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On determination of Sun-rise/set time and on construction of a Computer for the purpose

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ABSTRACT. A simple sliding computer to determine sun-rise/set times for any place within Indian latitudes is described. Its mathematical basis is discussed. A rough and ready method based on variation of sun-rise/set times with latitude is also mentioned. Variation in the observed times of sun-rise/set with altitude is discussed and its importance to aircraft pilots indicated.

1. Information about the sun-rise/set for destination airfield is fairly useful to aircraft pilots in the preparation of their flight plans. This information pertaining to any aerodrome station is readily available at the meteorological office at the station, but that for other places has to be computed. Where the Nautical Almanac or the relevant portion of it dealing with sun-rise/set is not available, the information cannot be supplied. Further, the computational work involves some time.

2. In order to be able to provide this information accurately without the aid of the Almanac and with minimum computational work, the author has devised a simple computer described below. Since sun-rise/set on any day remains appreciably the same over a long period of years, the computer is valid for any calendar year.

(*a*) The computer consists of two concentric discs A and B. The inner disc A (Fig. 1) can rotate over the outer disc B. For the sake of easy reference the disc B is prepared in two parts, B_1 (Fig. 2a) and B_2 (Fig. 2b)—one for use when the sun is to the north of the equator and the other when the sun is to its south. These discs B_1 and B_2 consist of radial lines along each of which are marked (1) two calendar dates, (2) sun-rise and sun-set times and (3) correction in minutes (*i.e.*, equation of Time) for each of the dates to be applied to these timings. The sun-rise and set timings are given along each radial line. On the two sides of these figures are marked the time corrections—positive correction in ordinary and negative in Roman numerals. Finally, the dates (with names of months given alongside the dates) are written adjoining corresponding time correction. The dates and the appropriate corrections are separated by concentric arcs of circles. Also, for ease of reference arrows are drawn at the end of the circular arcs along which the dates are written to indicate the range of dates as well as the directions in which they occur in the chronological order.

(*b*) The inner disc consists of markings for latitudes at each degree interval from 04°N to 25°N and for each odd degree latitude upto 35°N thereafter. An inner circle marked on this disc gives the time correction in minutes for longitudes from 68°E to 97°E, the correction being positive or negative according as the longitude is to the west or east of 82½°E, as indicated by words 'positive' and 'negative' on this disc. A zero pointer is also marked on this disc. (A moving pointer is attached to the central pin of this dial. This is not shown in the figure.)

3. The use of the Computer

(*a*) Choose the disc B_1 or B_2 appropriate to the date for which sun-rise/set is to be determined. Pass the moving pointer over the date in question. For intermediate dates not marked on B_1 or B_2, simple mental interpolation would be necessary. Also note the time correction for this date to be applied later. Rotate the inner disc till the zero pointer comes exactly under the moving pointer.

(b) Holding the pointer in *this position*, bring the moving pointer over the latitude of the place marked on the inner disc A. For intermediate latitudes not marked on this disc, simple mental interpolation would be necessary.

(c) Note the position of this pointer between the two consecutive radial lines (on the outer disc) between which it passes and the timings of sun-rise/set given along these lines. Determine by simple mental interpolation the timings corresponding to the position of the moving pointer. In case it occupies a position beyond the last radial line, the timings (*viz.*, 0557 and 1803) corresponding to this last line are to be taken.

(d) Apply the following corrections to these timings—

(i) Time correction corresponding to the date as found out in (a) above.

(ii) The time correction for the longitude of the place, as determined from the inner circle on disc A (by simple interpolation for intermediate values of longitudes).

(e) The result is the sun-rise/set time in IST.

4. Mathematical basis of the Computer

It is well known that the zenith distance Z of the sun at any place is given by—

$$\cos Z = \cos \phi \cos \delta \cos \omega t + \sin \phi \sin \delta \quad (1)$$

where ϕ is the latitude of the place, δ is the declination of the sun (which is taken as positive when the sun is to the north of the equator and negative when to its south). ω is the rate of conversion of time into arc, t is the time reckoned from the noon.

The condition for rise or set is that $Z = \pi/2$. Hence, for rise or set

$$\cos \phi \cos \delta \cos \omega t + \sin \phi \sin \delta = 0 \quad (2)$$

If t is measured from 0600 hrs (or 1800 hrs in case of sunset), the above equation can be written as—

$$\cos \phi \cos \delta \sin \omega t + \sin \phi \sin \delta = 0$$

$$\text{or } \sin \omega t = -\tan \phi \tan \delta \quad (3)$$

for numerical values of t, the —ve sign may be discarded and numerical values of ϕ and δ (without sign) only need be considered.

