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Fitting the seasonal distribution of wind direction in central region of Kerala using Bernstein Polynomials

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सार – चक्रीय डेटा के लिए गैर प्राचलिक बर्नस्टीन बहुपद अनुमानक दैनिक पवन की दिशाओं को सुनिश्चित करने के लिए लागू किया गया। चूंकि केरल के त्रिशूर जिला में नवंबर से फरवरी तक की ऋतु में एक विशेष प्रकार का पवन चलता है, जिसे वृश्चिकाकाटू कहा जाता है, और वर्ष-दर-वर्ष ऐसी पवन की गति में कमी आ रही है, अत: पवन की दिशा में विविधता है या नहीं, इसकी जांच करने के लिए एक विश्लेषण किया गया। 1983 से 1997 और 2003 से 2017 की अवधि के लिए सुबह (7.25 बजे) और शाम (2.25 बजे) के दौरान पवन की दिशाओं के ऋतुवार वितरण को अलग से सुनिश्चित किया गया। अध्ययन से पता चला है कि ऋतुनिष्ठ वितरण में कोई बदलाव नहीं आया है।

ABSTRACT. Non parametric Bernstein polynomial estimator for circular data was applied for fitting daily wind directions. Since Thrissur District of Kerala faces a special type of wind called Vruschikakkaatu in November to February season and there is a decrease of such wind speed year after year, an analysis was conducted to check whether there is variation in wind direction. The season wise distribution of wind directions during morning (7.25 am) and evening (2.25 pm) for the period from 1983 to 1997 and 2003 to 2017 were separately fitted. Study revealed there is no change in the seasonal distributions.

Key words – Bernstein Polynomials, Circular data, Wind direction, Climate change.

1. Introduction

Weather variable, wind direction is an important parameter in meteorological data analysis. This comes under the classification of circular data which does not have the properties of linear structure. The data can be represented as a point in the circumference of a circle. Here the usual arithmetic mean of the two points say 5° and 355° is 180°, which is absurd as the variables are circular which move only between 355° and 5°. This means that linear statistical measures for wind directions in case of meteorological data analysis are misleading.

Mardia (1972); Batscelet (1981); Fisher (1993); Mardia and Jupp (1999) and Jamalamadaka and Sen Gupta (2001) described various Statistical Measures for circular data analysis. Kamisan *et al.* (2010) found the best circular distribution for fitting the wind direction for South-westerly monsoon in Malaysia. Four types of distributions were used and found that von Mises distribution is the right one for fitting the distribution. Wrapped exponential and Lapalace distributions were introduced by Jammalamadaka and Kozubowski (2001, 2003 and 2004). Pewesy (2000 and 2006) introduced wrapped skew normal distribution. Kato and Shimizu (2005) introduced wrapped *t* distribution. Dattathreya *et al*. (2007) developed pdf for Circular log normal, Circular logistic, Circular weibull and Circular Extreme value distributions. Reed and Pewsey (2009) introduced the Wrapped Normal - Laplace distribution. Sophy (2012) explained wrapping of Linnik, Wrapped Geometric Stable, Wrapped Geometric, Wrapped Laplace distributions for integers as part of the Ph.D. research. Carnicero *et al*. (2018) introduced how Bernstein polynomials can be used to approximate various distributions defined on a closed interval and he used it to fit circular distributions. Vitale (1975) used Bernstein polynomials for the first time to produce smooth density estimates. Babu *et al.* (2002) studied the asymptotic properties of Bernstein polynomials in approximating bounded and continuous density functions. Kakizawa (2004) explained the use of Bernstein polynomials for continuous densities and Leblanc (2010) introduced a bias reduction technique using a Bernstein-polynomials.

