating is obtained in terms of the known and observable quantities.

## 1. Introduction

Various multi-level dynamical diagnostic models have been recently proposed for the study of vertical motion and energy transformations etc in weather systems. Notable among them, especially for the tropics, are by Krishnamurthi and Baumhefner (1966), Hawkins (1967) and Yanai and Nitta (1967) etc. However, it is seen that none of them are free from the limitations of the modelling approximations and parameterization of diabatic heating. The defect of having the first approximation for divergent part of the wind, derived from the rotational component of the wind only, and then iterating for a convergent solution is common to all the methods and it is a little difficult, at present, to foresee the possibility of getting rid of this defect till the divergent component of the wind can be derived from the observed wind itself to a satisfactory degree of accuracy.

Krishnamurthi and Baumhefner (1966) have computed only the adiabatic vertical velocity from Balanced Baroclinic Omega Equation. The iteration scheme presented by Yanai and Nitta (1967) does not need the parameterization of diabatic heating because it makes use of the vorticity equation and continuity equation only. Hawkins (1967) has computed the velocity from diabatic thermodynamic energy equation parameterizing the diabatic heating in a way similar to the one suggested by Kuo (1965). The time dependent term of thermodynamic energy equation is evaluated by integrating the vorticity equation for one time step and getting the geopotential at both times by solving the reverse balance equation. By doing so, one is able to avoid the usual criticism against the computation of time dependent term from the observations (Murakami 1957).

The iteration scheme presented in this paper does not require the parameterization of diabatic heating, however, the vertical velocity due to diabatic effects is also computed, knowing only the observed winds, which is the basic input for the model. In this proposed scheme, Balanced Baroclinic Omega Equation is broken in two components, one involving the forcing due to diabatic effects and the other involving forcing due to terms other than the diabatic heating. The former equation and thermodynamic energy equation constitute a pair of equations in two unknowns, *i.e.*, vertical velocity due to diabatic heating and the diabatic heating itself. From these two equations, the term containing the diabatic heating is eliminated and an expression for the vertical velocity due to diabatic heating is obtained. The time dependent term of this equation is proposed to be computed by the method suggested by Hawkins (1967). First approximation for vertical velocity and divergent component of the wind is proposed to be obtained from the solution of quasi-geostrophic omega equation and continuity equation.

## 2. Basic equations

The governing equation of the scheme are as follows. Table 1 gives the list of symbols. Neglecting the frictional effects the complete vorticity equations—

$$\partial\zeta/\partial t = -J(\psi,\eta) - \nabla\chi\cdot\nabla\eta - \eta\nabla^2\chi - \\ -\omega\partial\zeta/\partial p - \nabla\omega\cdot\nabla\partial\psi/\partial p \tag{1}$$

Vorticity equation in the approximate form —

$$\frac{\partial}{\partial p}\frac{\partial\zeta}{\partial t} = -\frac{\partial}{\partial p}\left|J(\psi,\eta) - \nabla\cdot f\nabla\chi\right| \tag{1a}$$

TABLE 1

List of Symbols

| Symbol | Explanation |
|---|---|
| $C_p$ | Coefficient of specific heat at constant pressure |
| $f$ | Coriolis parameter |
| $g$ | Acceleration due to gravity |
| $p$ | Atmospheric pressure |
| $Q$ | Heat supplied per unit mass per unit time |
| $R$ | Specific gas constant |
| $S$ | Static stability parameter |
| $\mathbf{V}$ | Horizontal velocity vector |
| $\eta$ | Absolute vorticity |
| $\zeta$ | Relative vorticity ($= \nabla^2\psi$) |
| $\phi$ | Geopotential |
| $\psi$ | Non-divergent stream function |
| $\chi$ | Irrotational velocity potential |
| $\omega$ | Vertical velocity in pressure co-ordinate |
| $\omega_g$ | Quasi-geostrophic vertical velocity |
| $\nabla$ | Isobaric del operator |
| $\nabla^2$ | Laplacian operator |
| $J$ | Jacobian operator |
| | $J\ (a, b) = \left(\frac{\partial a}{\partial x}\ \frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\ \frac{\partial b}{\partial x}\right)$ |

Thermodynamic energy equation —

$$\frac{\partial}{\partial t}\left(-\frac{\partial \phi}{\partial p}\right) + \mathbf{V}.\nabla\left(-\frac{\partial \phi}{\partial p}\right) - S\omega = \frac{R}{C_p \cdot p}\cdot Q \quad (2)$$

Balance equation —

$$\nabla^2\phi = f\nabla^2\psi + \nabla f.\nabla\psi + 2\ J\left(\frac{\partial \psi}{\partial x}\ ,\ \frac{\partial \psi}{\partial y}\right) \quad (3)$$

Equation of continuity —

$$\nabla^2\chi = -\ \frac{\partial \omega}{\partial p} \quad (4)$$

The quasi-geostrophic omega equation is —

$$\nabla^2\,(S\omega_g) + f^2\ \frac{\partial^2 \omega_g}{\partial p^2} = f.\ \frac{\partial}{\partial p}\ \Big\{J\ (\psi, \eta)\Big\} + $$
$$+\ \nabla^2\ \left\{\mathbf{V}\ .\ \nabla\ \left(-\ \frac{\partial \phi}{\partial p}\right)\right\} \quad (5)$$