Hence, $\log \sin \omega t = \log \tan \phi + \log \tan \delta$ (4)

where t is the numerical value corresponding to the latitude ϕ and declination δ. Thus, the problem of determination of t is reduced to the simple operation of addition of two logarithms—a well known principle of the slide rule. (Indeed, the computer can also be constructed in the form of a slide rule).

In the computer described above the radial lines drawn on discs B_1 and B_2 are to the scale of log tan δ for each degree of δ and also for particular value of $\delta = 23\frac{1}{2}°$ and near-zero values for some days near the equinoxes. The two dates corresponding to these values of $\pm\delta$ were found from the Nautical Almanac. (δ has been taken to be constant for a day, and is appreciably the same for any date over a long period of years). Values of t for these various values of δ were calculated from trigonometrical tables. For δ positive, they were subtracted from 0600 hours and added to 1800 hours and for δ negative they were added to 0600 hours and subtracted from 1800 hours to get apparent times of rise and set. The allowance of 3 minutes and 20 seconds for atmospheric refraction and sun's semi-diameter was incorporated by subtracting it from the apparent time of sun-rise and adding to the apparent time of set. Equation of time for each date (for which it is appreciably constant over a long period of years) was also noted from the Nautical Almanac. All these values, thus found, were marked along the radial lines on discs B_1 and B_2 as already described. The points corresponding to latitudes were also marked to the scale of log tan ϕ on disc A.

Thus, the rotation of the disc A in the manner described earlier actually means adding log tan ϕ to log tan δ. Hence t is obtained directly according to the equation (4).

The corrections for equation of time and longitude are for conversion of the apparent time t into Local Mean (civil) Time first and then to IST.