The climate of Thrissur is typical to the climate of Kerala,*i.e*., Tropical monsoon (humid) type. The district is characterized by dry weather during summer months (March -May) with intermittent summer showers, blessed with two rainy seasons, *i.e*., southwest monsoon (June-September) and post monsoon (October -November) and pleasant winter months (December - February). The district is blessed with pleasant and periodic wind from November fortnight to February fortnight, known as the *vrischikakaattu* (the wind blowing during the local calendar *Vrischikam though Makaram*). The wind is characterised with heavy speed and having no moisture content as it passes through the Palakkad Gap of the Western Ghats and subsequently gains momentum. The district has a tropical humid climate with an oppressive hot season and plentiful of seasonal rainfall. Philip and Kumar (2010) compared the extreme wind speeds estimated based on NCEP/NCAR reanalysis data for 100 years return period for Goa, Visakhapatnam and Machilipatnam in the north Indian Ocean and found there is a trend. Sihilja (2011) made an attempt to understand the nature of wind direction of vruschikakkat based on the windrose constructed during the year 2006 to 2011 and found similar patterns. Shana *etal*.(2020) analysed trend in evaporation across central regions of Kerala and studied wind speed of vruschikakkat. Unnikrishnan *etal*.(2019) reported the average wind speed from 1983 onwards during the *Vrischikakaattu* period (November - February, 2018) was 7.1 km/hr, which is decreasing year after year over the region.

2. Materials and method

Bernstein (1912) introduced the concept of Bernstein polynomials for a probabilistic proof of the Weierstrass Approximation Theorem. He showed using this concept that any continuous function, on a closed interval can be uniformly approximated with these polynomials. Non Parametric density estimation using Bernstein Polynomials was tried to fit the wind direction using the data collected from the Principal Agricultural Meteorological Station and maintained by the Department of Agricultural Meteorology, Kerala Agricultural University (IMD Station). The wind direction is being recorded in 16 directions based on compass using wind vane. Here North (N) is taken as 0°, then the other multiples of 22.5° like NNE, NE ENE, E, ESE, SE, SSE, S, SSW, SW, SW, W, WNW, NW and NNW respectively. Daily wind direction data collected from the Department of Agricultural Meteorology, College of Horticulture, Kerala Agricultural University, Vellanikkara, Thrissur District and Kerala for the period from 1/1/1983 to 31/12/2017 formed the material for the study.

where, if θ_i be the angle, then $X = \sum \cos\theta_i$ and $Y = \sum \sin\theta_i$, So that tan θ_i = Y/X. Then the resultant circular mean of the $\theta_i$istan^{-1}(Y/X).

Corresponding to the random variable X with finite support in [0,1] and distribution function $F_x(.)$, $F_x(.)$ can be approximated using the Bernstein Polynomial distribution order k which is defined by,

$$B_k(x) = \sum_{j=0}^{k} f_X\left(\frac{j}{k}\right)\binom{j}{k} x^j (1-x)^{k-j}, 0 \le x \le 1$$ and k is a natural number.

$B_k(x)$ is a distribution function and it is well known that it converges uniformly to the distribution function $F_x(x)$ as k tends to infinity. Analysis using this function was estimated simply even by using Microsoft Excel – vb environment.

The associated Bernstein density function is

$$B_k(x) = \sum_{j=0}^{k} \left[F_X\left(\frac{j}{k}\right) - F_X\left(\frac{j-1}{k}\right)\right]\beta(x/j, k-j+1)$$

where, β is the beta density function $\beta(x/a,b) = \sum_{j=0}^{k} \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}$,

The $B(a,b) = \frac{(a-1)!(b-1)!}{(a+b-1)!}$, where, a and b are positive integers.

These polynomials were approximated to circular densities by assuming the circle as a closed interval.

3. Results and discussion

The mean annual wind speed over the region is 4.9 km/hr. The highest speed was noticed during January (9.8 km/hr) followed by December (8.5 km/hr), February (6.9 km/hr) and November (5.2 km/hr). The least wind speed was observed in October (3.1 km/hr) followed by September (3.2 km/hr).