The Balanced Baroclinic Omega Equation is —

$$\nabla^2 (S\omega) + f^2 \frac{\partial^2 \omega}{\partial p^2} = f \frac{\partial}{\partial p} \left\{ J(\psi, \eta) \right\} + \nabla^2 \left\{ J \left( \psi, -\frac{\partial \phi}{\partial p} \right) \right\} - \beta \frac{\partial}{\partial p} \frac{\partial}{\partial y} \frac{\partial \psi}{\partial t} - 2 \frac{\partial}{\partial t} \frac{\partial}{\partial p} \left\{ J \left( \frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y} \right) \right\} + f \frac{\partial}{\partial p} \times \left( \nabla \omega \cdot \nabla \frac{\partial \psi}{\partial p} \right) + f \frac{\partial}{\partial p} \left\{ \omega \frac{\partial \zeta}{\partial p} \right\} + f \frac{\partial}{\partial p} \left( \nabla \chi \cdot \nabla \eta \right) + f \frac{\partial}{\partial p} \left( \zeta \cdot \nabla^2 \chi \right) + \nabla \chi \cdot \nabla \left( -\frac{\partial \phi}{\partial p} \right) - \frac{R}{C_p . p} \cdot \nabla^2 Q \qquad (6)$$

Eq. (6) may be broken into the following two components —

$$\nabla^2 (S\omega_1) + f^2 \frac{\partial^2 \omega_1}{\partial p^2} = F_1 \qquad (7)$$

$$\nabla^2 (S\omega_2) + f^2 \frac{\partial^2 \omega_2}{\partial p^2} = F_2 \qquad (8)$$

where, $F_1 = -\dfrac{R}{C_p \cdot p} \nabla^2 Q$

and $F_2$ = all the terms on R.H.S. of eq. (6) except $F_1$

and
$$\omega = \omega_1 + \omega_2 \qquad (9)$$

Substituting eq. (9) in eq. (2) and eliminating $Q$ from eq. (2) and eq. (7) and by taking the Laplacian of eq. (2) and adding to eq. (7) we have —

$$\frac{\partial^2 \omega_1}{\partial p^2} = \frac{1}{f^2} \nabla^2 \left| S\omega_2 + \frac{\partial}{\partial t}\left( \frac{\partial \phi}{\partial p} \right) + \mathbf{V} \cdot \nabla \left( \frac{\partial \phi}{\partial p} \right) \right| \qquad (10)$$

Thus, we get an expression for vertical velocity due to diabatic heating effects, $\omega_1$ in terms of known and measurable quantities.

## 3. The Iteration Scheme

The following iteration procedure is proposed to obtain the vertical velocity.

1. Solve eq. (5) to obtain $\omega_g$ as the first approximation of $\omega$

2. Solve eq. (4) to get first approximation of $\chi$

3. Solve eq. (1a) using $\psi$ and $\chi^{(N-1)}$ to obtain

$$\frac{\partial}{\partial p} \left( \frac{\partial \psi}{\partial t} \right)^{(N)}$$

4. Solve eq. (8) to get $\omega_2^{(N)}$ using $\psi$, $\chi^{(N-1)}$ and

$$\frac{\partial}{\partial p} \left( \frac{\partial \psi}{\partial t} \right)^{(N)}$$

5. Solve eq. (4) to get $\chi^{(N)}$ using $\omega_2^{(N)}$. (It may be noted that the procedure till now is exactly similar to the one used by Krishnamurthi and Baumhefner).

6. Integrate eq. (1) to get $\xi^{t+\Delta t}$ using $\chi^{(N)}$ and $\omega^{(N)}$ on the R.H.S.

7. Solve eq. (3) to get $\phi^t$ and $\phi^{t+\Delta t}$ from $\xi^t$ and $\xi^{t+\Delta t}$ and obtain

$$\frac{\partial}{\partial t} \left( -\frac{\partial \phi}{\partial p} \right)$$

8. Solve eq. (10) to get $\omega_1$ using $\omega^{(N)}$ (for $\omega_2$) on the R.H.S.

9. Obtain the new value of $\omega$ by adding $\omega_1$ to $\omega_2^{(N)}$ and go back to step 6. Iterate between steps 6 and 9, such that

$$\omega^{(M)} = \omega_2^{(N)} + \omega_1^{(M-1)}$$

10. Obtain $\chi^{(M)}$ from $\omega^{(M)}$ by solving (4).

The above procedure may give a fairly good approximation to $\omega_1$ and $\omega_2$. However, the iteration may be further extended by going back to step 2 and repeating the whole procedure upto step 10 till a convergent solution is obtained.

## 4. Conclusion

The purpose of writing this note is to convey this idea to the readers and invite the comments and criticisms. Meanwhile, the question of proper boundary conditions and convergence criteria for this scheme are being studied. It is also proposed to apply this scheme to study the vertical velocity distribution and energy transformation in typical monsoon situations.

## 5. Acknowledgements

The author expresses his gratitude to Dr. H. F. Hawkins of N.H.R.L., Miami and Dr. T. Murakami of M.R.I., Tokyo for the valuable discussions he had with them. The author also records his thanks to Dr. K. R. Saha and Shri C. M. Dixit of I.T.M., Poona for their useful comments and suggestions.

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