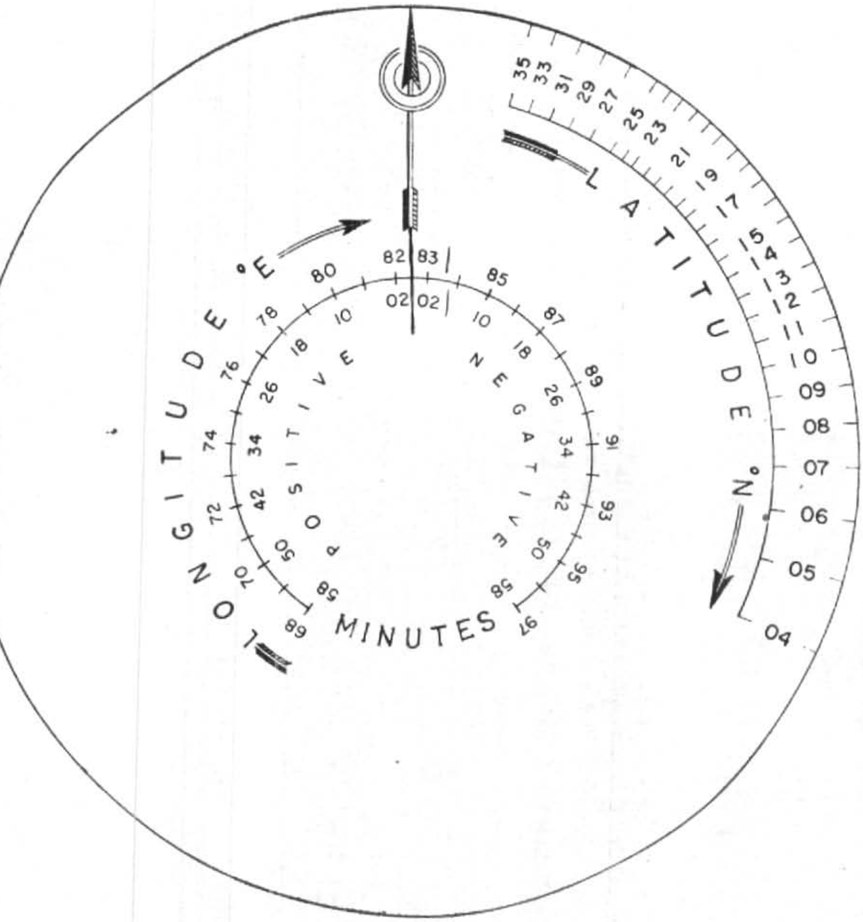

Fig. 1. Disc A

5. Another rough and ready method to get time of sun-rise/set

At a place of known latitude for which the local mean times of sun-rise/set for any day are known, the timings (LMT) for other places on different latitudes can be determined to a fair degree of accuracy by knowing the variation of sun-rise/set per degree of latitude for that day. In Fig. 3, curve I gives the per degree variation in sun-rise for latitudes 20° to 05°N and curve II for latitudes 20° to 35°N. The variation will have the sign opposite to that indicated in the figure when it is to be considered in opposite direction (*i.e.*, from lat. 05° to 20°N and from lat. 35° to 20°N). Again, for sun-set also opposite sign should be taken. The per degree variation is to be multiplied by the difference of latitudes (in degrees) of the two places and then applied to the times of sun-rise/set for the known place. If the original times are in IST, then a further correction for the difference of longitudes of the two places is to be applied at the rate of 4 minutes per degree (positive when the new place is to the west and negative for east) to get the time in IST. The data for Fig. 3 have been computed from the information in the Nautical Almanac.

6. Variation of sun-rise/set with altitude

At any place, at an altitude h above ground, the sun (or any heavenly body for

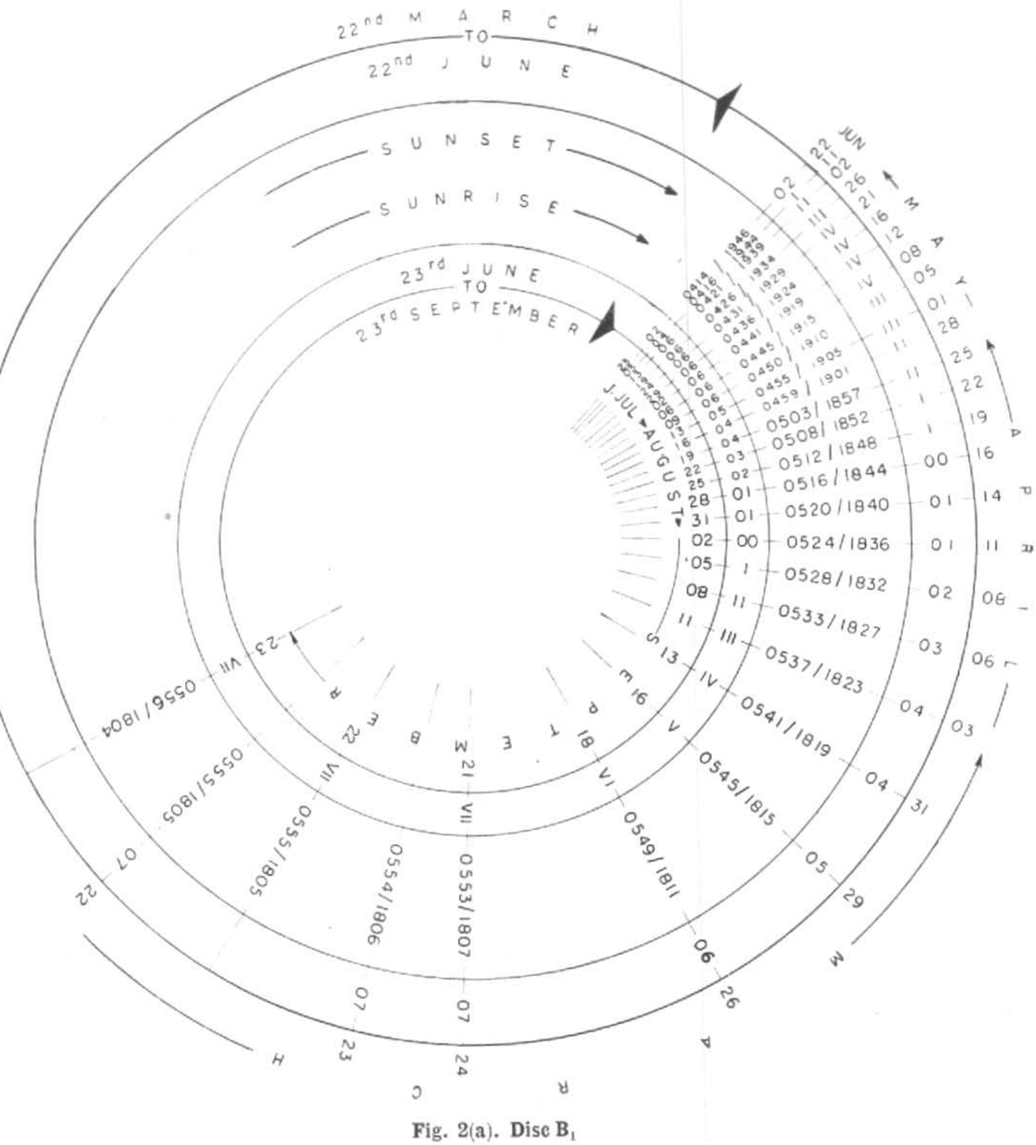

Fig. 2(a). Disc B_1

that matter) would appear to rise earlier and set later than at the ground. It would be useful for an aircraft pilot to have an idea of the variation. This is discussed below.