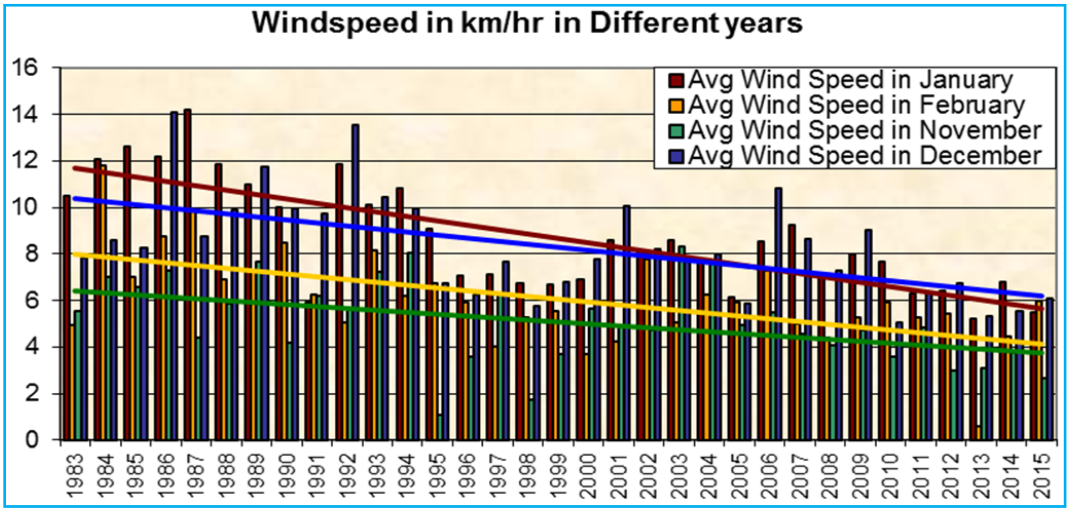

Fig. 1. Decreasing windspeed during vrischikakkattu

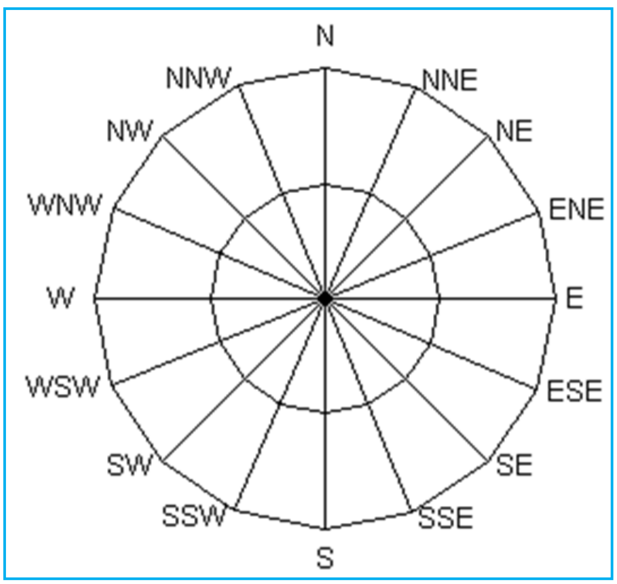

Fig. 2. Sixteen directions for measuring wind direction

TABLE 1

Descriptive Statistics for the wind directions

Variable	Wind in morning	Wind in evening
Grand Mean Vector (GM)	4.346°	217.906°
Length of Grand Mean Vector (r)	4669.404	410.368
95% Confidence Interval (-/+) for GM	3.656°	214.269°
	5.041°	221.777°
99% Confidence Interval (-/+) for GM	3.491°	213.425°
	5.208°	222.747°

It also revealed that the highest wind speed was noted during winter season, *i.e*., December to February (7.86km/hr), which coincides with *Vrischikakaattu* season in this parts of the State, followed by southwest monsoon (June to September). The wind speed during southwest monsoon period was 4.9 km/hr. Summer (March-May) recorded a wind speed of 4.3 km/hr while the least speed was noticed during post monsoon period (October-November). The average wind speed during the *Vrischikakaattu* period (November - February) was 7.1 km/hr, which is higher than the mean annual wind speed over the region.

Anemometer is used to measure the wind speed and wind vane for measuring wind direction. The average wind speed in Thrissur district during November, December, January and February is plotted in the Fig. 1.

