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Biexponential Model of Radio Refractive Index over India

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ABSTRACT. Based on the radiosonde data during 1956-1960 biexponential model of radio refractive index has been derived at 12 stations in India for the four representative months recommended by International Radio Consultative Committee (CCIR), *viz.*, February, May, August and November. It is found that the biexponential model of radio refractive index is an excellent representation upto the lowest three kilometres over Madras and Trivandrum and upto 1·5 km at Port Blair. The agreement is fairly good over the inland stations of north and central India at 850-mb level in all the representative months except August.

1. Introduction

1.1. Models of radio refractive index structure of the troposphere are developed to estimate the systematic refraction of radio waves and the calculation of theoretical radio field strengths at satellite heights. In U.S.A., the effective earth's radius model was shown to be very useful in emphasizing the departure from standard of the atmosphere over southern California (Smyth and Trolese 1947). However, it was found that 4/3 earth distribution has an approximately correct slope in the first kilometre over the earth's surface but decreases much too rapidly above that height. This led to the development of a three-part model of the atmosphere in which the modified radio refractive index N is assumed to decay linearly with height from the surface to one kilometre above the earth's surface. The model also assumes exponential decrease of N above one kilometre becoming a constant value of 105 at 9 km a.s.l. and above that level a single exponential decrease of N with height. The three-part model of the atmosphere has the advantage of the effective earth's radius approach, particularly for such applications as point to point radio relaying over distance upto say 100 miles, where the radio energy is generally confined to the first kilometre and is also in reasonably good agreement with the average N-structure of the atmosphere. However, this model is rather inconvenient for theoretical studies and hence a single exponential model which could be used as an entire function from the ground level upwards was preferred. The simple exponential model has been found to represent, to a first approximation, the average refractive index structure within the first few kilometres above the ground for the United States (Bean and Thayer 1959), France (Misme 1958), Japan (Tao and Hirao 1960) and England (Lane 1961) but doubts have been expressed regarding its validity over tropical countries (Misme 1961).

1.2. *Model Radio atmospheres over India* — The radio characteristics of a tropical standard atmosphere were utilized to study the propagation conditions associated with some synoptic situations over India (Srivastava 1967). The correlation coefficient between the suface refractivity, N_S and the gradient in the lowest kilometre showed that a straight line fit between them is slightly better than the exponential model in the lowest one kilometre over India (Srivastava and Chatterjee 1967). On the basis of the homoclimes of radio climate, Srivastava and Pathak (*See* Ref.) found that the simple exponential model holds good satisfactorily over inland stations of north India and partially during winter over Assam and the coastal stations of West Bengal and Gujarat. The agreement was rather poor over the coastal stations of the Peninsula where the influence of tropical and equatorial maritime air masses are predominant. The biexponential model (Bean and Dutton 1966) wherein the contribution of the dry and wet terms of the radio refractive index may be examined separately, offered another possibility of studying its applicability over the country with special reference to the southern parts of the Peninsula wherein the single exponential model fails.

The object of the paper is, therefore, to study and discuss the biexponential model of radio refractive index based on the radiosonde data during the years 1956-1960 over India for the representative months recommended by International Radio Consultative Committee (CCIR), *i.e.*, February, May, August and November.

TABLE 1

Dry and wet terms of surface refractivity (N units) and scale height (km) over India

(Based on afternoon radiosonde data during 1956-1960)

	February				May				August				November			
	Nd_0	Nw_0	Hd	Hw	Nd_0	Nw_0	Hd	Hw	Nd_0	Nw_0	Hd	Hw	Nd_0	Nw_0	Hd	Hw
Allahabad	262	46	8·9	1·0	244	50	9·3	3·0	251	131	9·3	2·7	261	60	9·0	1·3
Bombay	261	89	9·0	1·4	256	124	9·1	2·3	259	126	8·7	2·3	260	105	9·0	1·7
Calcutta	262	64	9·1	1·7	254	127	9·1	2·2	256	137	8·8	3·0	264	86	8·8	1·7
Gauhati	264	65	9·0	1·7	256	115	9·1	2·8	255	135	8·9	3·2	265	97	9·0	1·8
Jodhpur	255	30	9·2	2·0	241	48	9·5	3·0	247	111	9·2	2·6	254	34	9·4	2·2
Madras	261	96	9·1	1·5	256	127	9·1	2·1	255	115	9·0	3·0	259	111	9·0	3·1
Nagpur	249	40	9·6	3·0	239	54	9·5	4·8	249	119	9·1	3·2	253	61	9·2	2·4
New Delhi	259	37	9·4	1·9	242	47	9·3	4·0	248	119	9·3	3·1	257	48	9·1	1·8
Port Blair	260	108	9·0	2·0	258	126	9·0	2·7	259	126	9·0	2·7	260	116	9·1	2·1
Trivandrum	257	105	9·2	1·8	257	124	9·4	2·4	259	116	9·1	3·5	259	116	9·1	2·6
Veraval	263	90	8·9	1·9	258	139	9·0	1·6	259	134	8·9	1·3	259	100	9·1	1·5
Visakhapatnam	261	109	9·0	1·7	257	150	9·1	2·8	256	142	9·0	2·5	264	103	8·9	1·5