In Fig. 4 the place A is on the surface of the earth. C is a point at an altitude h above A. S_1 is the position of the sun on the horizon for A, say at the sun-set. The sun would appear to set at C when it is at S_2 where CS_2 is tangential to the earth at B, *i.e.*, the sun-set time for C would be that corresponding to the sun-set at B. The time difference T between the sun-set at A and that at B would depend on the difference in longitudes of A and B as well as the latitude difference between them. The arc AB is a portion of a great circle, and if α is the deviation of the

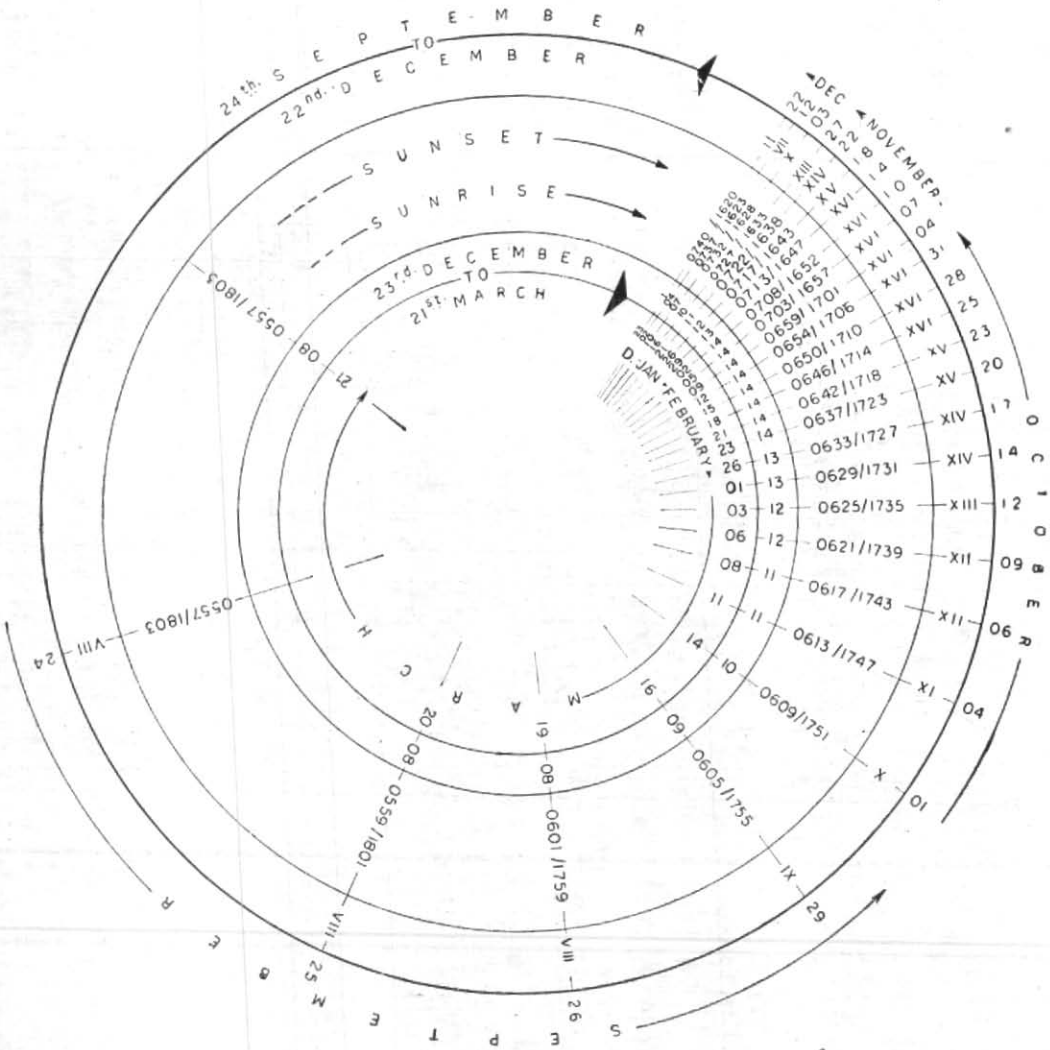

Fig. 2(b). Disc B_2

sun from the true west at sun-set (or from east at sun-rise), the difference in longitudes of A and B is AB $\cos\alpha \sec\phi$. If AB is in miles, the difference in longitudes is (AB $\cos\alpha \sec\phi$)/69 degrees (since $1°$ arc of earth's great circle measures 69 miles approximately). Hence the time difference between sun-set at A and that at B is $T = 4 \times (\text{AB} \cos\alpha \sec\phi / 69)$ minutes. It can be shown that when h is small in comparison to the radius of the earth and is measured in feet,