Vrischikakkattu is a seasonal wind blowing in certains regions of Thrissur and Palakkad districts. The easterly wind as it passes through the Palghat gap of the Western Ghats, gains momentum and blows with greater speed. The wind speed of *Vrischikakka*t is more in January and followed by December. From the data it can be noted that the average windspeed of Vrischikakkattu is decreasing year after year. This may be due to the low

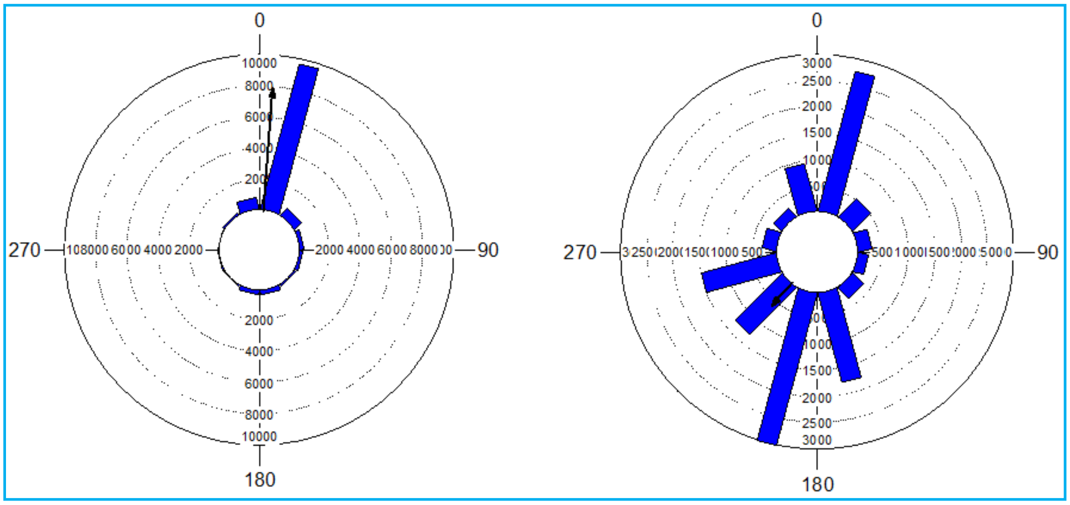

Fig. 3. Frequency of Wind I and Wind II

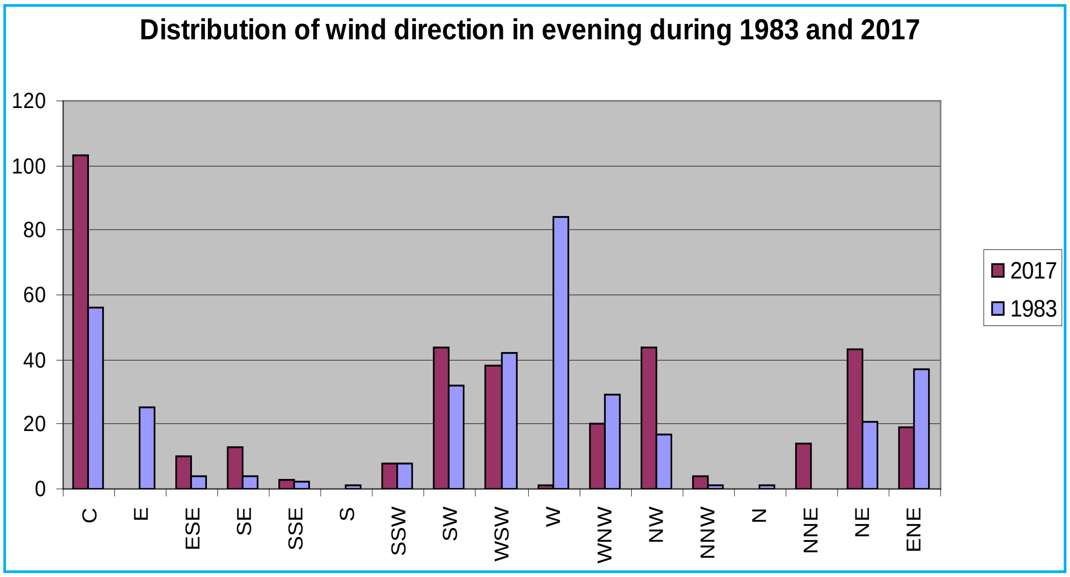

Fig. 4. The wind direction in evening during 1983 and 2017

temperature gradient during north east monsoon in Tamilnadu and Kerala in recent years. Under the climate change and warming scenario, wind speed and direction in Thrissur District are likely to change. Since the *vrischikakkaatu* is a seasonal wind in Thrissur and Palakkad district, the study of wind direction deserves utmost importance.