TABLE 2

Percentage errors (per cent) for the four representative months of the twelve Indian radiosonde stations

	850 mb				700 mb			
	Feb	May	Aug	Nov	Feb	May	Aug	Nov
Allahabad	5	4	7	4	8	10	20	5
Bombay	3	3	7	3	10	15	20	8
Calcutta	3	3	6	5	5	10	20	10
Gauhati	8	7	5	5	10	10	20	10
Jodhpur	6	6	6	5	8	12	15	8
Madras	1	1	4	1	2	1	5	2
Nagpur	6	4	7	6	5	7	6	4
New Delhi	4	5	8	5	4	6	10	4
Port Blair	3	4	4	5	10	15	15	10
Trivandrum	4	3	2	1	3	2	5	4
Veraval	3	6	10	5	10	12	15	10
Visakhapatnam	1	7	10	8	10	11	15	10

2. Modified Radio Refractive Index and data

Modified radio refractive index, $N(h)$ is given by—

$$N(h) = \frac{77 \cdot 6}{T}\left(P + \frac{4800\,e}{T}\right) \qquad (1)$$

$$= \frac{77 \cdot 6P}{T} + 4800 \times \frac{77 \cdot 6\,e}{T^2}$$

$$= \underset{\text{(dry term)}}{Nd} + \underset{\text{(wet term)}}{Nw} \qquad (2)$$

where P is the atmospheric pressure in millibars, e is the water vapour pressure in millibars and T is the temperature in degree absolute.

Mean monthly values of P, T and e at 12 stations in India based on the afternoon radiosonde data during 1956-1960 were substituted in equation (1) and the values Nd and Nw were calculated for surface, 850, 700, 600, 500, 400, 300, 200, 150 and 100-mb levels. It may, however, be mentioned that the wet term Nw of equation at 500-mb level was evaluated only in those cases for which sufficient number of observations were available.

3. Biexponential Model of Radio Refractive Index

The biexponential model of radio refractive index is expressed by assuming that both Nd and Nw in Eq. (1) can be represented by exponential function of height, Z (Bean and Dutton 1966)

$$N(h) = Nd_0 \exp\left(-\frac{Z}{Hd}\right) + Nw_0 \exp\left(-\frac{Z}{Hw}\right) \quad (3)$$

$$= Nd + Nw$$

where Nd_0 and Nw_0 are the refractivities (modified radio refractive index) at the surface for the dry and wet terms respectively. *Hd* and *Hw* are the scale heights of the dry and wet parts. It may be mentioned that scale height is defined as the height at which $N(h)$ reduces to $(1/e)^{\text{th}}$ of the surface value Nd_0 or Nw_0

Since,

$$\left.\begin{aligned} \log_{10} Nd &= \log_{10} Nd_0 - \frac{Z}{Hd}\log_{10} e \\ \text{and} \\ \log_{10} Nw &= \log_{10} Nw_0 - \frac{Z}{Hw}\log_{10} e \end{aligned}\right\} (4)$$

Hd and *Hw* can be determined by least squares method for all the stations separately for the representative months.

4. Results and Discussion

4.1. The values of *Hd* and *Hw* computed with the help of Eq. (3) are given in Table 1 for 12 radiosonde stations. Surface refractivities have also been given in the same table to facilitate the computation of the biexponential model of radio refractive index over India.

4.1.1. It is well known that the radio refrative index structure of the lowest three kilometres in the troposphere is generally considered sufficient for the estimation of refraction and basic transmission loss. The data presented in Table 1 was, therefore, utilized to compute the dry and wet parts (N cal) of the right hand side of Eq. (3) at 850 and 700-mb levels and the percentage error

$$\frac{N_{\text{obs}} - N_{\text{cal}}}{N_{\text{obs}}} \times 100$$

was determined for 850 and 700-mb levels. The results for all the twelve radiosonde stations are shown in Table 2.