$$\text{AB (in miles)} = 1\cdot23\sqrt{h}$$

Hence, $T = (4/69) \times 1\cdot23\sqrt{h} \times \cos\alpha \sec\phi$

Also it can be shown that

$$\cos\alpha = \frac{[\cos(\phi-\delta)\cos(\phi+\delta)]^{\frac{1}{2}}}{\cos\phi}$$

Cos α decreases with increasing values of δ and ϕ. Within Indian latitudes, however, $\cos\alpha$ is never less than $\cdot87$. Hence $T = \cdot074\sqrt{h}$ (very nearly), taking the value for $25°$ latitude as an average value.

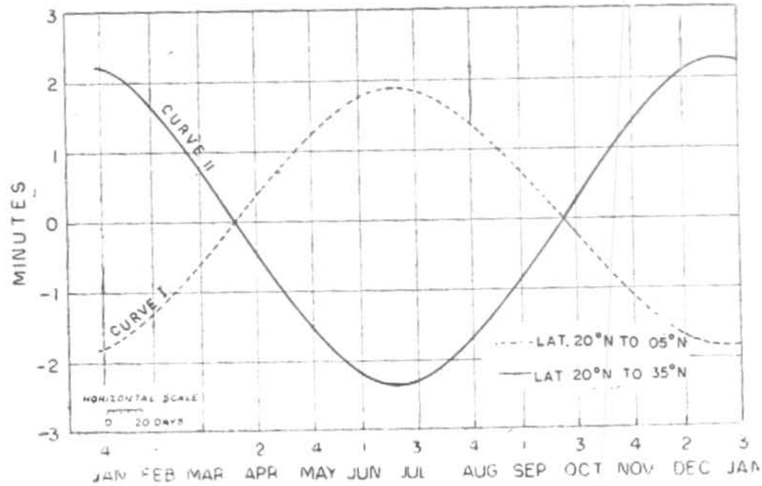

Fig. 3. Time correction per degree latitude

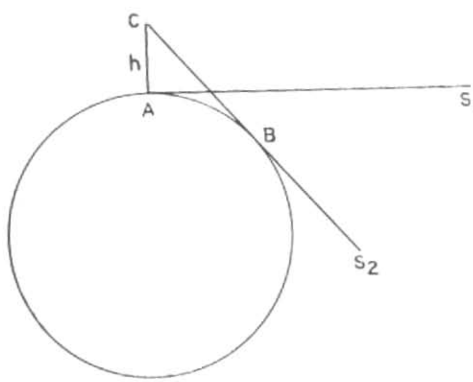

Fig. 4

It can be shown that the correction due to altitude difference is $c=|.018\sqrt{h}\times \sin \alpha\times l|$, where l is the variation of sun-rise/set per degree latitude, h and α having the same meaning as before. It can also be shown that only the numerical value of c need be considered and added to the numerical value of T to get the total time correction. It is easily seen that c has the maximum value when $|\delta|$ is large (*i.e.*, round about December and June) and increases with ϕ. It is of the order of only 2 minutes even for $\phi=35°$ (northern-most Indian latitude) and for as large values of h as 20,000 ft. Hence, c can be neglected for all values of α and reasonably high values of h. If the effect of atmospheric refraction in further bending the sun's rays from B to C is taken into account, then the co-efficient in the expression for T would be slightly increased and it will take the form $T=0{\cdot}08\sqrt{h}$ minutes, where h is the elevation in feet.

Thus for example, if $h=20{,}000$ ft, $T=11$ minutes approximately. The importance of this time difference to aircraft pilot is at once apparent when it is observed that if an aircraft flying at 20,000 ft over an aerodrome, observes the sun setting at that height, it can descend to the airfield only 11 minutes after the sun has set at the airfield excluding the time for descent. And if the time for descent is more than 15 minutes it can land only after the civil twilight is over — not a very happy situation if the lighting for the airfield has not been previously arranged.

REFERENCE

India met. Dep. 1958 *Indian Ephemeris and Nautical Almanac.*