Fig. 2. gives the 16 directions of wind in Kerala. With out loss of generality, North (N) direction was considered 0°, then the subsequent anglular directions with multiples of 22.5° like NNE, NE ENE, E, ESE, SE, SSE, S, SSW, SW, WSW, W, WNW, NW and NNW respectively.

Daily wind direction data collected from the Department of Agricultural Meteorlogy, College of Horticulture, Kerala Agricultural University, Vellanikkara, Thrissur District, Kerala for the period from 1/1/1983 to 31/12/2017 formed the material for the study. The descriptive statistics like Circular mean was computed by treating the angular points as points on the circumference of the unit circle for the data corresponding to wind direction are given in Table 1.

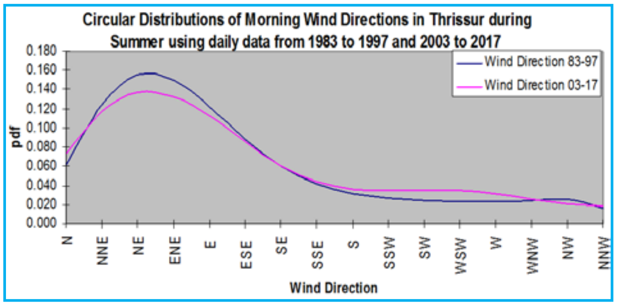

Fig. 5. Non parametric distributions with Bernstein polynomials for morning wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for summer season

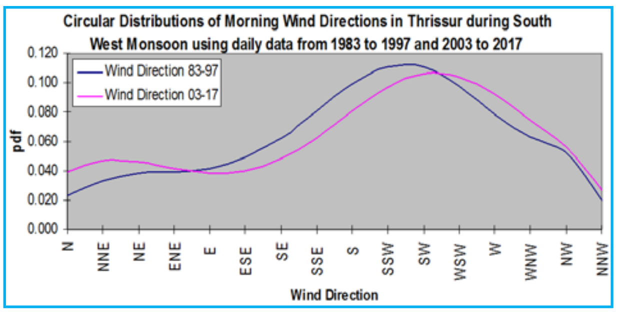

Fig. 6. Non parametric distributions with Bernstein polynomials for morning wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for southwest monsoon season

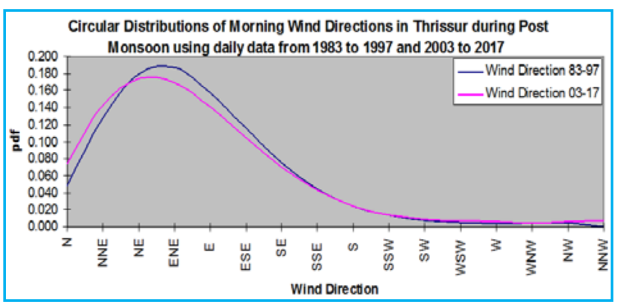

Fig. 7. Non parametric distributions with Bernstein polynomials for morning wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for post-monsoon

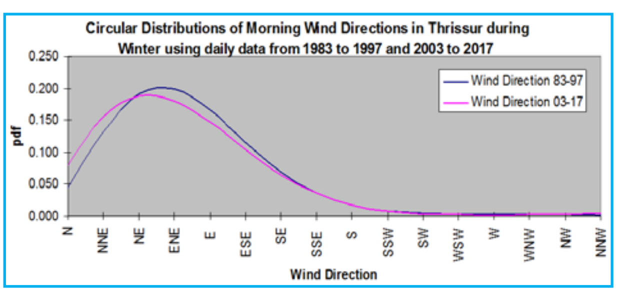

Fig. 8. Non parametric distributions with Bernstein polynomials for morning wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for winter season

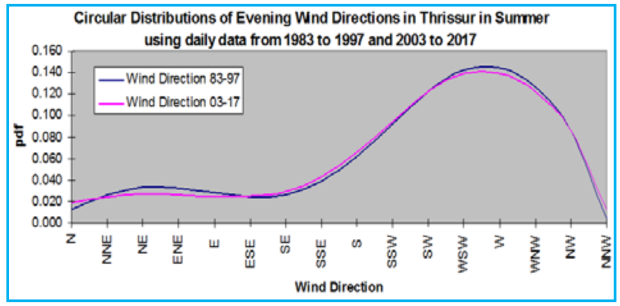

Fig. 9. Non parametric distributions with Bernstein polynomials for evening wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for summer season