4.1.2. Taking into consideration the uncertainties in the constants of equation (1) and also various errors in radiosonde observations, it appears satisfactory if 5 per cent errors is taken as the lowest limit for the biexponential model to hold good at any Indian station in the lowest three kilometres over India. Above three kilometres the radiosonde errors increase since the wet bulb reading of Indian radiosondes are unreliable near freezing level. Keeping this in view, it is obvious that the biexponential model is an excellent representation upto 3 km over Madras and Trivandrum where the single exponential model failed (Srivastava and Pathak *see* Ref.). The agreement with the biexponential model remained fairly good at Port Blair at 850-mb level but the large errors at 700-mb level do not favour the use of this model. No explanation is forthcoming at present as to why the model should not hold good at this station as well since the same air masses affect Madras, Trivandrum and Port Blair almost equally throughout the year. At the remaining stations over the country, the biexponential model of radio refractive index was reasonably good only at 850-mb except during August when, the errors were rather large. At the higher level, namely 700-mb, the errors were considerable forbidding the use of this model.

TABLE 3

Hd and *Hw* for Arctic and Tropical stations

	Hd (km)	*Hw* (km)
Canton Island	9·4	2·0
New Delhi (average)	9·3	2·7
Madras (average)	9·1	2·4
Isachsen, N.W.T. Canada } Arctic	6·3	6·5
U.S.A. (Year round country-wide average)	9·0	2·5

4.2. *Spatial distribution of Hd and Hw over India*

4.2.1. The monthly and geographical variation of *Hd* is quite small (Table 1). However, the wet term scale height, namely *Hw*, shows considerable geographical as well as seasonal variation. The spatial distribution of *Hw* during February, May, August and November, is shown in the Fig. 1. The diagrams are self explanatory.

4.2.2. Table 3 compares *Hd* and *Hw* over India with those at other places in the world. The scale heights over India appear to be quite consistent.

5. Limitations of the study

The investigations presented in the paper based on the radiosonde data are useful only for the study of average conditions of radio ray refraction over India. Knowledge of the five structure of the radio atmosphere requires microwave refractometers which are expensive and difficult to handle and are not available in India so far. The main limitations of the study arises from the use of two different types of radiosonde in India, namely C and F types, whose performance characteristics are different (Ananthakrishnan *et al.*

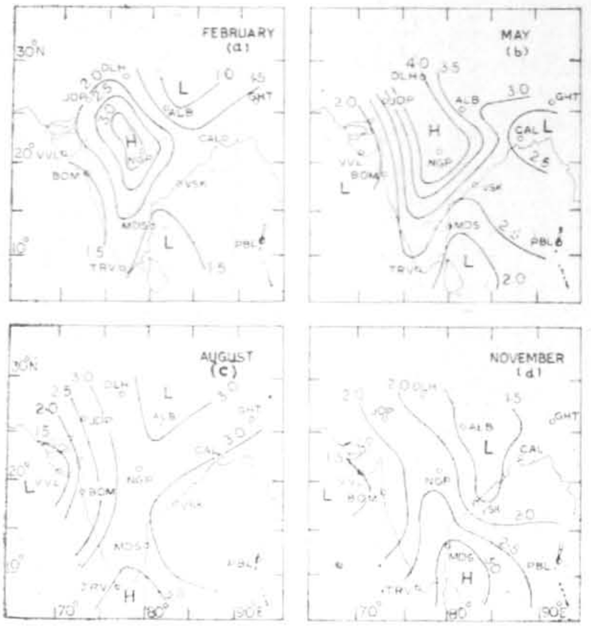

Fig. 1. Distribution of H_w over India during representative months

see Ref). According to these authors C-type radiosonde records systematically higher temperature than F-type sonde, the differences being of the order of 0·5° to 1°C in the lower troposphere and 3° to 5° C in the upper troposphere. The standard deviation of the daily temperature values, increases steadily from 850 mb to 100 mb for both C and F-type sondes. No correction term can be applied to N values over India since the errors due to dew point temperatures of C and F-type radiosondes which greatly influence N values, have not been determined so far.

However, since model radio atmospheres have been derived at twelve individual stations based on the data of 5 years and minimum error of 1 per cent in the computation of N values has been allowed for radiosonde observations to determine the goodness of 'biexponential fit', the general applicability of the results is not likely to be affected for the purpose of average properties of radio atmosphere inspite of the differences in C and F-type radiosondes.

6. Conclusions

The above study has shown that the biexponential model of radio refractive index is a good representation upto the lowest 3 kilometres over Madras and Trivandrum and upto 1·5 km over Port Blair. The agreement is fairly good over the inland stations of north and central India at 850-mb level except during August.

The dry and wet term scale heights over India are quite consistent when compared with the other places in the world.

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