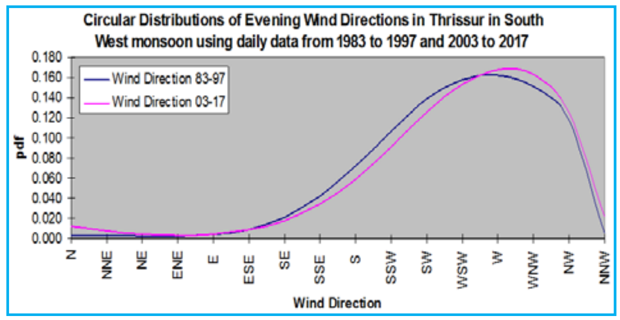

Fig. 10. Non parametric distributions with Bernstein polynomials for evening wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for southwest monsoon season

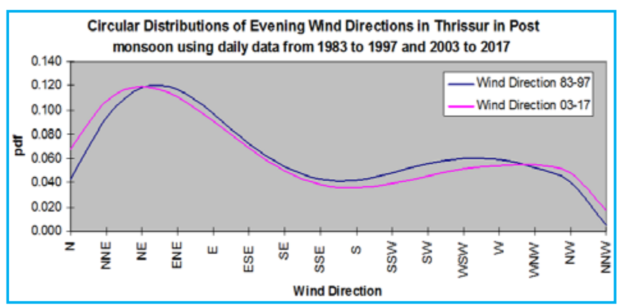

Fig. 11. Non parametric distributions with Bernstein polynomials for evening wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for post-monsoon Season

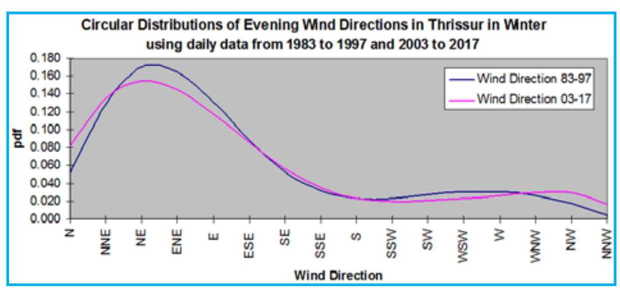

Fig. 12. Non parametric distributions with Bernstein polynomials for evening wind direction data of Thrissur from 1983 to 1997 and 2003 to 2017 for winter season

TABLE 2

Comparison of seasonal indices of wind directions for the period 1983-97 with that of the period 2003-2017 using paired t-test

Pair	Wind in morning				Wind in evening			
	Mean	SE (Mean)	*t*	Sig. (2-tailed)	Mean	SE (Mean)	*t*	Sig. (2-tailed)
Summer1983-97 - Summer 2003-2017	0.01	0.30	0.04	0.97	0.01	0.81	0.01	0.99
SW Monsoon 1983-97 - SW Monsoon 2003-17	0.00	0.46	0.00	1.00	0.01	1.15	0.01	1.00
Post Monsoon 1983-97 - Post Monsoon 2003-17	-0.01	0.77	-0.01	0.99	0.01	0.80	0.01	0.99
Winter 1983-97 - Winter 2003-2017	-0.01	0.85	-0.01	0.99	-0.01	0.94	-0.01	0.99

Wind direction is usually measured in 16 directions. In addition to this calm (no wind at the time of observation) is noted by 'C' and variable (fluctuating wind at the time of observation) by 'V'. The winddirection variables were coded from 0 to 16 for easiness of the analysis. From Fig. 3, it is clear that the pattern of wind direction in morning is unimodal and most frequent in NNE direction with a mean value of 4.346°. But when it was studied in the evening, it was bimodal with clustering near NNW, N, NNE, NE directions and SSE, S, SSW, SW, WSW directions.

When wind speed and wind direction are observed at a time in morning, NNW, N, NNE directions shows maximum windspeed. E, ESE, SE, SSE, S, SSW, SW, WSW, W direction shows maximum windspeed below 10km/h. The same pattern is observed in the case of evening winds also.

Analysing the daily wind direction, it is found that there is high variation in wind direction after 2008. Simply plotting the distribution of wind direction categorically using the data corresponding to the years 1983 and 2017 the variations are clear from the charts given in Fig. 4. But if we consider a long period it is not so.

Though the frequency of wind from South West direction has increased from 1983 to 2017, there was considerable decrease in the frequency of wind from west direction (Westerly wind). Frequency of 'calm' wind direction (calm wind direction denoted that there is no wind speed is noticed at the time of observation) has increased well.

Though there may be small amount of variation year after year, such variations are not seen if we consider two groups of continuous years after a gap. Such studies are very common in econometric analysis in comparing the heterogeneity of groups. Hence an attempt is made to study the distributions of wind directions in two groups of continuous years after removing some middle years. Here daily wind directions from 1/1/1983 to 31/12/1997 were taken as former group and 1/1/2003 to 31/12/2017 were considered as latter group. The middle portion was removed to test whether there is an overall shift is realized from the data using bernstein polynomials.

3.1. *Probability density function through Bernstein polynomials*

Bernstein polynomial distribution of order k is generally defined by,

$$B_k(x) = \sum_{j=0}^{k} f_X\left(\frac{j}{k}\right)\binom{j}{k} x^j (1-x)^{k-j}, 0 \le x \le 1 \text{ and } k \text{ is}$$

a natural number.

We know that $B_k(x)$ is a distribution function and it converges uniformly to the distribution function $F_x(x)$ as k tends to infinity. Analysis using the empirical function was by using Microsoft Excel – vb environment.

This polynomials were approximated to circular densities by assuming the circle as a closed interval.

Carnicero *et al.* (2010) proposed a modified formula for circular data as :

$$\hat{f}_k(\theta) = \frac{1}{2\pi}\left[\begin{array}{l} \frac{1}{2}\left[\hat{F}_X\left(\frac{2\pi}{k}\right)+1-\hat{F}_X\left(\frac{2\pi(k-1)}{k}\right)\right] \\ \beta(\frac{\theta}{2\pi}/1,k)+\sum_{j=2}^{k-1}\frac{1}{2}\hat{F}_X\left(\frac{2\pi j}{k}\right)+1 \\ -\hat{F}_X\left(\frac{2\pi(j-1)}{k}\right)\beta(\frac{\theta}{2\pi}/j,k-j+1) \\ +\left[\hat{F}_X\left(\frac{2\pi}{k}\right)+1-\hat{F}_X\left(\frac{2\pi(k-1)}{k}\right)\right] \\ \beta(\frac{\theta}{2\pi}/k,1) \end{array}\right]$$

where, the weight for the first and last density will be equal and hence the fitted density will be circular. Here we have 16 directions for wind and k can be taken as 16. Empirical distribution functions were estimated using the formula and fitted. The newly developed tool for finding the probability density function for wind directions observed in the morning using Bernstein polynomials were plotted in Figs. 5-8.

The probability density function for wind directions observed in the evening using Bernstein polynomials were plotted in Figs. 9-12.

Table 2 clearly explain that there is no significant changes in proportion of frequency of wind direction in the region in any season while comparing the two periods. Hence the study reveals the concept that distribution of wind direction is constant over time even though there is varying wind speed year after year.

Even though there are large studies on changing the climatic scenario, there are very rare studies on wind directions. Here the analysis reports reveal that the seasonal distribution of wind direction are not changing significantly even if large variation in the seasonal wind speed in the central regions of Kerala.

4. Conclusions

Thrissur experiences a special wind called Vrischikakkaattu (an easterly strong wind), blows during the period November to February with high speed than normal each year. Also there are reports that the wind speed is continuously changing year after year. Hence a study was conducted to test whether there is shift in wind direction due to this factor. Non parametric Bernstein polynomial estimator for circular data was introduced and applied for daily wind directions. The distribution of wind directions during morning (7.25am) and evening (2.25 pm) for the period from 1983 to 1997 and 2003 to 2017 were separately fitted. The systematic change in the pattern of wind direction during 1983 to 2017 is found meager and that it is not noticeable. The same data were analysed with paired *t*-test and found there is no significant changes in proportion of wind direction in the long run.

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